Government of **Western Australia**
School Curriculum and Standards Authority

ATAR course examination, 2023

Question/Answer booklet

MATHEMATICS SPECIALIST

Section Two: Calculator-assumed

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

WA student number: In figures

In words

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	48	35
Section Two: Calculator-assumed	11	11	100	89	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2023: Part II Examinations*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (89 Marks)**

This section has **11** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 9 **(6 marks)**

The Cartesian equation of a sphere is given as $x^2 + y^2 + z^2 - 4x + 2y - 6z + 5 = 0.$

(a) Write the equation of the sphere in vector form. (3 marks)

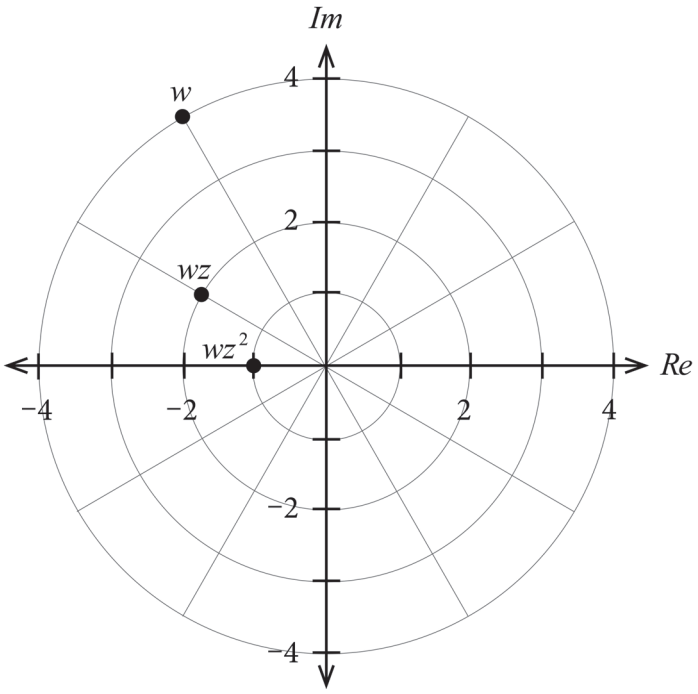
A line has vector equation $\underset{\sim}{r} = \begin{pmatrix} 7 \\ -1 \\ 9 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$.

(b) Determine the point(s) of intersection between the line and the sphere. (3 marks)

See next page

Question 10 **(7 marks)**

The complex number $w = 4cis\left(\frac{2\pi}{3}\right)$ is shown in the Argand diagram, along with the complex numbers wz and wz^2.



(a) Express wz and wz^2 in exact polar form. (2 marks)

Consider the geometric transformation(s) applied to transform $w \rightarrow wz \rightarrow wz^2$ etc.

(b) Describe the geometric transformation(s) performed by successive multiplication by z. (2 marks)

See next page

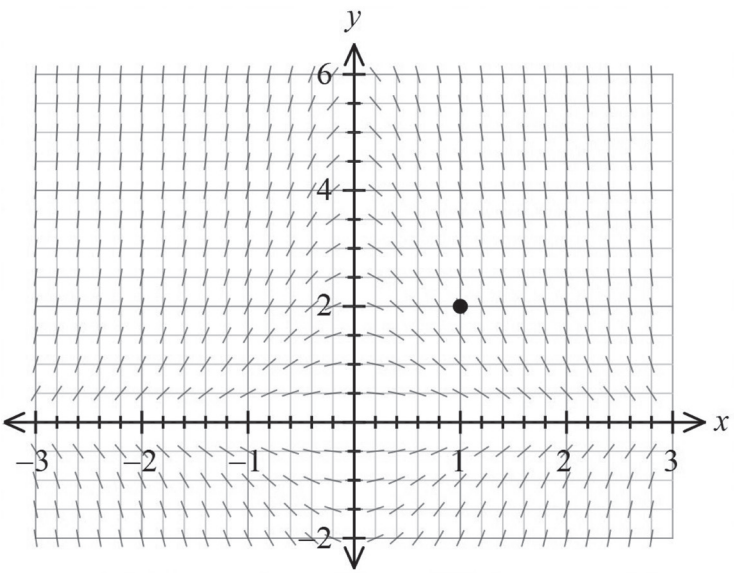
(c) Determine z in exact polar form. (1 mark)

(d) Describe the geometric transformation(s) performed by successive multiplication by z^{-1}. (2 marks)

See next page

Question 11 **(5 marks)**

A slope field is given by the equation $\frac{dy}{dx} = k(xy)$ where k is a constant.



