Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS APPLICATIONS

Calculator-assumed

ATAR course examination 2020

Marking key

Marking keys are an explicit statement about what the examining panel expect of candidates when they respond to particular examination items. They help ensure a consistent interpretation of the criteria that guide the awarding of marks.

2020/66971

Section Two: Calculator-assumed **65% (105 Marks)**

Question 7 **(6 marks)**

The world's tallest man was recorded as 60 cm long at birth. He grew 28 cm in his first year, 26 cm in his second year and so on, always 2 cm less than in the previous year until he stopped growing.

(a) Calculate his annual growth (in cm) in his fourth and fifth years. (1 mark)

Solution
Fourth year: 22 cm Fifth year: 20 cm
Specific behaviours
✓ calculates correct values

(b) Deduce the rule for his annual growth in the n^{th} year, until he stopped growing. (2 marks)

Solution
$T_n = 30 - 2n$
Specific behaviours
✓ determines the correct constant term ✓ determines the correct coefficient of n

(c) In which year did he first not grow any taller? (1 mark)

Solution
In the fifteenth year
Specific behaviours
✓ calculates correct value

(d) Calculate his maximum height. (2 marks)

Solution
60 + (28 + 26 + 24 +…+ 2) = 270 cm
Specific behaviours
✓ correctly sums terms from 28 to 2 ✓ correctly adds 60 to the sum

Question 8 **(9 marks)**

A farmer has a large lake on his farm and has started stocking it with fish of a variety that will flourish in the conditions in this lake. Monitoring has shown that the number of adult fish is increasing at a consistent rate of 9% per month and at the beginning of 2020 the lake holds 660 of the adult fish.

(a) Write a recursive rule to give the number of adult fish in the lake at the end of each month from the beginning of 2020. (2 marks)

Solution
$T_{n+1} = 1.09T_n \qquad T_0 = 660$
Specific behaviours
✓ correctly states recursive rule ✓ correctly states T_0

(b) Deduce a rule for the n^{th} term of this sequence. (2 marks)

Solution
$A_n = 660 \times 1.09^n$
Specific behaviours
✓ gives formula in exponential form ✓ states correct rule

The farmer plans to allow the general public to pay to fish in the lake. This will commence at the beginning of the next month after the adult fish population first reaches 4000.

(c) Determine how many months after the beginning of 2020 fishing will commence. (2 marks)

Solution
$4000 = 660 \times 1.09^n \quad n = 20.9$ Therefore fishing will commence 21 months after the beginning of 2020 **or** $T_{21} = 4031.8$ Therefore fishing will commence 21 months after the beginning of 2020
Specific behaviours
✓ correctly solves for n ✓ correctly states correct number of months after the beginning of 2020

Question 8 (continued)

(d) The farmer wishes to maintain a steady state in the adult fish population once fishing commences. Calculate how many adult fish can be taken from the lake each month.

(3 marks)

Solution
$T_{21} = 4031$ $T_{n+1} = 1.09T_n - x$ $4031 = 1.09 \times 4031 - x \qquad x = 362.79$ 362 fish per month
Specific behaviours
✓ correctly determines T_{21} = 4031 ✓ correctly determines monthly increase ✓ correctly rounds down for number of fish

Question 9 **(11 marks)**

Giuseppe wishes to set up an annuity. He is told that an annuity with quarterly investment returns and quarterly payments is modelled by the recursive rule:

$A_{n+1} = A_n \times 1.019 - P, A_0 = Q$ with the values of P and Q consistent with the spreadsheet below.

Quarter	Opening balance	Investment gain	Payment	Closing balance
1	\$648 000	\$12 312	\$15 000	X
2		Y	\$15 000	
3				

(a) Determine the values of P, Q, X and Y and write them in the table below. (4 marks)

P	Q	X	Y
\$15 000	\$648 000	\$645 312	\$12 260.93

Solution
See table above
Specific behaviours
✓ correctly determines the value of P ✓ correctly determines the value of Q ✓ correctly determines the value of X ✓ correctly determines the value of Y

(b) What is the annual compound interest rate for this investment? (1 mark)

Solution
1.9 × 4 = 7.6%
Specific behaviours
✓ correctly calculates the annual interest rate

When the balance in the annuity first falls below \$300 000, Giuseppe converts the payment to a perpetuity so that his children are left with some inherited benefits. The interest rate remains the same as that calculated in part (b).

