2023/74295

Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS APPLICATIONS

Calculator-assumed

ATAR course examination 2023

Marking key

Marking keys are an explicit statement about what the examining panel expect of candidates when they respond to particular examination items. They help ensure a consistent interpretation of the criteria that guide the awarding of marks.

Section Two: Calculator-assumed **65% (97 Marks)**

Question 7 **(9 marks)**

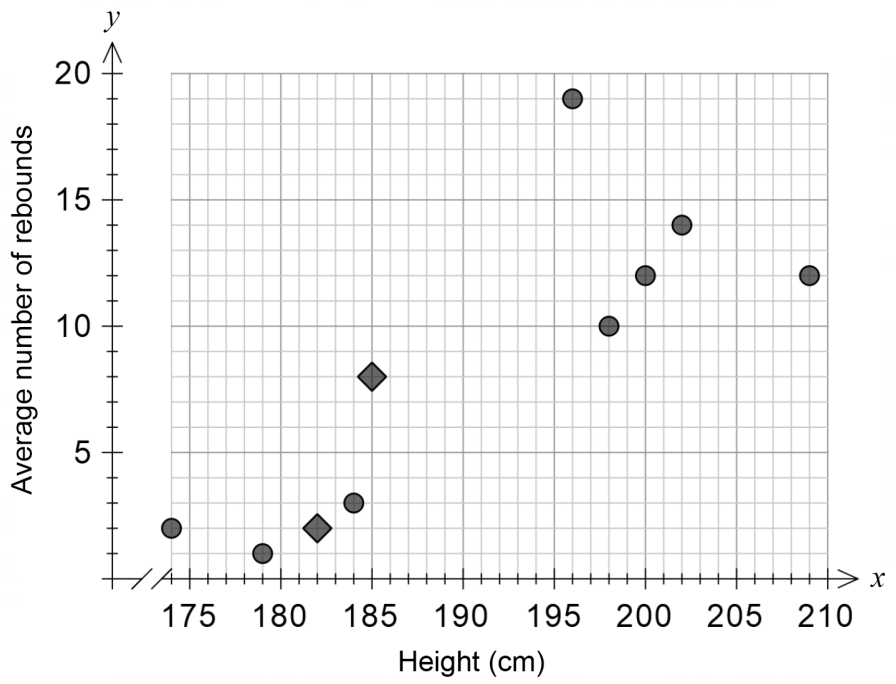
The heights of players and the average number of rebounds per game were recorded for a Basketball team over the course of a 30 game season. The data collected is shown in the table below.

Player	Height (x) (cm)	Average number of rebounds (y)
1	198	10
2	174	2
3	200	12
4	182	2
5	184	3
6	209	12
7	196	19
8	185	8
9	202	14
10	179	1

(a) Identify the explanatory variable. (1 mark)

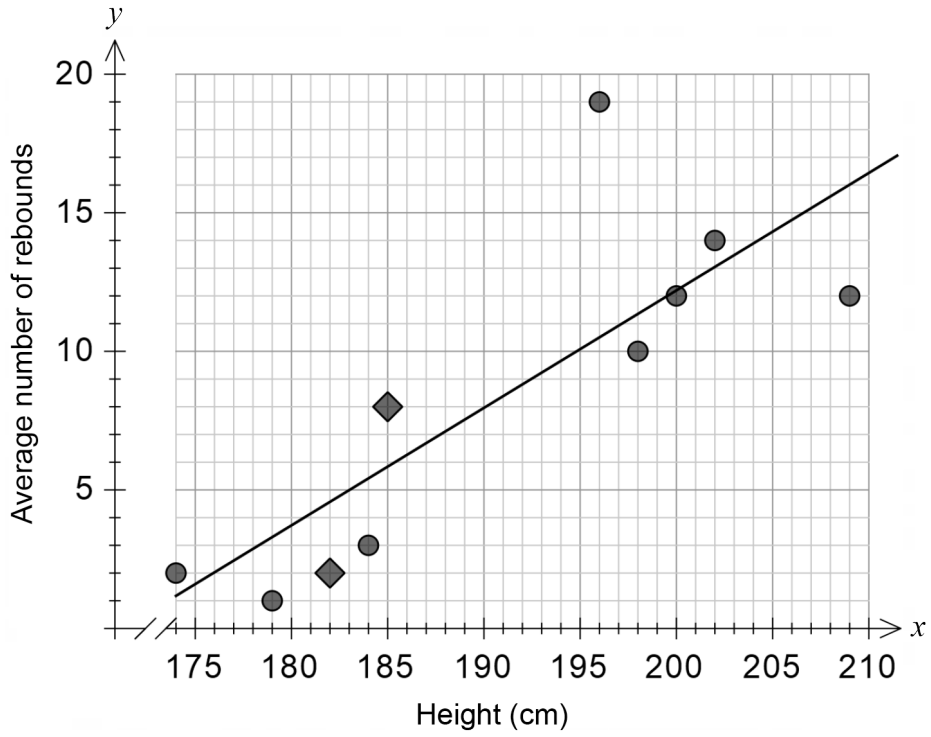
Solution
height
Specific behaviours
✓ identifies correct variable

