Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS APPLICATIONS

Calculator-free

ATAR course examination 2019

Marking key

Marking keys are an explicit statement about what the examining panel expect of candidates when they respond to particular examination items. They help ensure a consistent interpretation of the criteria that guide the awarding of marks.

2019/0063004

Section One: Calculator-free **35% (55 Marks)**

Question 1 **(8 marks)**

(a) Why is the graph planar? (1 mark)

Solution
No two edges cross
Specific behaviours
✓ states correct reason

(b) Show that the graph satisfies Euler's formula. (2 marks)

Solution
$v = 3$, $e = 4$, $f = 3$. $3 + 3 - 4 = 2$, verified.
Specific behaviours
✓ gives correct values for the number of vertices, edges and faces ✓ correctly verifies Euler's formula

(c) Construct the adjacency matrix for the graph. (3 marks)

Solution
$\begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$
Specific behaviours
✓ gives correct entry for A to A ✓ gives the remainder of correct entries ✓ correctly labels rows and columns

A student wishes to carry out closed walks of length two from Building A.

(d) List all his possible walks. (2 marks)

Solution
A-B-A A-C-A A-A-A A total of 3 walks
Specific behaviours
✓ lists at least two walks ✓ lists all 3 walks

Question 2 **(9 marks)**

Katie is a hobby farmer who has been experimenting with a species of tomato plant growing under the same soil and climatic conditions. She varied the amount of water (W), in millimetres, used during each week and recorded the total number of tomatoes (T) produced by each plant. The scatterplot showing her results is drawn below.

(a) Identify the response variable. (1 mark)

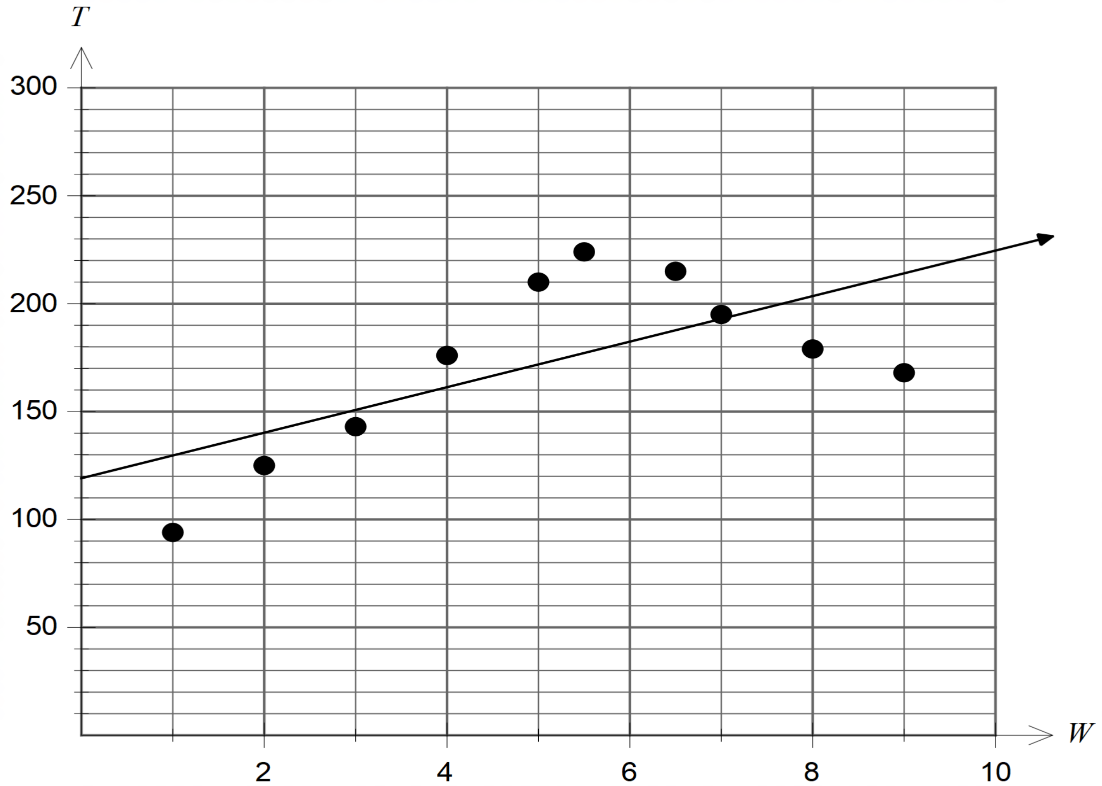
Solution
Number of tomatoes
Specific behaviours
✓ identifies correct variable