(c) Determine the number of years the annuity operates before the perpetuity starts. (2 marks)

Solution
Using the recursive rule, n = 66 and the balance is \$299 501.07, so it would last 66 quarters, i.e. 16.5 years.
Specific behaviours
✓ correctly determines the value for n = 66 ✓ correctly states the number of years

Question 9 (continued)

(d) What are the quarterly payments under this perpetuity? (2 marks)

Solution
I = 7.6, PV = –299 501.07, FV = 299 501.07, P/Y = C/Y = 4 which gives PMT = 5690.52 **or** 0.019 × 299 501.07 = 5690.52 Quarterly payments will be $5690.52
Specific behaviours
✓ uses correct values for I, PV, PMT, P/Y, C/Y ✓ correctly calculates the quarterly payments

(e) Giuseppe believes that his investment returns are at an effective interest rate of 7.93% p.a. Use a clear calculation to comment on the accuracy of this belief. (2 marks)

Solution
$i_{effective} = \left(1 + \frac{0.076}{4}\right)^4 = 1.0782$ i.e. effective interest rate is 7.82%, therefore his belief is not true
Specific behaviours
✓ correctly calculates the effective interest rate ✓ correctly concludes that the belief is not true

Question 10 **(15 marks)**

A football club records body measurements for all of their players. Shown below are the waistline measurements (cm) and percentage body fat for eleven players.

Player	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Waistline measurement (w)	89	100	87	96	94	83	81	83	84	97	98
Percentage body fat (p)	14	17	11	19	17	12	9	10	8	14	19

Research has shown that estimates for percentage body fat can be determined by using waistline measurements.

(a) Calculate the correlation coefficient r_{wp} for these data. (1 mark)

Solution
$r = 0.88$
Specific behaviours
✓ correct answer

(b) Determine the equation of the least-squares line for these data. (1 mark)

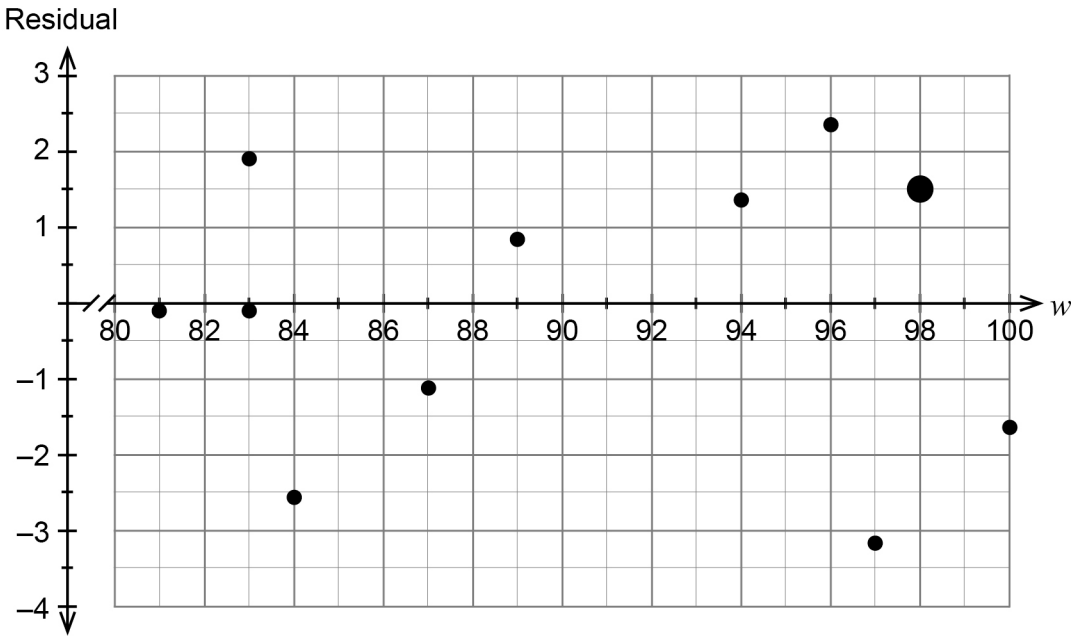
Solution
$p = 0.5w - 31.4$
Specific behaviours
✓ correct answer