(b) Complete the scatter graph below by plotting the missing data from the table. (2 marks)

Solution

Specific behaviours
✓ plots one correct coordinate ✓ plots both correct coordinates

The data has a correlation coefficient of 0.814, and the equation of the least-squares line is $y = 0.43x - 74.23$.

(c) Draw the least-squares line on the graph above. (2 marks)

Solution

Specific behaviours
✓ shows correct slope ✓ shows that line passes approximately through (200, 12)

(d) Describe the association between players' heights and average number of rebounds in terms of direction and strength. (2 marks)

Solution
strong positive association
Specific behaviours
✓ correctly states strong ✓ correctly states positive association

(e) Determine the coefficient of determination and state its meaning in the context of the question. (2 marks)

Solution
$r^2 = 0.663$ Therefore 66.3% of the variation in average number of rebounds can be explained by the variation of the height of the basketball players.
Specific behaviours
✓ determines correct coefficient of determination ✓ determines correct meaning

Question 8 **(16 marks)**

The table below shows the average price per kilogram of watermelon in a supermarket for each of the four seasons of the year over a four-year period.

(a) Determine the values **P**, **Q**, **R** and **S** in the table above. (4 marks)

Solution
$P = \dfrac{1.24 + 1.58 + 3.12 + 2.44}{4} = 2.10$ $Q = 2.75 \times 4 - (2.07 + 4.65 + 2.71) = 1.57$ or $Q = 2.75 \times 57.09\% = 1.57$ $R = \dfrac{\frac{3.48}{2} + 2.69 + 1.57 + 2.07 + \frac{4.65}{2}}{4} = 2.60$ $S = \dfrac{2.07}{2.75} \times 100 = 75.27$
Specific behaviours
✓ calculates correct P value ✓ calculates correct Q value ✓ calculates correct R value ✓ calculates correct S value

(b) Complete the following table. (1 mark)

Season	**Summer**	**Autumn**	**Winter**	**Spring**
Seasonal index	58.83%	76.19%	**155.72%**	109.26%

Solution
see table above
Specific behaviours
✓ calculates correct value
Note: the value in the table is the sum to 400 155.55% is the value when the winter components are averaged.

(c) Interpret the seasonal index for Summer in the context of this question. (1 mark)

Solution
on average Summer season is 41.17% below trend value or 58.83% of the trend value
Specific behaviours
✓ interprets the context correctly

(d) Define 'deseasonalised data'. (1 mark)

Solution
Data where periodic variations due to seasonal effects have been removed.
Specific behaviours
✓ determines correct definition

(e) Using information from the tables on page 6, show how the deseasonalised data value for Autumn 2021 was calculated. (1 mark)

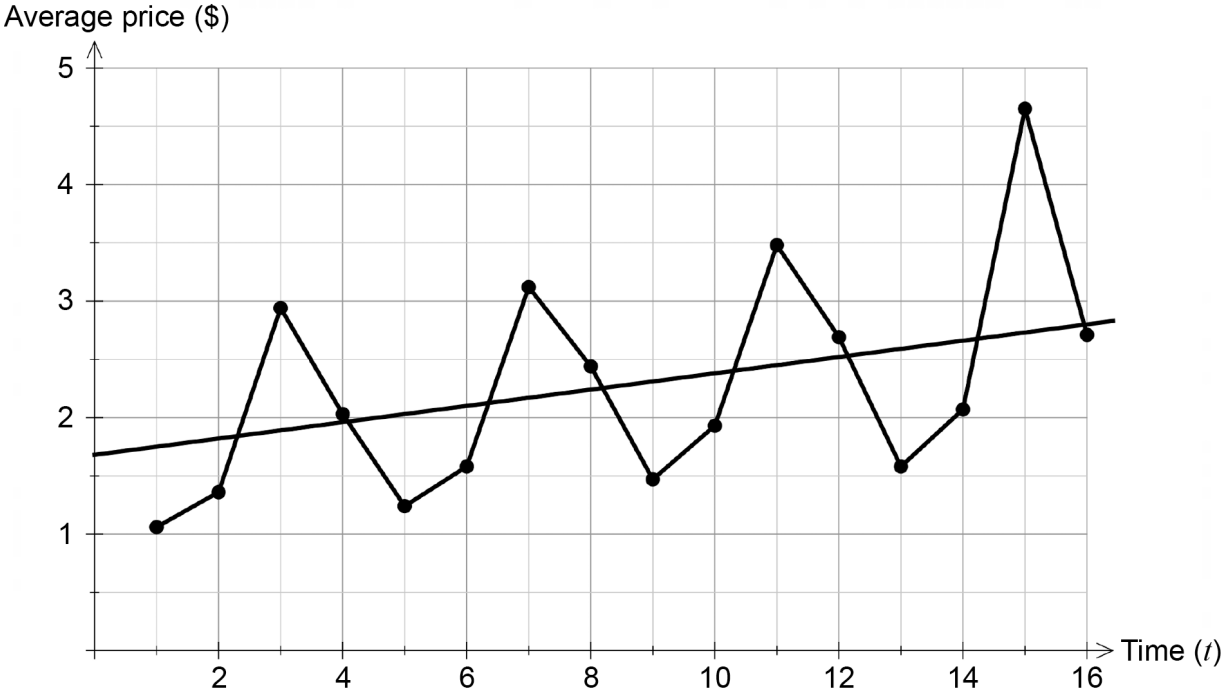
Solution
$\frac{1.93}{0.7619} = 2.53$
Specific behaviours
✓ determines correct calculation