(b) Use the equation of the least-squares line to predict the total number of tomatoes produced when 10 millimetres of water are given to a plant during each week. (2 marks)

Solution
$T = 10.55 \times 10 + 119.11$ $T = 105.5 + 119.11$ $T = 224.61 \approx 224 / 225$
Specific behaviours
✓ correctly substitutes 10 into least-squares line ✓ rounds correctly to a whole number of tomatoes

Question 2 (continued)

(c) Fit the least-squares line to the scatterplot. (2 marks)

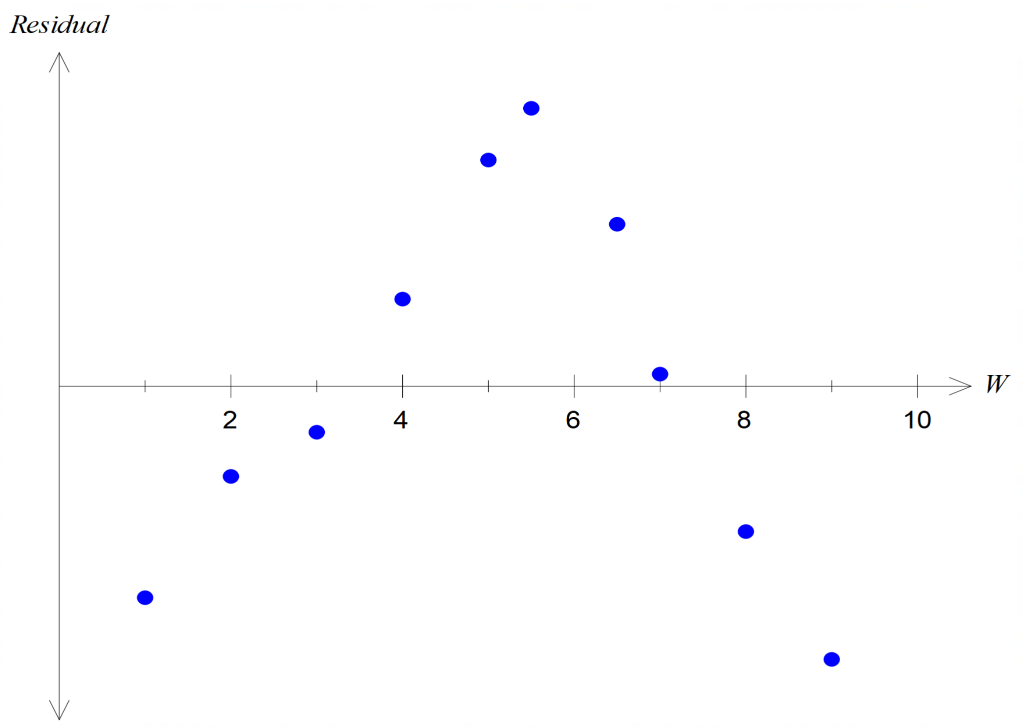
Solution


Specific behaviours
✓ plots one correct point ✓ plots a second correct point and draws least-squares line

Katie decided to draw a residual plot to gather more information about her results.

(d) (i) Sketch a residual plot she would have likely drawn for the given data.
Note: you do not have to calculate actual values. (2 marks)

Solution



Specific behaviours
✓ correctly shows shape of residual plot
✓ plots all points on correct side of horizontal axis

(ii) Use your residual plot to discuss the appropriateness of fitting a linear model to the data. (2 marks)

Solution
A linear model is not appropriate as a pattern is evident in the residual plot.
Specific behaviours
✓ states linear model is not appropriate
✓ states a valid reason

Question 3 **(10 marks)**

A company has four small workshops that each produce four different types of outdoor furniture. The annual cost of production of the furniture at each workshop is shown in the table below, with all values in thousands of dollars.