(c) In the context of this question, interpret the slope of the line found in part (b). (2 marks)

Solution
For each 1 cm increase in waistline measurement, the percentage body fat increases by 0.5.
Specific behaviours
✓ matches waistline measurement to percentage body fat ✓ average 0.5 increase in percentage body fat for each 1 cm increase in waistline measurement

Question 10 (continued)

(d) The residual plot shown below is for the first 10 players' data. Calculate the residual for player number 11 and plot this point on the graph. (2 marks)

Solution
 Residual = 1.5
Specific behaviours
✓ correct residual ✓ correctly plotted

(e) Comment on the appropriateness of fitting a linear model to the data. Justify your answer. (2 marks)

Solution
Appropriate as there is no clear pattern in the residuals
Specific behaviours
✓ correctly states that fitting a linear model is appropriate ✓ correctly states there is no clear pattern in residuals

(f) What percentage of the variation in the percentage body fat measurements is **unexplained** by the variation in the waistline measurements? (2 marks)

Solution
$r^2 = d = 0.77$ $\therefore$ 23% unexplained
Specific behaviours
✓ correctly determines coefficient of determination ✓ correctly gives unexplained variation percentage

(g) Wayne is player number 12 and has a waistline measurement of 105 cm.

(i) Determine his predicted percentage of body fat. (1 mark)

Solution
$p = 0.5 \times 105 - 31.4 = 21$
Specific behaviours
✓ correct answer

(ii) Comment on the validity of the prediction and give a justification for your answer. (2 marks)

Solution
Not valid, extrapolation
Specific behaviours
✓ correctly states prediction is not valid ✓ correctly states it is due to extrapolation

(h) Player number 13 has a residual of –2.6. What information does this provide about the percentage body fat for this player? (2 marks)

Solution
The percentage body fat for player 13 is 2.6 below their predicted percentage body fat.
Specific behaviours
✓correctly states that percentage body fat is below/under … ✓correctly relates actual percentage body fat measurement to the predicted percentage body fat (from the least-squares line)

Question 11 (9 marks)

Shari requires a loan of $325 000 for the purchase of a new house. She wishes to make two equally-spaced repayments of $700 each month.

Shari is offered a choice of two loan options for the first three years, both of which have interest calculated daily.

- Option 1 An introductory compound interest rate of 2.55% per annum for the first year which changes to 2.99% per annum for the next two years.
- Option 2 A compound interest rate of 2.85% per annum fixed for the first three years.

(a) Describe briefly the benefit of making two repayments of $700 each month instead of one repayment of $1400 at the end of each month. (1 mark)

Solution
Loan amount reduces at a quicker rate or less interest is paid
Specific behaviours
✓ correct answer

(b) For Option 1, calculate

(i) the loan balance at the end of the first year. (3 marks)

Solution
N = 24, I = 2.55, PV = 325 000, PMT = –700, P/Y = 24, C/Y = 365 Loan balance = $316 386.79
Specific behaviours
✓ uses at least 4 correct values for N, I, PV, PMT, P/Y, C/Y ✓ uses all correct values for N, I, PV, PMT, P/Y, C/Y ✓ correctly determines balance

(ii) the loan balance at the end of the third year. (2 marks)

Solution
N = 48, I = 2.99, PV = 316 386.79, PMT = –700, P/Y = 24, C/Y = 365 Loan balance = $301 279.69
Specific behaviours
✓ uses correct values for N, I, PV ✓ correctly determines balance

(c) Determine which option gives the best result for Shari after three years and by how much. (3 marks)

Solution
N = 72, I = 2.85, PV = 325 000, PMT = –700, P/Y = 24, C/Y = 365 Loan balance = $301 422.81 Option 1 best by 301 422.81 – 301 279.69 = $143.12
Specific behaviours
✓ uses all correct values for N, I, PV, PMT, P/Y, C/Y ✓ correctly determines balance for option 2 ✓ correctly determines Option 1 is best by $143.12

Question 12 **(10 marks)**

Jessica wants to borrow $15 000 from her parents to purchase a car. They will be charging her compound interest at the rate of 4% per annum, with interest added yearly.