(f) Determine the equation of the least-squares line using the deseasonalised data. (1 mark)

Solution
$y = 0.07t + 1.68$
Specific behaviours
✓ determines correct equation

Question 8 (continued)

(g) The graph below is the time series plot for these data. On this graph draw the least-squares line determined in part (f). (2 marks)



Solution
see graph above
Specific behaviours
✓ shows correct y-intercept ✓ draws line with correct slope

(h) Predict the average price per kilogram of watermelon from the supermarket in Spring 2024. (3 marks)

Solution
$t = 24$ $y = 0.07(24) + 1.68 = 3.36$ $3.36 \times 1.0926 = 3.67$
Specific behaviours
✓ uses $t = 24$ ✓ predicts trend value ✓ seasonally adjusts

(i) Discuss the reliability of the prediction in part (h). (2 marks)

Solution
prediction not reliable as it is extrapolation
Specific behaviours
✓ states prediction not reliable ✓ states it is extrapolation

Question 9 **(16 marks)**

Sonia secures a bank loan to buy a professional gaming computer. The loan has reducible interest. Information about the loan is shown below.

Loan issued: Start of October 2023.
Starting balance: \$9200.
Interest: Compounded monthly.
Repayments: \$290 per month.

After the first monthly payment at the end of October 2023, Sonia's balance is \$8992.80.

(a) Use the information above to show that the annual interest rate is 10.8%. (2 marks)

Solution
Interest for the first month = $(8992.80 + 290) - 9200 = 82.80$ Annual interest rate = $\frac{82.80}{9200} \times 100 \times 12 = 10.8\%$
Specific behaviours
✓ correctly calculates the interest for the first month ✓ correctly calculates annual interest rate

(b) Determine a recursive rule to model the balance of the loan at the end of each month. (2 marks)

Solution
$T_{n+1} = 1.009T_n - 290, T_0 = 9200$
Specific behaviours
✓ states correct rule ✓ states correct initial value

(c) Determine

(i) the balance of the loan at the end of November 2023. (1 mark)

Solution
\$8783.74 (Term 2 in the sequence)
Specific behaviours
✓ determines correct balance

(ii) the total amount of interest incurred in the first three months. (2 marks)

Solution
82.80 + (0.009 × 8992.80) + (0.009 × 8783.74) = \$242.79
Specific behaviours
✓ determines correct interest for each month ✓ sums each interest

(iii) the balance of the loan at the end of May 2024. (1 mark)

Solution
\$7489.24 (Term 8 in the sequence)
Specific behaviours
✓ determines correct balance

Question 9 (continued)

(d) Determine how many months it takes to repay the loan. (1 mark)

Solution
38 months
Specific behaviours
✓ determines correct value

(e) Determine the final repayment and the total amount repaid. (2 marks)

Solution
$T_{37} = 150.64$ Therefore, the final repayment is \$150.64 × 1.009 = \$152 Total amount repaid = $37 \times 290 + 152 = \$10\,882$ or 38 × 290 – 138.00 = \$10 882
Specific behaviours
✓ determines correct final repayment ✓ determines correct total amount repaid

(f) Calculate the total interest paid on the loan. (1 mark)

Solution
10 882 – 9200 = \$1682
Specific behaviours
✓ determines correct value

(g) Sonia is paid every fortnight in her employment. Instead of monthly repayments of \$290 she is now considering making fortnightly repayments of \$145, with the interest calculated fortnightly. Use mathematical evidence to show what difference this would make and advise Sonia what her savings might be. (4 marks)

