Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS APPLICATIONS

Calculator-assumed

ATAR course examination 2016

Marking Key

Marking keys are an explicit statement about what the examining panel expect of candidates when they respond to particular examination items. They help ensure a consistent interpretation of the criteria that guide the awarding of marks.

2016/64865

Section Two: Calculator-assumed 65% (100 Marks)

Question 7 (9 marks)

Julie buys a car with a purchase price of $13 000. However, she has been told to expect the car to depreciate in value. The value of the car after n years can be determined by using the recursive rule.

$$T_{n+1} = 0.85T_n, \quad T_0 = 13\,000$$

(a) Complete the table below to show the value of the car at the end of each year, to the nearest dollar. (2 marks)

n	**0**	**1**	**2**	**3**
Value of car after n years ($)	13 000	**11 050**	**9393**	**7984**

Solution
see table above
Specific behaviours
✓ completes entries without rounding ✓ completes entries correctly rounding

(b) Use the information above to determine the rate of depreciation of Julie's car per year. (1 mark)

Solution
15% per year
Specific behaviours
✓ states the correct rate of depreciation

(c) Determine a rule for the n^{th} term of the sequence of values found in part (a). (2 marks)

Solution
$T_n = 13000 \times 0.85^n$
Specific behaviours
✓ states the correct coefficient ✓ states the correct base

(d) Determine the value of Julie's car after eight years, correct to the nearest dollar. (2 marks)

Solution
$T_n = 13000 \times 0.85^8 = 3542.38 = \$3542$
Specific behaviours
✓ calculates the correct value ✓ correctly rounds to the nearest dollar

(e) Julie decides that she will sell her car at the end of the year in which its value drops to half of the purchase price. After how many years should she sell her car? (2 marks)

Solution
$6500 = 13000 \times 0.85^n \Rightarrow n = 4.265$ Therefore Julie will sell her car at the end of the fifth year.
Specific behaviours
✓ calculates the correct value of n ✓ correctly states to sell car at end of fifth year

Question 8 **(17 marks)**

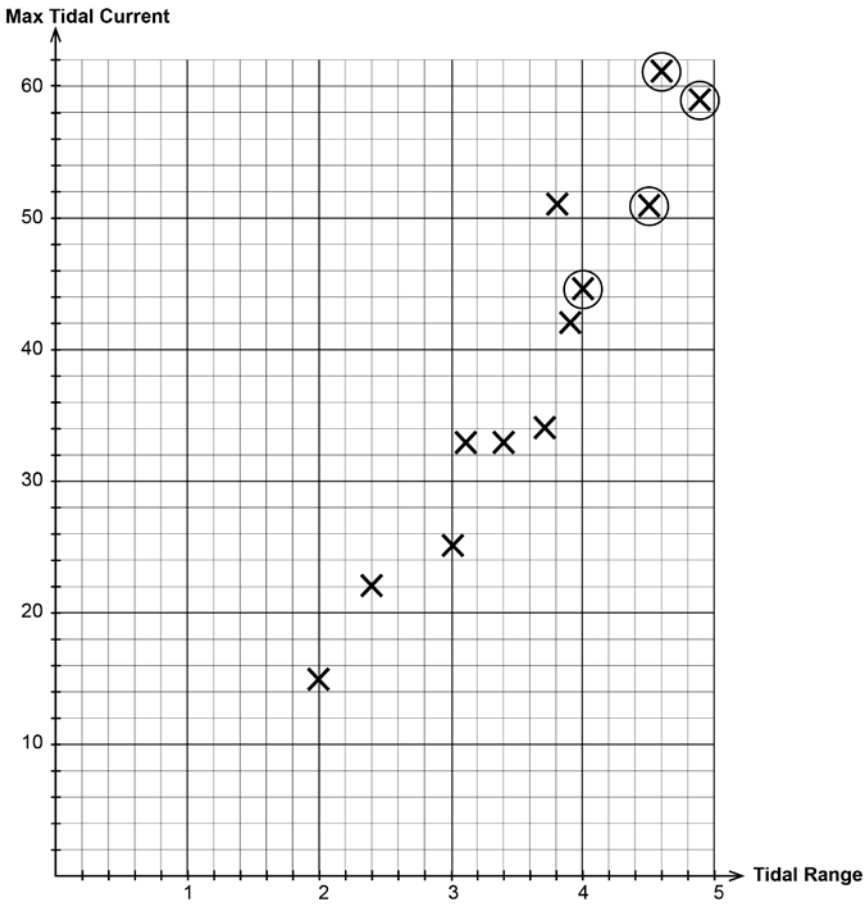
An experiment was conducted to determine whether there was any relationship between the maximum tidal current, in centimetres per second, and the tidal range, in metres, at a particular marine location. (The tidal range is the difference between the height of high tide and the height of low tide.) Readings were taken over a period of 12 days and the results are shown in the following table.