(a) Jessica is currently studying so she will not want to be making any regular repayments.

(i) Complete the table below to show the amount she will owe her parents at the end of each year. (2 marks)

Solution				
Number of years (n)	0	1	2	3
Amount owing ($)	15 000	**15 600**	**16 224**	**16 872.96**
Specific behaviours				
✓ correctly determines amount owing after 1 year ✓ correctly completes the table				

(ii) Write a recursive rule to calculate the amount owing at the end of each year. (2 marks)

Solution
$A_{n+1} = 1.04A_n, \quad A_0 = 15\,000$
Specific behaviours
✓ states recursive part of rule ✓ states starting value

Jessica's parents are encouraging her to get a part-time job so that she can make repayments along the way. Jessica estimates that she will be able to earn enough money to pay off $2400 each year.

(b) If interest is charged yearly and she repays the $2400 at the end of each year, write a recursive rule to calculate the amount owing at the end of each year. (1 mark)

Solution
$T_{n+1} = 1.04T_n - 2400, T_0 = 15\,000$
Specific behaviours
✓ states the correct recursive rule

(c) If interest is charged monthly and she makes equal monthly repayments,

(i) write a recursive rule to calculate the amount owing at the end of each month. (1 mark)

Solution
$B_{n+1} = 1.0033B_n - 200, B_0 = 15\,000$
Specific behaviours
✓ states the correct recursive rule

(ii) calculate how many months it will take to repay the loan. (1 mark)

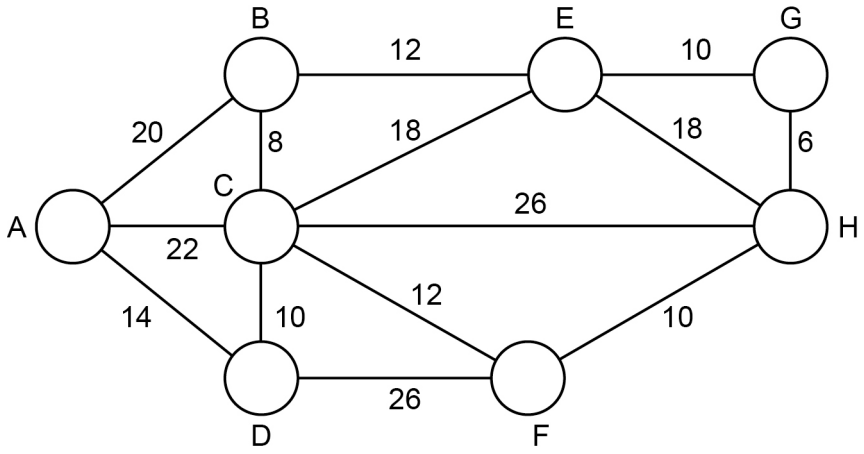
Solution
87 months
Specific behaviours
✓ states the correct number of months

(iii) calculate the total amount Jessica would pay over the duration of the loan. (3 marks)

Solution
Amount owing at end of 86th month = \$89.46 Interest for 87th month = $89.46 \times \frac{0.04}{12} = 0.30$ Total paid = (86 $\times$ 200) + (89.46 + 0.30) = \$17 289.76
Specific behaviours
✓ correctly determines amount owing at end of 86th month ✓ correctly determines interest for 87th month ✓ correctly states total amount paid

Question 13 **(9 marks)**

The graph below represents a road transport network from a warehouse at A to seven retail outlets B, C, D, E, F, G and H. The number on each edge represents the distance, in kilometres, along each road.