Solution
Interest each fortnight $\frac{10.8}{26} = 0.41538\%$ (5 d.p.) $A_{n+1} = \left(1 + \frac{0.108}{26}\right) A_n - 145, A_0 = 9200$ 74 repayments and total repaid = 74 × 145 – 28.92 = \$10 701.08 Sonia should use fortnightly repayments to save 10 882 – 10 701.08 = \$180.92 She would also pay off the loan quicker.
Specific behaviours
✓ determines correct number of repayments ✓ calculates the total repaid ✓ determines correct final savings ✓ gives correct advice

Question 10 **(18 marks)**

Data concerning rental properties have been collected from 10 suburbs of a city. The data is for median property value (p) ($'000), median weekly rent ($\$w$) and percentage vacancy rate ($v\%$) within each suburb.

The data in the table below show the median property value and the median weekly rent for the 10 suburbs.

Median property value (p) ($'000)	395	470	550	725	580	780	700	740	690	585
Median weekly rent (w) ($)	445	460	590	630	530	850	680	690	640	575

(a) Calculate the correlation coefficient and the equation of the least-squares line for these data. (3 marks)

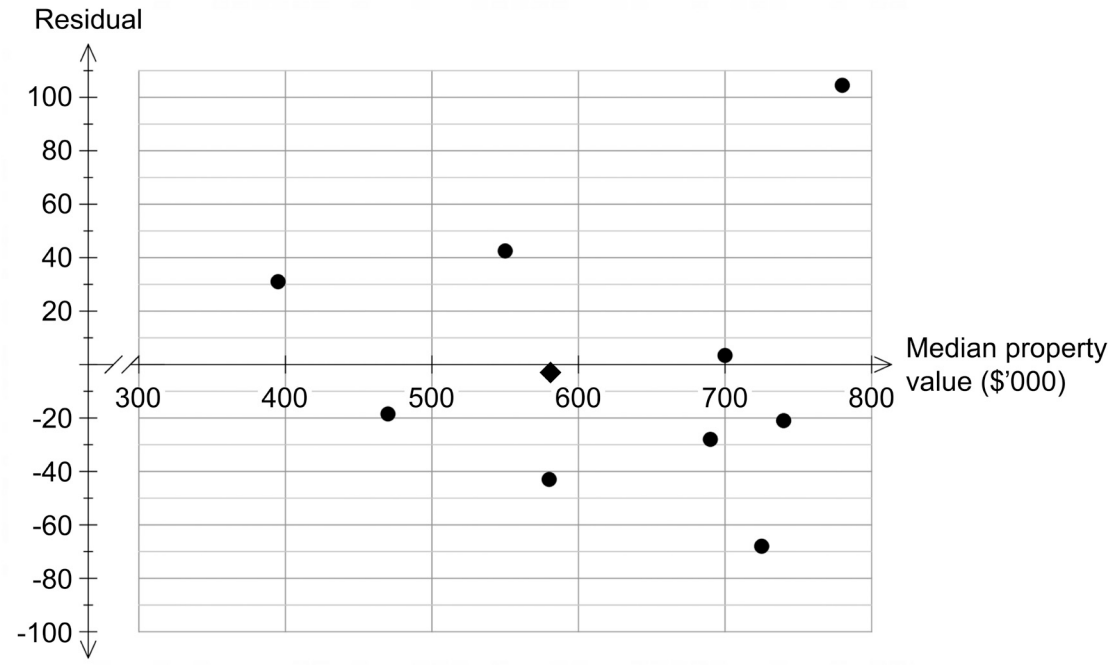
Solution
r = 0.91 $\quad w = 0.861p + 73.84$
Specific behaviours
✓ determines correct correlation coefficient ✓ determines correct least squares line ✓ uses correct variables

(b) In the context of this question, interpret the gradient of the least-squares line determined in part (a). (2 marks)

Solution
For each $1000 increase in median property value, median weekly rent will increase by $0.86
Specific behaviours
✓ uses median property value and median weekly rent ✓ uses correct values

Question 10 (continued)

(c) The graph below shows the residual plot for the first nine suburbs as given in the table. Determine the residual for the 10th suburb and plot this value on the graph. (2 marks)



Solution
Residual: (585, –2.6) See graph above for plot
Specific behaviours
✓ determines correct point ✓ plots point correctly

(d) State a conclusion that can be drawn from the residual plot. (1 mark)

Solution
It is appropriate to use a linear model for these data as there is no clear pattern in the residual plot
Specific behaviours
✓ states correct conclusion

(e) The predicted weekly rent of a property was calculated to be \$612. What property value was this based on? (2 marks)

Solution
$612 = 0.861p + 73.84 \quad \Rightarrow p = 625.04$ Property value \$625 000
Specific behaviours
✓ solves for p ✓ expresses answer in thousands of dollars

(f) If the data point (780, 850) was removed from all calculations, would the gradient of the least-squares line determined in part (a) increase, decrease or stay the same? (1 mark)

Solution
decrease
Specific behaviours
✓ gives correct answer

Bivariate data analysis between percentage vacancy rate and median weekly rent produced the following: $r_{vw}^2 = 0.85$ and $w = -82.64v + 940.64$.