Tidal range	2.0	2.4	3.0	3.1	3.4	3.7	3.8	3.9	4.0	4.5	4.6	4.9
Maximum tidal current	15.2	22.0	25.2	33.0	33.1	34.2	51.0	42.3	45.0	50.7	61.0	59.2

(a) State the explanatory variable. (1 mark)

Solution
tidal range is the explanatory variable
Specific behaviours
✓ correctly states tidal range

(b) Complete the scatterplot below by plotting the last four data points and labelling the horizontal axis and the vertical axis clearly. (2 marks)



Solution
see graph above
Specific behaviours
✓ correctly plots points ✓ correctly labels axes

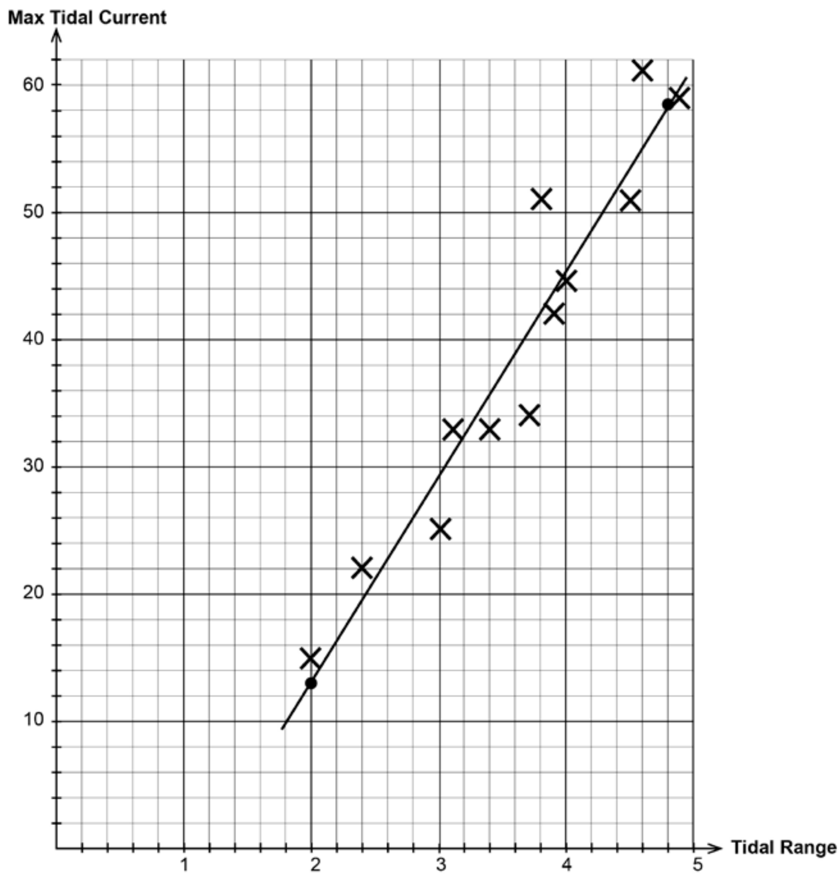
(c) Calculate the correlation coefficient for the data, and comment briefly on your answer with reference to the appearance of the scatterplot in part (b). (2 marks)

Solution
correlation coefficient = 0.956 (three decimal places) strong positive linear relationship
Specific behaviours
✓ correctly calculates correlation coefficient ✓ correctly mentions strength and direction

(d) (i) Determine the equation for the least-squares line that models these data. State the slope and vertical-intercept correct to one decimal place. (2 marks)

Solution
$y = 15.98x - 18.33 \therefore y = 16.0x - 18.3$ correct to one decimal place
Specific behaviours
✓ states correct equation of the least-squares line ✓ correctly rounds to one decimal place

(ii) Draw this line on the scatterplot in part (b) by showing two calculated points on the graph. (2 marks)



Solution
see graph above
Specific behaviours
✓ shows two points on the graph ✓ draws straight line through these points

(iii) Interpret the slope of the least-squares line. (2 marks)