(a) The value of the slope field at the point $(1, 2)$ is equal to -4. Determine the value of the constant k. (2 marks)

(b) Determine the equation for the solution curve that contains the point $(1, 2)$ and draw this curve on the diagram above. (3 marks)

A spare diagram is provided at the end of this Question/Answer booklet. If you need to use it, cross out this attempt and indicate that you have redrawn it on the spare diagram.

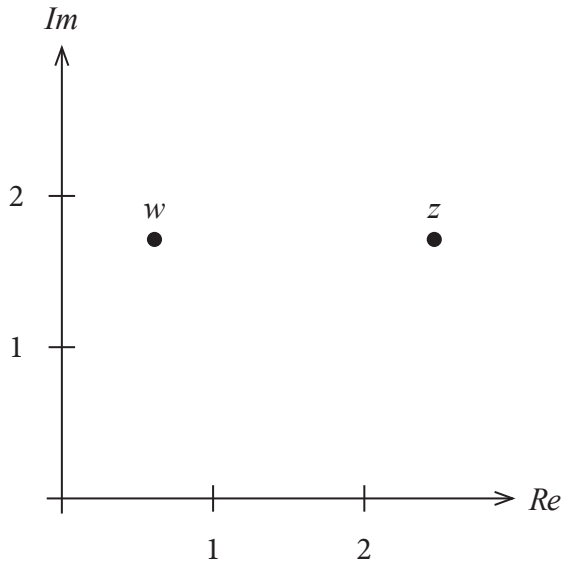
See next page

Question 12 **(6 marks)**

Complex numbers z and w are shown in the Argand diagram below. It is known that:

$|z| = 3,\ Arg\,(z) = \theta$ where $0 < \theta < \frac{\pi}{4}$

$w = z - k$ such that $Arg\,(w) = 2\theta$ where $Im\,(k) = 0,\ k > 0.$



(a) Represent the given information on the Argand diagram. (3 marks)

A spare diagram is provided at the end of this Question/Answer booklet. If you need to use it, cross out this attempt and indicate that you have redrawn it on the spare diagram.

(b) Determine a simplified expression for k in terms of θ. Justify your answer. (3 marks)

Question 13 **(11 marks)**

A factory produces boxes of breakfast cereal with a labelled weight of 1.00 kg.

Let μ denote the population mean and σ denote the population standard deviation of the weights of the boxes. The factory sets the packaging process to a mean weight $\mu = 1.01$ kg with a standard deviation $\sigma = 0.05$ kg.

To maintain quality, a random sample of 400 boxes is taken each day and weighed. Let $\overline{X}$ denote the sample mean weight.

(a) State the distribution for $\overline{X}$ and its parameters. (3 marks)

(b) Determine the probability that the sample mean is more than 5 g above the labelled weight. (2 marks)

The sample mean on a particular day is $\bar{x} = 1.05$ kg, while the sample standard deviation is $s = 50$ g.

(c) Determine a 95% confidence interval, correct to 0.001 kg, for the population mean weight based on this sample. (2 marks)

See next page

Anja, a quality control officer, wants a 95% confidence interval based on a sample size of 100 with a width of no more than 0.1 kg.

(d) What is the maximum standard deviation for this confidence interval? (2 marks)

Over the next 50 days, Ben, who is a data collection agent, takes random samples of size 100 each day and a 95% confidence interval is calculated for each sample. Ten of these 50 intervals (20% of the intervals) have a lower bound that is less than 1.00 kg. Ben claims that this indicates that the mean weight of the packaging is set too low.