(g) Explain why $r_{vw} = -0.92$. (2 marks)

Solution
$\sqrt{0.85} = 0.92$, but the gradient of least squares line is negative therefore r is negative.
Specific behaviours
✓ determines square root ✓ explains why negative

(h) A property has a vacancy rate of 4.1% and a median property value of $605 000. Predict the median weekly rent using the most reliable predictor. Justify which predictor is used. (2 marks)

Solution
the strength of $r_{vw} > r_{pw}$, therefore use vacancy rate predicted value $\approx \$602$
Specific behaviours
✓ explains why vacancy rate is the best predictor ✓ determines correct predicted value

(i) Calculate the expected change in the weekly rent if the percentage vacancy rate increases by 0.4%. (1 mark)

Solution
$-82.64 \times 0.4 = -\$33.06$ (A decrease of $33.06)
Specific behaviours
✓ determines correct change

(j) Comment on the statement 'it is clear both property price and vacancy rate will cause changes to the median weekly rent'. Justify your answer. (2 marks)

Solution
statement incorrect, cause not established
Specific behaviours
✓ comments statement incorrect ✓ gives correct justification

Question 11 **(10 marks)**

A ferry service connects a group of islands in the Pacific Ocean.

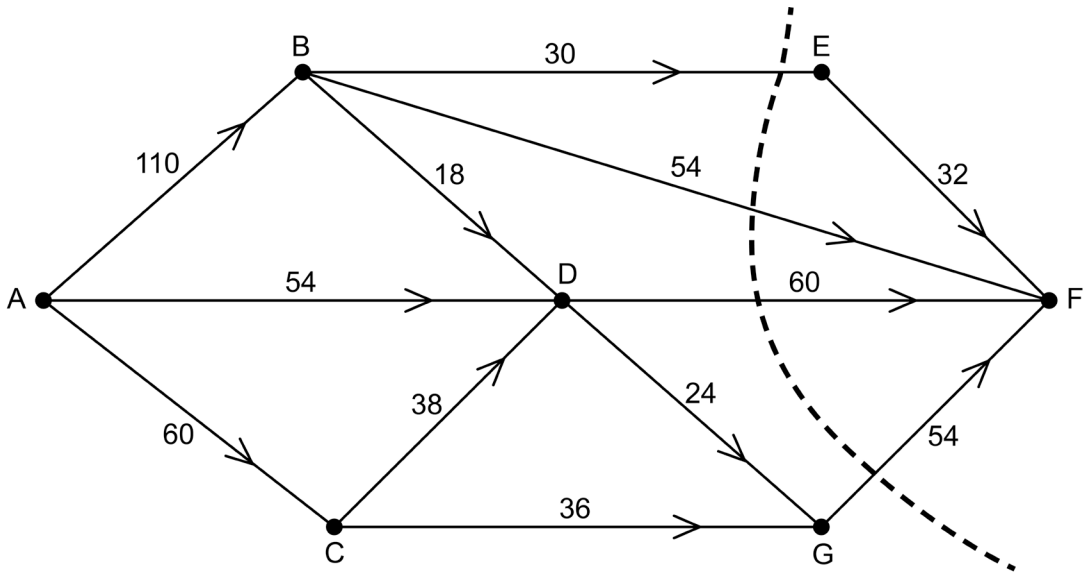
(a) The network below is for the hourly departures from A and shows the number of passengers that can be carried between each island, A, B, C, D, E, F and G. Determine the maximum number of passengers that can be carried from A to F each hour. Show systematic working. (4 marks)

Solution			
Possible solutions:			
ABEF	30	ABEF	30
ABF	54	ACGF	36
ABDF	18	ADF	54
ADF	42	ABF	54
ADGF	12	ABDF	6
ACDGF	12	ACDGF	18
ACGF	30		
Total passengers = 198			

Specific behaviours
✓ shows systematic working
✓ lists at least four correct paths with passenger numbers
✓ lists complete set of paths with passenger numbers
✓ states correct total