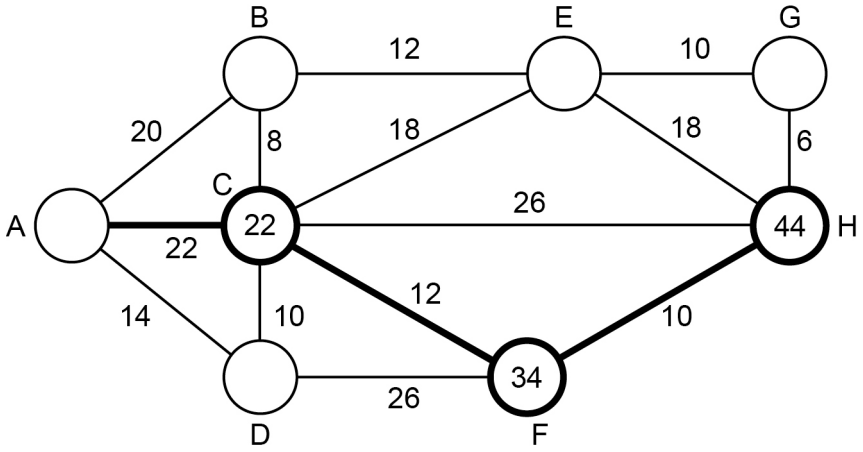


(a) Identify the shortest Hamiltonian path from the warehouse and state its length. (3 marks)

Solution
ADCBEGHF 14 + 10 + 8 + 12 + 10 + 6 + 10 = 70 km
Specific behaviours
✓ gives a Hamiltonian path to all vertices ✓ determines correct Hamiltonian path ✓ states correct length of path

A special delivery must be made from the warehouse to retail outlet H.

(b) Determine the shortest path and the distance travelled for this delivery. Working **must** appear on the network to show an appropriate method has been used. (3 marks)

Solution
 Shortest path is ACFH Distance = 44 km
Specific behaviours
✓ shows correct working on the network ✓ states correct path ✓ states correct distance

Road CH presently goes around what is now a dry salt lake. It is proposed that a direct road be constructed that will reduce the distance between retail outlets C and H.

(c) By how much can the direct road between C and H be reduced, so that the shortest path from the warehouse to H includes the direct road CH? (3 marks)

Solution
$22 + (26 - x) < 44 \quad \Rightarrow x > 4$ $also \quad x < 26$ $4 < x < 26$
Specific behaviours
✓ determines that the path ACH is less than 44 ✓ correctly states that direct road needs to be reduced by more than 4 ✓ correctly states that direct road needs to be reduced by less than 26

Question 14 **(14 marks)**

The table below shows the number of sprinkler systems installed by a local reticulation business over the past four years.

Year	Season	n	Number of systems	Seasonal mean	Number of systems as a percentage of the seasonal mean	Seasonally adjusted figures
2017	Summer	1	**A**	14	71.4	10.4
	Autumn	2	18		**B**	15.7
	Winter	3	11		78.6	14.7
	Spring	4	17		121.4	14.7
2018	Summer	5	15	**C**	105.3	15.7
	Autumn	6	16		112.3	14.0
	Winter	7	11		77.2	14.7
	Spring	8	15		105.3	13.0
2019	Summer	9	13	11.75	110.6	13.6
	Autumn	10	12		102.1	10.5
	Winter	11	8		68.1	10.7
	Spring	12	14		119.1	12.1
2020	Summer	13	16	—	—	—
	Autumn	14	15		—	—

(a) Calculate the value of **A**, **B** and **C**. (3 marks)

Solution
$\frac{A+18+11+17}{4}=14 \Rightarrow A=10$ $B=\frac{18}{14}\times 100=128.6$ $C=\frac{15+16+11+15}{4}=14.25$
Specific behaviours
✓ correctly calculates the value of A ✓ correctly calculates the value of B ✓ correctly calculates the value of C

(b) Complete the table showing the seasonal index for each season. (2 marks)