(e) Is Ben correct? Justify your response. (2 marks)

Question 14 **(11 marks)**

Plane P_1 has Cartesian equation: $z = 2x + y + 4$.

Line L has equation given by: $\underset{\sim}{r} = \begin{pmatrix} 2 - \lambda \\ 1 + \lambda \\ 2\lambda \end{pmatrix}$.

(a) Determine a vector that is perpendicular to plane P_1. (2 marks)

(b) Write the equation for plane P_1 in vector form. (2 marks)

See next page

(c) Determine the acute angle, correct to the nearest degree, between plane P_1 and line L. (3 marks)

(d) Obtain the Cartesian equation of the plane P_2 that contains the line L and is perpendicular to plane P_1. (4 marks)

See next page

Question 15 **(9 marks)**

The WeLuvYas Bank extends personal loans to approved customers. A random sample of n personal loans is taken. A 99% confidence interval for the population mean loan μ (in thousands of dollars) based on this sample is $10.2 < \mu < 25.4$.

(a) What is the mean personal loan $\bar{x}$ for this sample? (2 marks)

(b) Calculate the standard deviation of the sample mean. (2 marks)

Ali exclaims excitedly 'everyone here at WeLuvYas is 99% certain that the true population mean is within the interval $10.2 < \mu < 25.4$'.

(c) State **two** reasons why Ali is not correct. (2 marks)

See next page

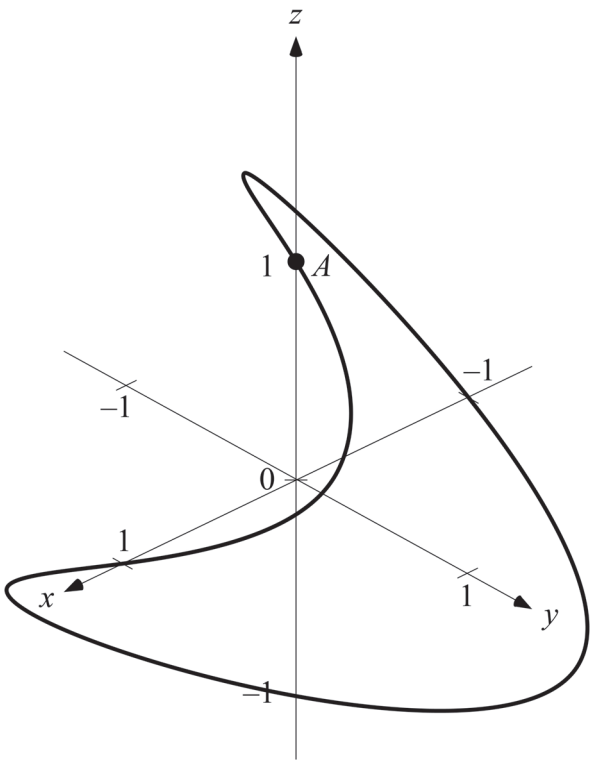
A data analyst discovers that the sample size was actually $2n$. In addition to this, the sample mean was actually \$2000 more than that originally determined.

(d) Re-calculate the 99% confidence interval for the population mean on the basis of the updated information. (3 marks)

See next page

Question 16 **(5 marks)**

A fly moves around a path given by a three dimensional curve. The fly's path begins at point A and is shown below. Its position vector is specified by $\underset{\sim}{r}(t) = \begin{pmatrix} \sin t \\ \sin 2t \\ \cos t \end{pmatrix}$ metres where $0 \leq t \leq 2\pi$ seconds.