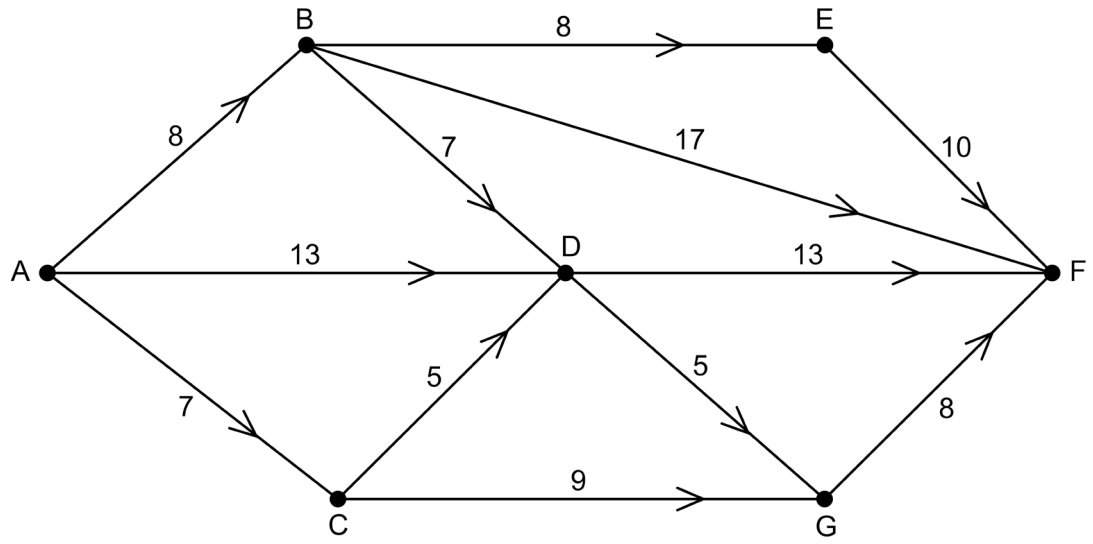
(b) On the network below, draw the minimum cut that corresponds to the maximum flow determined in part (a). (1 mark)

Solution



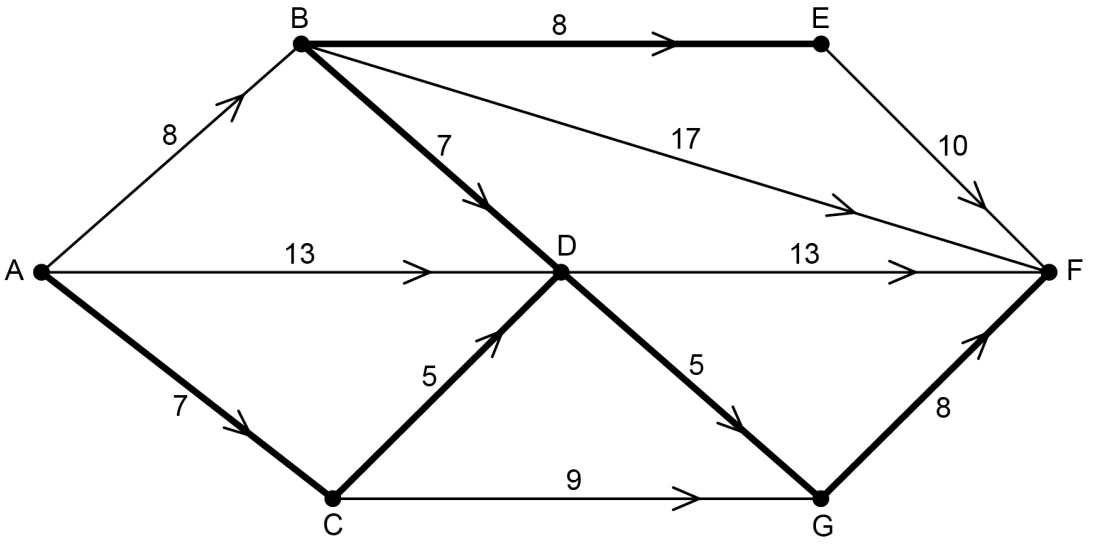
Specific behaviours
✓ draws correct cut

(c) The network below shows the distance in kilometres between the islands. Ferry passengers are charged a \$10 booking fee plus \$1.50 for each kilometre they plan to travel. Calculate the minimum cost of travelling from A to F. (3 marks)