	Type 1 $'000	Type 2 $'000	Type 3 $'000	Type 4 $'000
Workshop A	25	43	50	39
Workshop B	33	31	56	39
Workshop C	28	47	59	38
Workshop D	36	32	56	41

The cost matrix is given by

$$\begin{bmatrix} 25 & 43 & 50 & 39 \\ 33 & 31 & 56 & 39 \\ 28 & 47 & 59 & 38 \\ 36 & 32 & 56 & 41 \end{bmatrix}$$

The company is interested in knowing what the minimum annual cost would be if each furniture type was allocated to its own individual workshop. The Hungarian Algorithm is to be used to determine the allocation and the minimum annual cost. The first step of the Hungarian Algorithm, where the smallest number in each row is subtracted from all other numbers in that row, is shown below.

(a) Continue the steps of the Hungarian Algorithm to determine the appropriate allocation of workshops to furniture type and state the **minimum** annual cost. (5 marks)

$$\begin{bmatrix} 0 & 18 & 25 & 14 \\ 2 & 0 & 25 & 8 \\ 0 & 19 & 31 & 10 \\ 4 & 0 & 24 & 9 \end{bmatrix}$$

Solution
$\begin{bmatrix} 0 & 18 & 1 & 6 \\ 2 & 0 & 1 & 0 \\ 0 & 19 & 7 & 2 \\ 4 & 0 & 0 & 1 \end{bmatrix}$ Subtracts smallest number in each column from each element Least amount of strike-throughs is 3, therefore need to add smallest element not crossed out (i.e. one) to the intersections and subtract the smallest element from the uncrossed elements to give: $\begin{bmatrix} 0 & 17 & 0 & 5 \\ 3 & 0 & 1 & 0 \\ 0 & 18 & 6 & 1 \\ 5 & 0 & 0 & 1 \end{bmatrix}$ Least amount of strike-throughs is now four Therefore, the allocation matrix is $\begin{bmatrix} 0 & 17 & \boxed{0} & 5 \\ 3 & 0 & 1 & \boxed{0} \\ \boxed{0} & 18 & 6 & 1 \\ 5 & \boxed{0} & 0 & 1 \end{bmatrix}$

Type	Type 1	Type 2	Type 3	Type 4
Workshop	**C**	**D**	**A**	**B**

Total minimum annual cost is $149 000

Specific behaviours
✓ subtracts smallest number in each column from other numbers in that column ✓ correctly makes adjustments to matrix ✓ correctly allocates workshops ✓ states the sum of the allocations correctly ✓ states minimum annual cost in thousands of dollars

Question 3 (continued)

The revenue matrix, in thousands of dollars, for the sale of the furniture produced annually at each workshop is given by

$$\begin{bmatrix} 37 & 61 & 60 & 53 \\ 45 & 52 & 73 & 50 \\ 38 & 65 & 75 & 55 \\ 44 & 54 & 76 & 45 \end{bmatrix}$$

(b) Given that *Profit* = *Revenue* – *Cost*, complete the Profit matrix below. (1 mark)

Solution
$\begin{bmatrix} 12 & 18 & 10 & 14 \\ 12 & 21 & 17 & 11 \\ 10 & 18 & \boxed{16} & \boxed{17} \\ \boxed{8} & \boxed{22} & \boxed{20} & \boxed{4} \end{bmatrix}$
Specific behaviours
✓ correctly completes all entries

(c) Use the Hungarian Algorithm to determine the appropriate allocation of workshops to furniture type that will produce the **maximum** annual profit. (4 marks)

Solution

$$\begin{bmatrix} 10 & 4 & 12 & 8 \\ 10 & 1 & 5 & 11 \\ 12 & 4 & 6 & 5 \\ 14 & 0 & 2 & 18 \end{bmatrix}$$ Subtracts each element in the profit matrix from 22

$$\begin{bmatrix} 6 & 0 & 8 & 4 \\ 9 & 0 & 4 & 10 \\ 8 & 0 & 2 & 1 \\ 14 & 0 & 2 & 18 \end{bmatrix}$$ Subtracts smallest number in each row from each element

$$\begin{bmatrix} \boxed{0} & 0 & 6 & 3 \\ 3 & \boxed{0} & 2 & 9 \\ 2 & 0 & 0 & \boxed{0} \\ 8 & 0 & \boxed{0} & 17 \end{bmatrix}$$ Subtracts smallest number in each column from each element