(a) Determine the initial acceleration vector and indicate this clearly on the diagram above. (3 marks)

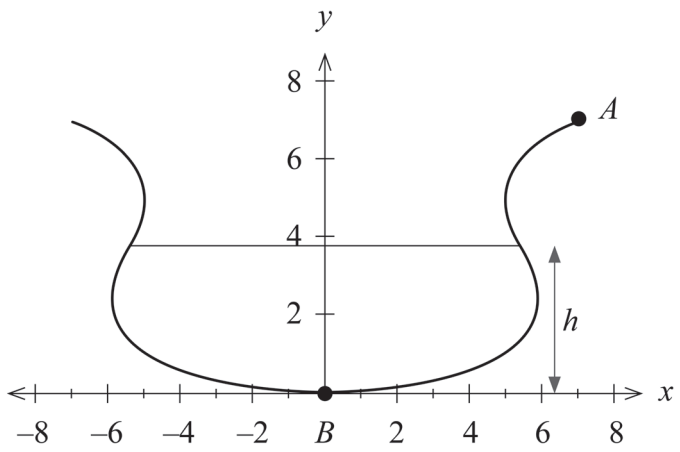
See next page

(b) Calculate the length of the path taken by the fly, correct to 0.001 metres. (2 marks)

See next page

Question 17 **(7 marks)**

The shape of a decorative vase is modelled by revolving the curve AB about the y axis where $x = \sqrt{y\left(y^2 - 11y + 35\right)}$ with $0 \leq y \leq 7$. All dimensions are in centimetres.



(a) Determine an integral expression, in terms of h, for the volume of water in the vase if it is filled to a depth of h cm. (2 marks)

Water is poured into the initially empty vase at a constant rate of 50 cm^3/s.

(b) Determine the time taken to fill the vase to a depth of 6 cm. (2 marks)

See next page

With the depth at 6 cm, another 30 cm^3 of water is added to the vase.

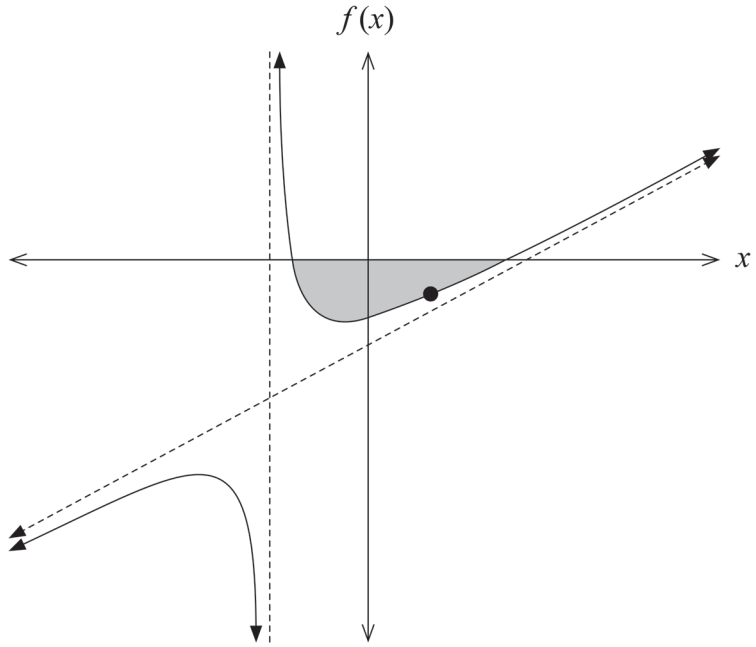
(c) Using the increments formula, calculate the approximate change in depth of water in the vase. (3 marks)

See next page

Question 18 **(6 marks)**

Function $f(x)$ is a rational function of the form $\frac{x^2 + bx + c}{x + d}$ with the following properties:

- $f(2) = -2$
- $f(x)$ has a vertical asymptote at $x = -3$ and another asymptote with equation $y = x - 5$.



(a) Show that $b = -2,\ c = -10$ and $d = 3$. (3 marks)