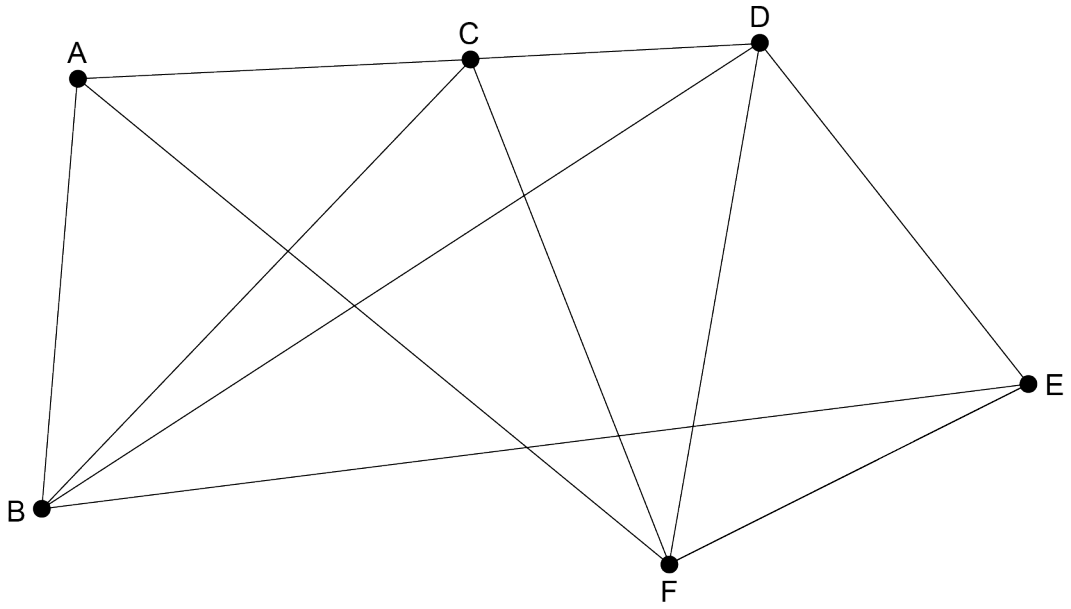
Solution
Shortest path is ACGF, distance = 24 km Cost = 10 + 1.50 × 24 = \$46
Specific behaviours
✓ determines shortest path ✓ determines shortest distance ✓ determines minimum cost

(d) During the off-season when there are fewer passengers, the ferry company will only maintain the services that form the minimum spanning tree. On the network below, highlight clearly the services that will be maintained. (2 marks)

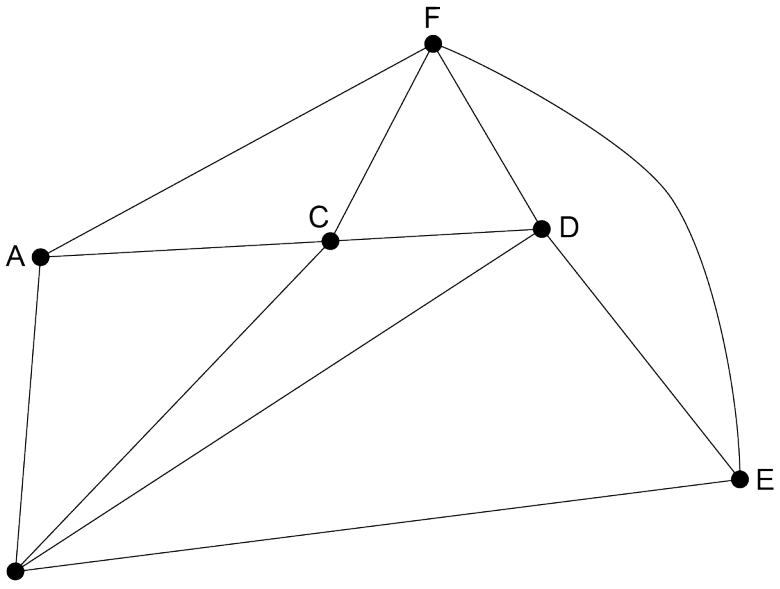
Solution

Specific behaviours
✓ determines a spanning tree with four correct edges ✓ determines all correct edges

Question 12 **(7 marks)**

An event organiser is setting up a music festival with multiple stages. The graph below shows the stages as vertices, and the edges are the walkways that connect each stage.



(a) To avoid congestion, the organisers want to move the stages and re-position some connecting walkways so no walkways intersect. Redraw the graph to meet these conditions. (2 marks)

Solution

Specific behaviours
✓ draws the graph with no edges crossing ✓ draws the graph with all edges maintained
Accept other relevant answers.

(b) Use Euler's formula, stating the number of vertices, edges and faces, to verify that this graph is planar. (2 marks)

Solution
7 faces, 6 vertices and 11 edges $v+f-e=6+7-11=2$ which verifies Euler's formula.
Specific behaviours
✓ correctly states the number of vertices, edges and faces ✓ verifies Euler's formula

(c) State a reason why the graph is not Eulerian. (1 mark)

Solution
it contains odd vertices
Specific behaviours
✓ states that the graph contains odd vertices

One attendee wishes to visit every stage to make the most of the festival.