Solution
The Max Tidal Current increases by 16 cm per second for every 1 m increase in Tidal Range.
Specific behaviours
✓ correctly states Tidal Current increases by 16 cm per second ✓ correctly states for every 1 m increase in Tidal Range

(e) Calculate the coefficient of determination and interpret it. (2 marks)

Solution
$r^2 = 0.915$ i.e. 91.5% of the variation in max tidal current can be explained by the variation in tidal range
Specific behaviours
✓ correctly calculates coefficient of determination ✓ correctly interprets coefficient of determination

(f) (i) Estimate the maximum tidal current on a day when the tidal range is 4.2 m, and comment on the reliability of this estimate. (3 marks)

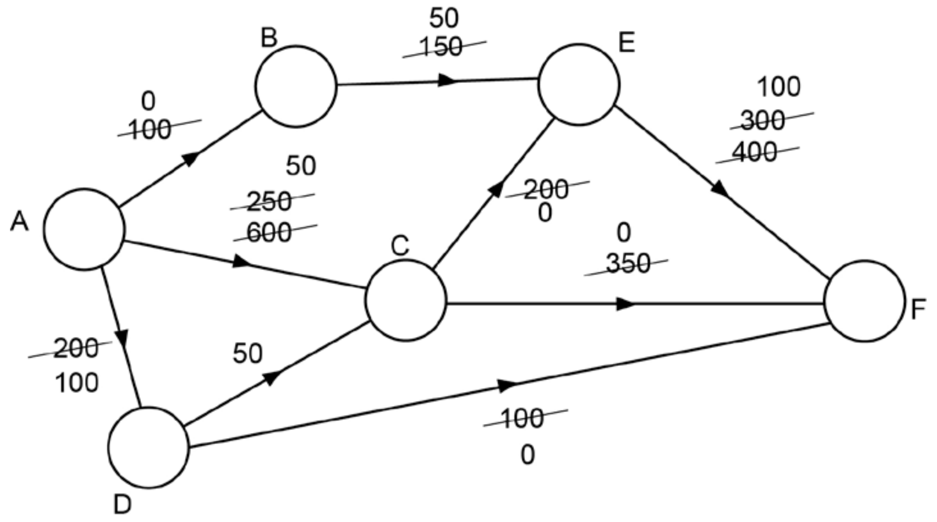
Solution
x = 4.2, y = 48.8 cm per second – reliable as it is interpolation and the correlation coefficient is strong
Specific behaviours
✓ correctly estimates tidal current ✓ correctly states reliability of this value (i.e. interpolation) ✓ correctly states reliability of this value (i.e. strong correlation coefficient)

(ii) It is suggested that the equation found in part (d)(i) could be used to predict the maximum tidal current on a day when the tidal range is 15 m. Comment briefly on the validity of this suggestion. (1 mark)

Solution
(x = 15, y = 221.4 cm per second) not required as part of solution not valid since it is extrapolation
Specific behaviours
✓ correctly states the reason it is not valid

Question 9 **(8 marks)**

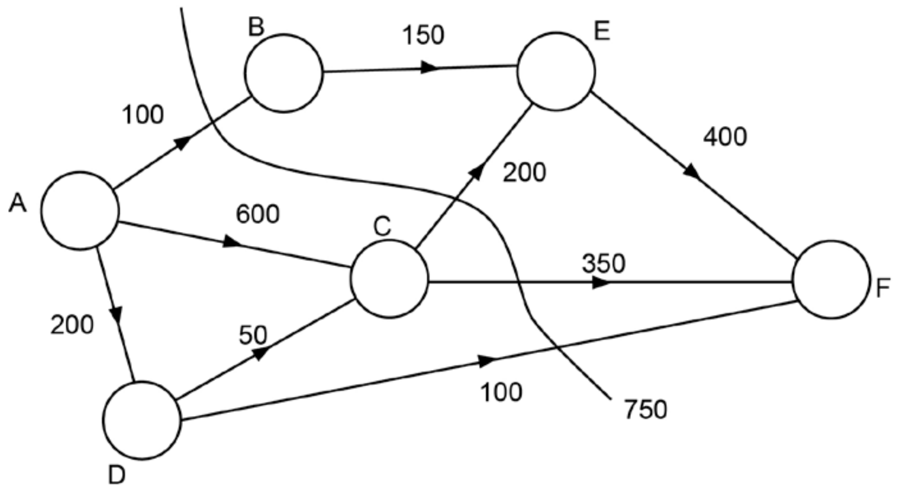
The network below shows the maximum rate of water flow (in litres per minute) through a system of water pipes from a source at A.