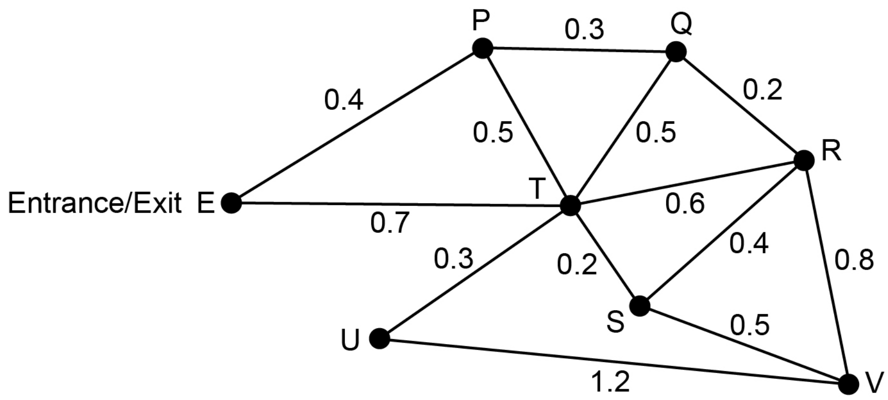
Type	Type 1	Type 2	Type 3	Type 4
Workshop	**A**	**B**	**D**	**C**

Specific behaviours

✓ correctly obtains first matrix
✓ correctly obtains second matrix
✓ correctly obtains third matrix
✓ correctly allocates workshops

Question 4 **(11 marks)**

A marine park has attractions with paths connecting them. The vertices on the graph represent the attractions and the numbers on the edges represent the path distances (km) between the attractions. Visitors can either walk around the park or take one of the many shuttle buses that run between attractions.



The manager of the marine park leaves his office, which is located at the entrance/exit (E) and walks to attraction V.

(a) (i) Determine the shortest distance from E to V. (1 mark)

Solution
1.4 km
Specific behaviours
✓ correctly determines shortest distance

(ii) If the manager needs to pick up some tools left at U on the way, determine the route he should take and the corresponding distance, given he wants to take the shortest route from E to V. (2 marks)

Solution
ETUTSV Total = 2 km
Specific behaviours
✓ correctly states shortest route ✓ correctly determines shortest distance

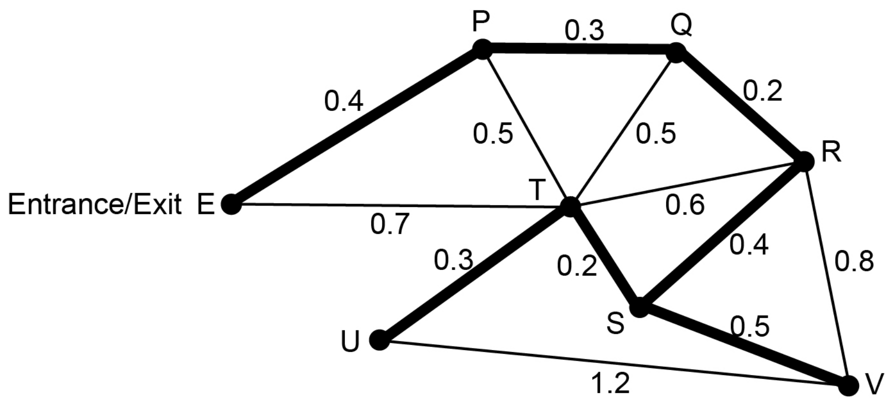
Rachel arrives at the entrance. She wants to complete a Hamiltonian cycle.

(b) State the route she should take. (2 marks)

Solution
EPQRSVUTE
Specific behaviours
✓ states a path containing all vertices ✓ states the correct Hamiltonian cycle (starting and finishing at the entrance/exit)

(c) (i) Use Prim's algorithm, or otherwise, to determine the minimum total length of pipelines. Highlight the required pipelines on the diagram below. (4 marks)

Solution
EP(0.4), PQ(0.3), QR(0.2), RS(0.4), VS(0.5), ST(0.2), UT(0.3) The minimum length is 2.3 km.
Specific behaviours
✓ gives at least 4 correct connections ✓ gives all correct connections ✓ correctly states minimum length