(d) (i) Write down a pathway the attendee can take to visit every stage exactly once if they start at Stage F. (1 mark)

Solution
FACDEB or FEBACD or FEDCAB
Specific behaviours
✓ states a correct pathway
Accept other relevant answers.

(ii) What is the name given to this path? (1 mark)

Solution
Hamiltonian
Specific behaviours
✓ states correct name

Question 13 (10 marks)

Chau purchased a brand-new motorcycle valued at $17 000. Once Chau drove the motorcycle out of the dealer's yard, it immediately depreciated in value by 15%. By the end of the first year, it depreciated a further 5%. After the first year the motorcycle depreciated at a constant rate of 7% per year.

(a) (i) Show how the value of the motorcycle at the end of the first year was calculated. (2 marks)

Solution
$17\,000 \times 0.85 \times 0.95 = 13\,727.50$
Specific behaviours
✓ multiplies by 0.85 ✓ multiplies answer by 0.95

(ii) Complete the remainder of the table. (2 marks)

Solution				
***n* (years)**	1	2	3	4
Value of the motorcycle ($)	13 727.50	**12 766.58**	**11 872.91**	**11 041.81**
Specific behaviours				
✓ determines one correct value ✓ correctly completes table				

(b) Determine the percentage by which the motorcycle depreciated in the first year. (2 marks)

Solution
$\frac{3272.50}{17\,000} \times 100 = 19.25\%$
Specific behaviours
✓ calculates the actual decrease of 3272.50 ✓ calculates correct percentage

(c) Write a recursive rule that represents the amount the motorcycle will be worth at the end of each year, after the first year. (2 marks)

Solution
$T_{n+1} = 0.93T_n,\ T_0 = 13\,727.50$
Specific behaviours
✓ states correct value for T_0 ✓ states correct recursive rule

(d) Determine how much the motorcycle will be worth at the end of eight years. (1 mark)

Solution
$T_7 = 8259.848$ therefore the motorcycle will be worth \$8259.85 to the nearest cent
Specific behaviours
✓ determines correct value

(e) When will the motorcycle be first worth less than \$5000? (1 mark)

Solution
$T_{13} = 5344.04$ $T_{14} = 4969.96$ therefore during the 15th year
Specific behaviours
✓ determines correct time

Question 14 (11 marks)

Ravi retired at the beginning of the month with a superannuation balance of $945 864. He has a written guarantee of a 7.5% per annum return on his superannuation with interest added at the end of each month. Ravi will receive an annuity of $3200 paid at the end of each fortnight.

Ravi has also committed to purchasing a new car for a price of $37 000. There is a 12-month waiting period for the delivery of the car and he plans to pay for the car by withdrawing the cost from his superannuation account at time of delivery, when payment is due.

(a) Calculate the balance in the superannuation account at the end of two years. (6 marks)

Solution
N = 26, I = 7.5, PV = –945 864, PMT = 3200, P/Y = 26, C/Y = 12 ⇒ FV = 933 028.76 Balance after 1 year = 933 028.76 – 37 000 = $896 028.76 N = 26, I = 7.5, PV = –896 028.76, PMT = 3200, P/Y = 26, C/Y = 12 ⇒ FV = 879 324.68 Balance after 2 years = $879 324.68
Specific behaviours
✓ determines at least four correct parameters for first year calculation ✓ determines all correct parameters for first year calculation ✓ determines correct balance after one year ✓ determines correct starting value for second year ✓ determines all correct parameters for second year ✓ determines correct balance after two years

(b) After 15 years of retirement Ravi expects to have a less active lifestyle and his living expenses will not be as high. He understands that the balance of his superannuation needs to last for his remaining years. Ravi decides the most suitable plan for him is to place the balance of his superannuation into a perpetuity with a fortnightly payment.

Assuming the 7.5% interest rate is maintained, what fortnightly amount can Ravi expect from the perpetuity? (5 marks)

Solution
After 13 more years N = 338, I = 7.5, PV = –879 324.68, PMT = 3200, P/Y = 26, C/Y = 12 ⇒ FV = 498 326.85 Perpetuity: N = 26, I = 7.5, PV = –498 326.85, FV = 498 326.85, P/Y = 26, C/Y = 12 ⇒ PMT = 1435.07 Fortnightly payment of $1435.07
Specific behaviours
✓ determines thirteen extra years needed ✓ determines all correct parameters ✓ determines correct value after fifteen years ✓ determines all correct parameters for perpetuity ✓ determines correct fortnightly payment

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