(a) What is the maximum amount of water that could be delivered to F, in litres per minute? (List each path used and the corresponding flow). (3 marks)

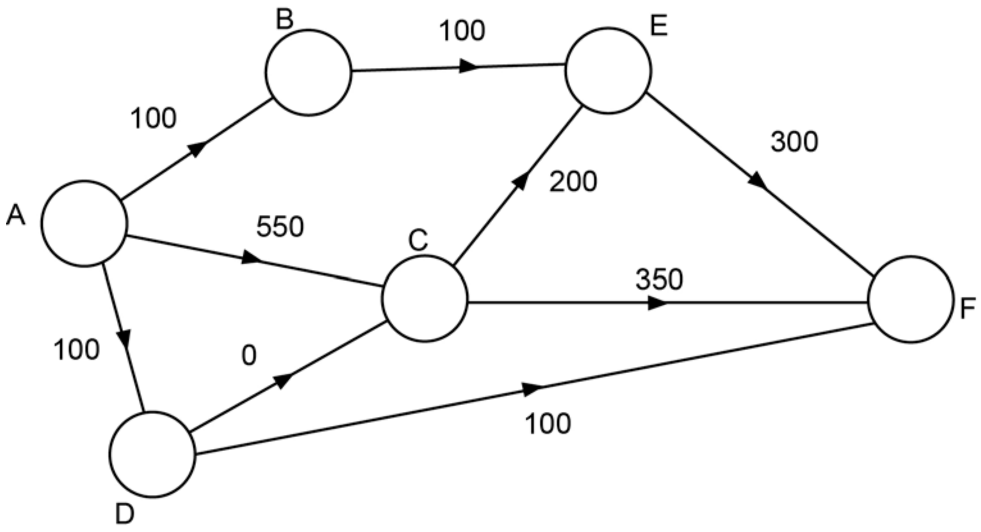
Solution
ABEF – 100 ADF – 100 Note: other paths are possible ACF – 350 ACEF – 200 gives a total of 750 i.e. 750 litres per minute
Specific behaviours
✓ correctly determines at least two paths with correct flow contribution ✓ correctly determines all paths with correct flow contribution ✓ correctly determines the maximum flow

(b) Verify the maximum flow obtained in part (a) by showing a minimum cut on the given network. (1 mark)



Solution
see above graph
Specific behaviours
✓ correctly shows minimum cut

(c) Relabel the network below, showing the flow you would direct along each pipe in order to achieve the maximum flow found in part (a) to point F. (1 mark)



Solution
see above graph
Specific behaviours
✓ correctly labels graph

(d) When the maximum flow occurs from A to F, how much of the water, in litres per minute, passes through C? (1 mark)

Solution
550 litres per minute
Specific behaviours
✓ correctly determines amount of water which passes through C

(e) The flow through C, as calculated in part (d), is reduced to a maximum of 480 litres per minute. In order to maintain the same maximum flow as that obtained in part (a), the capacity of a single pipe (arc) is to be increased by the least amount. Which pipe should be chosen, and by how much should its capacity be increased? (2 marks)

Solution
550 – 480 = 70 Therefore increase arc DF from 100 to 170
Specific behaviours
✓ correctly calculates the increase required ✓ correctly identifies arc DF

Question 10 **(11 marks)**

A school canteen manager recorded the number of ice-creams sold for three weeks. The data are recorded in the table below, together with some calculations.

	Sales day (d)	Ice-cream sales	Weekly mean	Percentage of weekly mean
Monday	1	210		132.9%
Tuesday	2	230		145.6%
Wednesday	3	100	$\boldsymbol{B}$	63.3%
Thursday	4	90		57.0%
Friday	5	160		101.3%
Monday	6	190		128.4%
Tuesday	7	230		155.4%
Wednesday	8	90	148	60.8%
Thursday	9	80		54.1%
Friday	10	150		101.4%
Monday	11	180		126.8%
Tuesday	12	220		154.9%
Wednesday	13	$\boldsymbol{A}$	142	$\boldsymbol{C}$
Thursday	14	70		49.3%
Friday	15	150		105.6%

(a) Determine the values of $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$, giving the value of $\boldsymbol{C}$ correct to one decimal place. (4 marks)

Solution
$\dfrac{180+220+A+70+150}{5}=142 \quad \Rightarrow A=90$ $\dfrac{210+230+100+90+160}{5}=158 \quad \Rightarrow B=158$ $C = \dfrac{90}{142}\times 100 = 63.4\%$
Specific behaviours
✓ correctly calculates the value of A ✓ correctly calculates the value of B ✓ correctly calculates the value of C ✓ correctly rounds the value of C to one decimal place