Season	Summer	Autumn	Winter	Spring
Seasonal index	95.8	114.3	**74.6**	**115.3**

Solution
See table above
Specific behaviours
✓ correctly calculates value for winter ✓ correctly calculates value for spring

(c) Show how the seasonally adjusted figure of 13.6 for Summer 2019 was calculated. (2 marks)

Solution
$\frac{13}{0.958} = 13.57 \approx 13.6$
Specific behaviours
✓ correctly uses division ✓ uses correct values

(d) During which season could more employees be given annual holidays with least disruption to sprinkler installations? Use mathematical evidence to support your answer (2 marks)

Solution
Winter, as the seasonal index (74.6) is the lowest
Specific behaviours
✓ chooses correct season ✓ gives valid reason

(e) Determine the least-squares line using the seasonally adjusted figures. (1 mark)

Solution
$y = -0.24n + 14.88$
Specific behaviours
✓ states correct equation

(f) Using your line from part (e), estimate the number of sprinkler systems that will be installed in Summer 2021. (2 marks)

Solution
$y(17) = 10.8$ Therefore, predicted value is $10.8 \times 0.958 = 10.35$ i.e. 10 sprinkler systems.
Specific behaviours
✓ calculates the value of y when $n = 17$ ✓ correctly multiplies by seasonal index and rounds appropriately

(g) Comment on the long-term prospects of the business. (2 marks)

Solution
The number of systems installed is declining since the gradient of the least-squares line is negative.
Specific behaviours
✓ correctly mentions declining number of sprinkler systems installed ✓ correctly refers to gradient of least-squares line

Question 15 **(8 marks)**

The graph below shows a network of sewage pipes. The numbers on the edges indicate the number of litres per minute that can flow along each pipe.

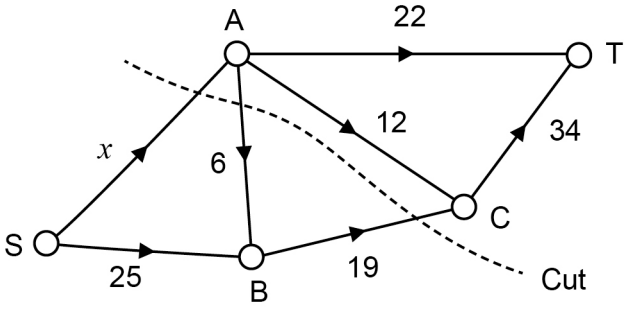


Diagram 1

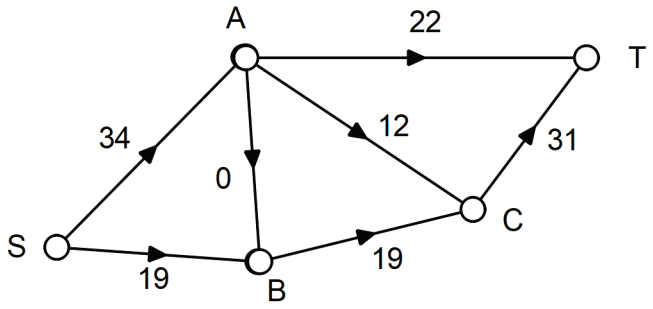
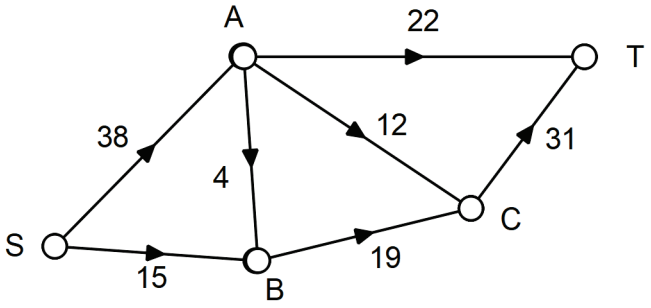
(a) Show that the value of x is 38, given that the value of the cut is 57. (2 marks)

Solution
$x + 19 = 57$ $x = 38$
Specific behaviours
✓ interprets the cut correctly to establish an equation ✓ shows that $x = 38$