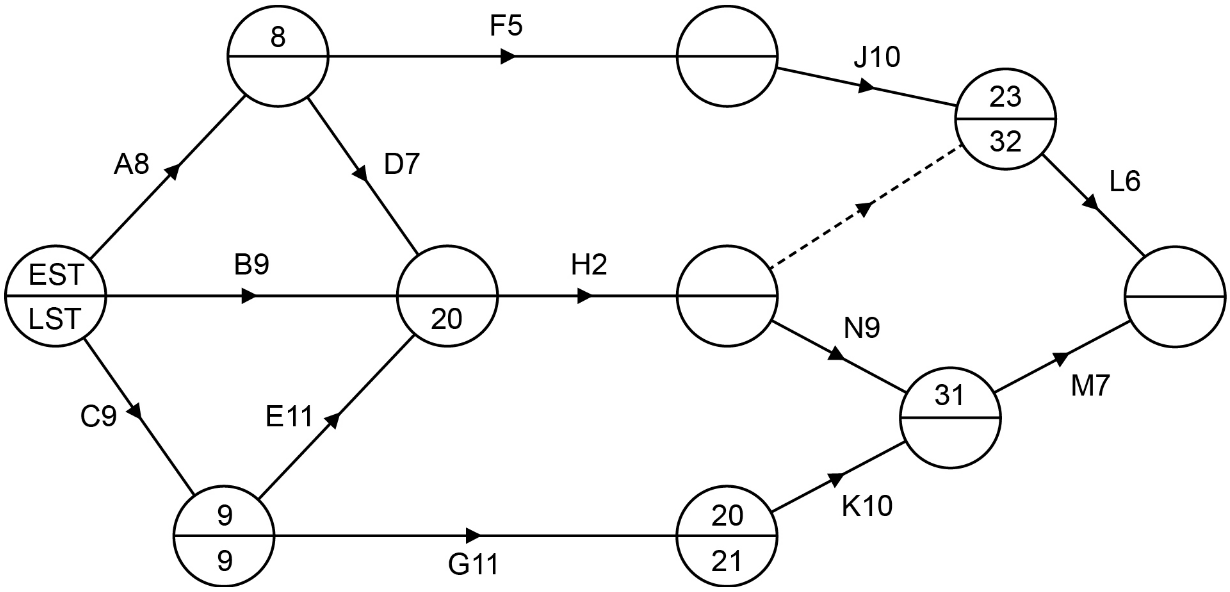
Solution

Specific behaviours
✓ highlights correct spanning tree

(ii) The manager has been told that a pipeline of length 0.2 km could be laid from S to U. How, if at all, will this affect the total length of pipelines that should be laid in order to maintain a minimum length? (2 marks)

Solution
The minimum length will decrease by 0.1 km (as SU would be used instead of TU).
Specific behaviours
✓ states it is a decrease ✓ gives the decrease as 0.1 km

Question 5 **(11 marks)**

The network below represents a construction project. The number on each edge gives the time, in hours, to complete the activity. Each activity requires one worker.



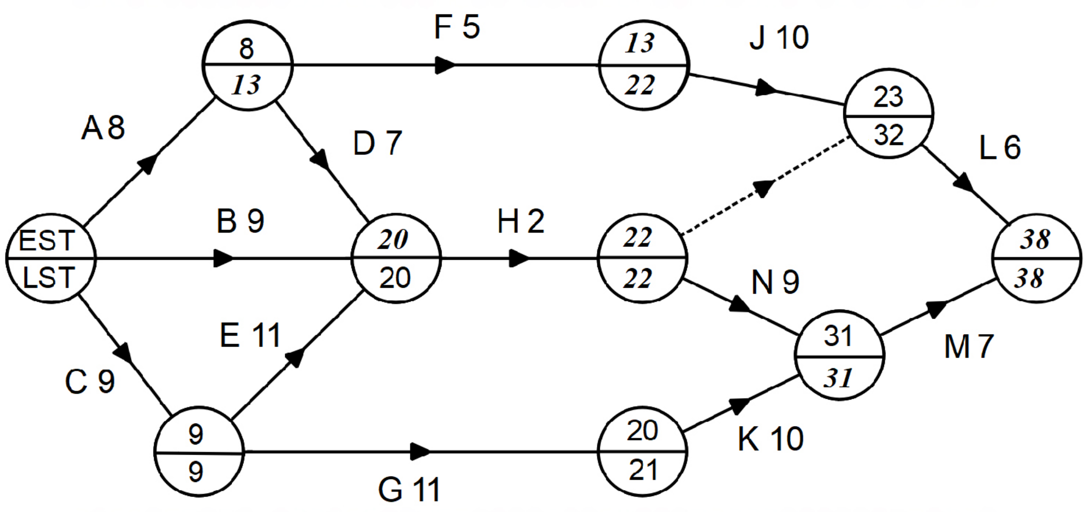
(a) Complete the precedence table below. (2 marks)