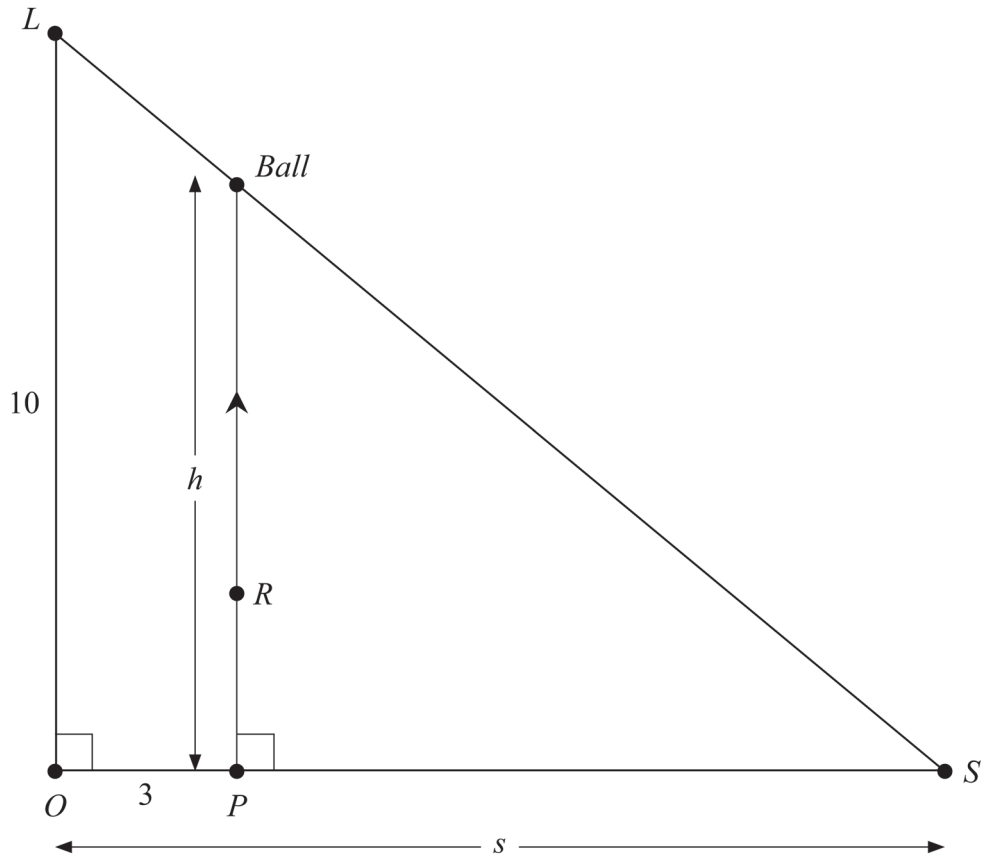
(b) Calculate the exact area of the shaded region. (3 marks)

See next page

Question 19 **(16 marks)**

A ball is projected vertically into the air from point R, so that it will eventually hit the ground at point P, 3 metres from the base of a 10 metre high light at L.

At any time t seconds, when the ball is h metres above the ground, it casts a shadow on the ground at point S at a distance s metres from the base of the light.



At any time t it can be shown that $s(10-h)=30$.

(a) Using implicit differentiation, show that $\frac{ds}{dt}=\frac{30}{(10-h)^2}\times\frac{dh}{dt}$. (3 marks)

See next page

At $t = 0.5$ seconds, it is found that the ball is 6.275 metres above the ground and moving upwards at 6.1 metres per second.

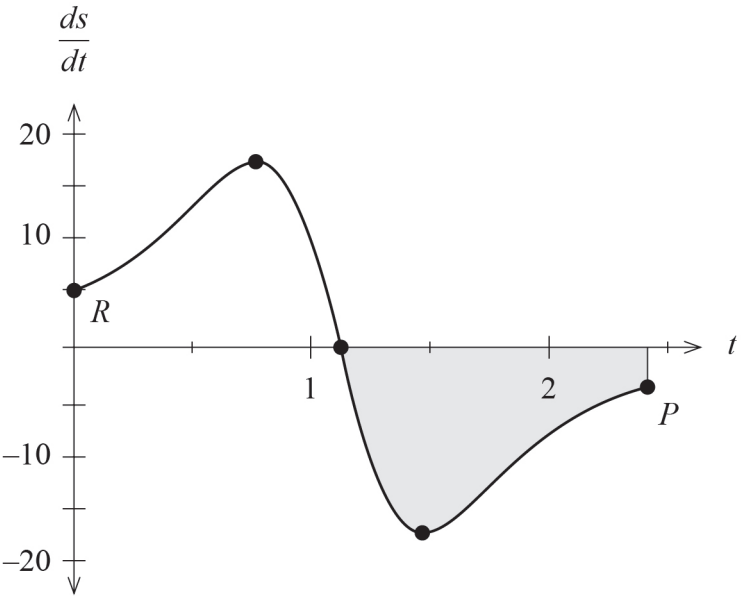
(b) By assuming $h''(t) = -9.8$ ms^{-2}, show that $h(t) = 2 + 11t - 4.9t^2$. (3 marks)