(b) Calculate the value of the maximum flow through the network. (2 marks)

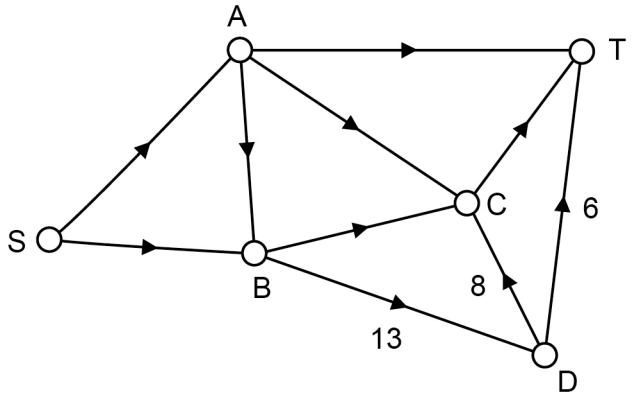
Solution
Listing: **1.** SBCT-19, SAT- 22, SACT-12 gives a total of 53 L per minute **2.** SACT-12, SAT- 22, SABCT- 4, SBCT-15 gives a total of 53 L per minute **or** minimum cut through AT, AC and BC = 53
Specific behaviours
✓ lists at least 2 correct paths **or** shows the minimum cut ✓ states correct maximum flow

(c) Indicate on **Diagram 2** below a possible flow along each pipe corresponding to the maximum flow calculated in part (b). (2 marks)

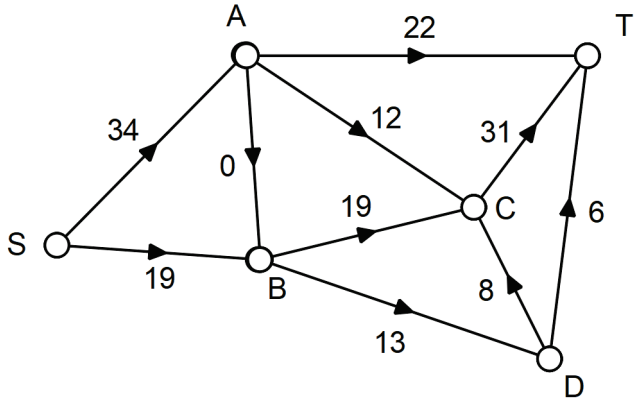
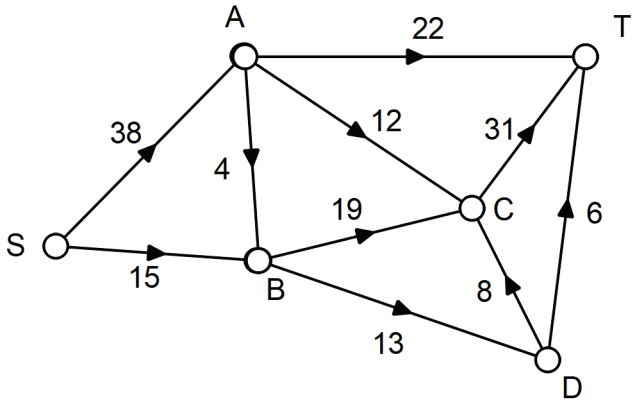
Solution
Using Listing **1**:    Using Listing **2:** 
Specific behaviours
✓ indicates at least 4 correct capacities ✓ indicates all correct capacities

Question 15 (continued)

Extra pipes, BD, DC and DT, are added to form a new system shown below. The capacities of the new pipes are indicated on the diagram. The original pipes have the same capacity as shown in **Diagram 1**.