Activity	A	B	C	D	E	F	G	H	J	K	L	M	N
Time (hours)	8	9	9	7	11	5	11	2	10	10	6	7	9
Immediate predecessor	-	-	-	A	C	A	C	B,D,E	**F**	**G**	**H,J**	**N,K**	**H**

Solution
See table above
Specific behaviours
✓ correctly allocates predecessors for activity L ✓ correctly allocates all predecessors

(b) Complete the network showing the earliest starting time (EST) and latest starting time (LST) for each node. (Note: the first node indicates which is the EST and the LST.)

(2 marks)

Solution

Specific behaviours
✓ completes at least 4 nodes correctly ✓ completes all nodes correctly

(c) Determine the critical path and the minimum completion time for the project. (2 marks)

Solution
Critical path is CEHNM. Minimum completion time is 38 hours
Specific behaviours
✓ states correct path ✓ states correct time

(d) Calculate the float times for Activities D and F. (2 marks)

Solution
Float time for D is 5 hours Float time for F is 9 hours
Specific behaviours
✓ gives correct float for D ✓ gives correct float for F

Question 5 (continued)

(e) Given that the sum of all the times of the activities is 104 hours, calculate the minimum number of workers required to complete the project in the minimum completion time. (1 mark)

Solution
three
Specific behaviours
✓ states correct number of workers required

(f) What is the latest time into the project that Activity F could start without affecting the minimum completion time? (1 mark)

Solution
Seventeen hours
Specific behaviours
✓ states correct latest time

(g) Explain the purpose of the dotted line on the network. (1 mark)

Solution
Activity L depends on activities H and J
Specific behaviours
✓ states correct purpose

Question 6 **(6 marks)**

The population of turtles in an artificial lake at a wildlife sanctuary is initially 32 and research has shown a natural decrease in population of 50% each year. Twenty extra turtles are introduced to the lake at the end of each year.

(a) Determine a recursive rule for the turtle population. (2 marks)

Solution
$T_{n+1} = 0.5T_n + 20, T_1 = 32$
Specific behaviours
✓ states correct rule ✓ states correct first term

(b) Determine the long-term steady state of the turtle population. (2 marks)

Solution 1
$T_2 = 0.5(32) + 20$ $= 36$ $T_3 = 0.5(36) + 20$ $= 38$ $T_4 = 0.5(38) + 20$ $= 39$ $T_5 = 0.5(39) + 20$ 39.5 Approaching a steady state of 40 turtles
Specific behaviours
✓ correctly calculates a further 4 terms ✓ correctly determines the long-term steady state

OR

Solution 2
$x = 0.5x + 20$ $0.5x = 20$ $x = 40$
Specific behaviours
✓ correctly writes steady state equation ✓ correctly determines the long-term steady state

Question 6 (continued)

(c) If the wildlife sanctuary preferred a long-term steady state of 80 turtles, what yearly addition of turtles would be required to produce this steady state? Assume all other conditions remain the same. (2 marks)

Solution 1
By trial and error. Example: if k = 30, $T_2 = 46, T_3 = 53, T_4 = 56.5$, not approaching 80 Example: if k = 40, $T_2 = 56, T_3 = 68, T_4 = 74, T_5 = 77$, approaching 80
Specific behaviours
✓ uses trial and error to correctly generate a sequence of at least 4 terms ✓ correctly determines the yearly addition of turtles

OR

Solution 2
$80 = 0.5 \times 80 + k$ $80 = 40 + k$ $k = 40$
Specific behaviours
✓ correctly writes steady state equation ✓ correctly determines the yearly addition of turtles

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