(c) Determine the initial speed of the ball's shadow, correct to the nearest 0.01 metres per second. (3 marks)

See next page

Question 19 (continued)

The graph of the function $\frac{ds}{dt}$ against time t is shown below. Point R of this graph corresponds to the ball being thrown into the air, while point P corresponds to the ball hitting the ground.



The definite integral $\int_a^b \left(\frac{ds}{dt}\right) dt$ was evaluated so that the area for the shaded region could be determined. This area is 13.4258 square units.

(d) Determine the values for a and b (correct to 0.01 seconds) and describe what this definite integral represents in terms of the motion of the shadow. (4 marks)

(e) Determine the fastest rate at which the shadow moves (correct to the nearest 0.01 metres per second) and the time when this occurs (correct to the nearest 0.01 seconds). (3 marks)

End of questions

Supplementary page

Question number: _____

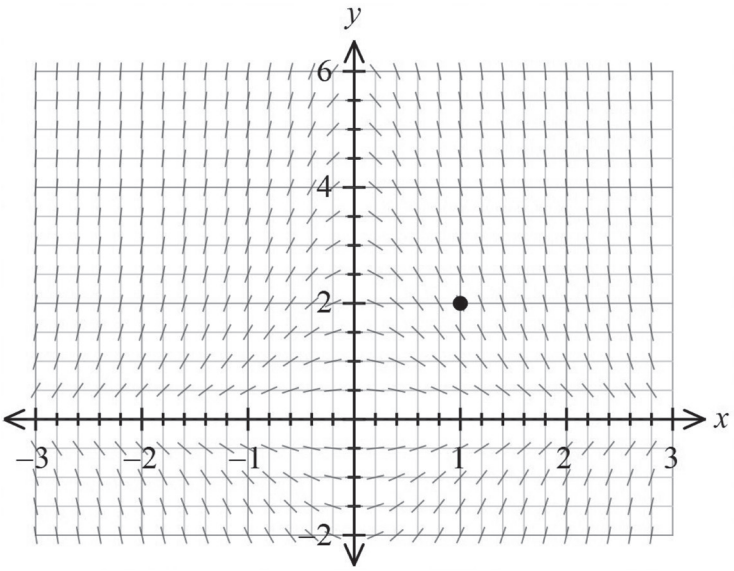
Supplementary page

Question number: _____

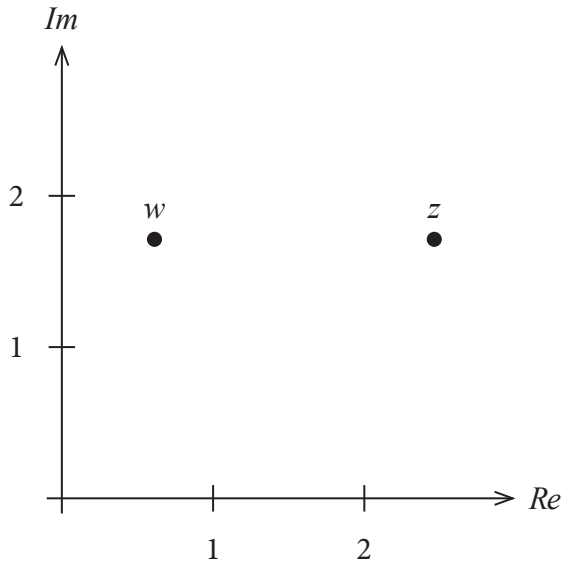
Supplementary page

Question number: _____

Spare diagram for Question 11(b).



Spare diagram for Question 12(a).



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