(d) How will the addition of these pipes affect the new maximum flow through the system?
(2 marks)

Solution
Using Listing **1**: SBDT = 6 SABDCT = 3 Therefore, max flow = 53 + 9 = 62 New max flow = 62 L per min This is an increase of 9 L per min  Using Listing **2:** SBDT = 6 SBDCT = 3 Therefore, max flow = 53 + 9 = 62 New max flow = 62 L per min This is an increase of 9 L per min **or** Min cut through AT, CT and DT gives 62 as before This is an increase of 9 L per min 
Specific behaviours
✓ correctly states the flow will increase ✓ correctly calculates increase of 9 L per min (from 53 to 62)

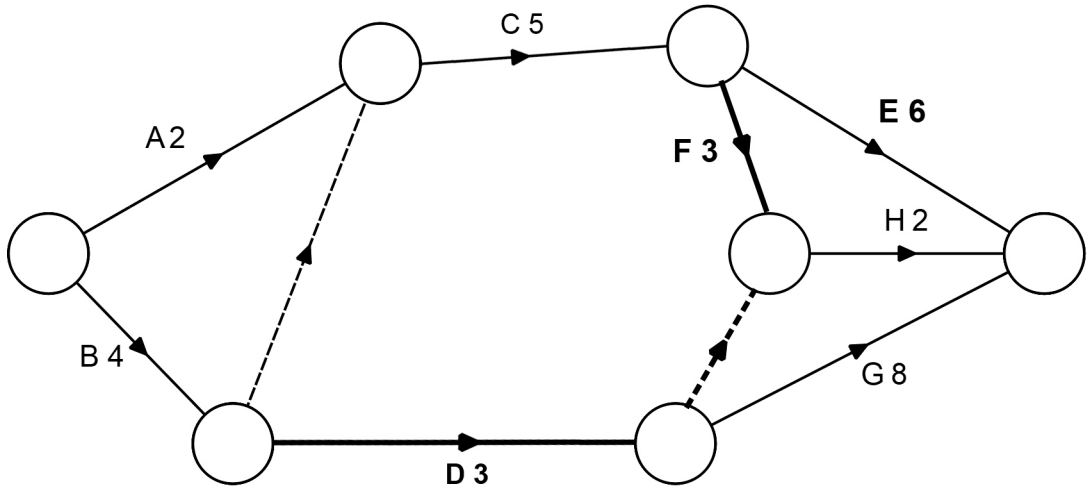
Question 16 **(14 marks)**

Valeska and her sister Katrin are planning a small building project. The table below shows the required activities, together with the times taken (in days) and the immediate predecessors for each activity.

Activity	A	B	C	D	E	F	G	H
Time (days)	2	4	5	3	6	3	8	2
Immediate predecessors	–	–	A, B	B	C	C	D	D, F

(a) Complete the project network below, showing all activities and durations. (3 marks)

Solution



Specific behaviours

✓ draws diagram with D following B as well as F following C
✓ contains a dotted line linking D with H
✓ network is labelled with tasks, durations and arrows

(b) Determine all critical activities and the minimum completion time for the project. (3 marks)

Solution

There are two critical paths: BCE and BDG, so the critical activities are B, C, E, D and G.
Minimum completion time = 15 days

Specific behaviours

✓ states at least 3 critical activities
✓ states all critical activities
✓ states correct minimum completion time

Question 16 (continued)

(c) Calculate the float times for each of the non-critical activities. (3 marks)

Solution
A has a float time of 2 days F has a float time of 1 day H has a float time of 1 day
Specific behaviours
✓ correctly states float time for A ✓ correctly states float time for F ✓ correctly states float time for H

(d) If Activity H is delayed by three days, what effect will this have on the minimum completion time and the critical activities? (2 marks)

Solution
H has a float time of 1 day so a delay of 3 days would increase the minimum completion time to 17 days. The critical activities would now be B, C, F and H (i.e. the new critical path is BCFH)
Specific behaviours
✓ states correct increase of 2 days ✓ states new critical activities

(e) Extra resources become available that can be used to shorten the duration of **one** of Activities B, E or F (on the original network) by one day. Which of these activities should be shortened and why? (3 marks)

Solution
Shorten B, which will reduce the minimum completion time by 1 day, as it is on both critical paths. Shortening F will have no effect since it is not a critical activity. Shortening E would only be effective if D or G were reduced as well, as E is only on one critical path.
Specific behaviours
✓ states correct activity to shorten ✓ states that B is on both critical paths ✓ provides a valid reason for not shortening E and F

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