(b) (i) Use the average percentage method to complete the table below by calculating the seasonal index for Wednesday. (1 mark)

Day	Seasonal index
Monday	129.4% = 1.294
Tuesday	152.0% = 1.520
Wednesday	**0.625 (0.590)**
Thursday	56.8% = 0.568
Friday	102.8% = 1.028

Solution
$\frac{63.4+63.3+60.8}{3} = 62.5\% = 0.625$ **or**   5 – (1.294 + 1.520 + 0.568 + 1.028)   = 0.590 (59.0%)
Specific behaviours
✓ correctly calculates the seasonal index for Wednesday

(ii) Use the seasonal index to determine the deseasonalised number of ice-cream sales for Tuesday of Week Three, correct to the nearest 10. (2 marks)

Solution
$220 \div 1.52 = 144.74 = 140$ (nearest 10)
Specific behaviours
✓ correctly calculates deseasonalised value   ✓ correctly rounds to nearest 10

(c) The equation of the least-squares line used to forecast the deseasonalised number of ice-cream sales is

$$\text{deseasonalised number of ice-creams} = -1.695d + 161.16.$$

(i) Describe the trend in the number of ice-cream sales over time. (1 mark)

Solution
decreasing over time (negative gradient)
Specific behaviours
✓ correctly states trend is decreasing

(ii) Predict the **actual** number of ice-cream sales for Friday of Week Four. (3 marks)

Solution
Using least-squares equation with $d = 20$ deseasonalised number of ice-creams = $-1.695(20)+161.16 = 127.26$ Therefore actual number of ice-creams = $127.26 \times 1.028 = 130.82 \approx 130$ or 131
Specific behaviours
✓ correctly states the value of d ✓ correctly calculates the deseasonalised number of ice-creams ✓ correctly multiplies by seasonal index

Question 11 **(10 marks)**

The data in the table below, taken from those surveyed by the Australian Bureau of Statistics, show estimates for the number of persons 15 years and over who participated in sport and physical recreation in Western Australia, Tasmania and Victoria.

Participation in Sport and Physical Recreation, 2013–14

	Persons Participating							
Western Australia	**15–17 years**	**18–24 years**	**25–34 years**	**35–44 years**	**45–54 years**	**55–64 years**	**65 years+**	**Total**
Males ('000)	43.3	80.9	149.1	113.7	116.6	85.6	74.0	663.2
% of males	71.6	67.6	70.9	66.4	68.2	63.8	51.4	65.6
Females ('000)	30.3	84.3	131.8	111.4	107.8	82.8	76.5	A
% of females	61.6	70.6	67.3	61.5	64.2	60.7	48.1	61.9
Total ('000)	73.6	165.2	280.9	B	224.4	168.4	150.5	1288.1
Tasmania	**15–17 years**	**18–24 years**	**25–34 years**	**35–44 years**	**45–54 years**	**55–64 years**	**65 years+**	**Total**
Males ('000)	7.6	16.5	23.2	23.6	25.4	19.5	23.5	139.3
% of males	73.4	71.4	81.3	77.7	72.7	55.7	57.4	68.5
Females ('000)	7.8	14.1	22.0	22.3	27.2	22.0	20.7	136.1
% of females	79.0	69.0	74.3	70.0	74.3	62.9	46.7	65.5
Total ('000)	15.4	30.6	45.2	45.9	52.6	41.5	44.2	275.4
Victoria	**15–17 years**	**18–24 years**	**25–34 years**	**35–44 years**	**45–54 years**	**55–64 years**	**65 years+**	**Total**
Males ('000)	74.5	202.3	267.9	268.7	218.2	192.6	164.1	1388.3
% of males	69.4	65.1	62.4	67.9	59.7	60.1	44.0	61.7
Females ('000)	79.9	159.7	276.3	296.7	240.5	184.6	211.9	1449.6
% of females	68.4	62.5	63.7	71.9	62.6	55.8	49.8	61.4
Total ('000)	154.4	362.0	544.2	565.4	458.7	377.2	376.0	2837.9

Use the information in the table to answer the following questions.

(a) Determine the values of A and B for the Western Australian data. (2 marks)

Solution
$A = 1288.1 - 663.2 = 624.9$ $B = 113.7 + 111.4 = 225.1$
Specific behaviours
✓ correctly calculates the value of A ✓ correctly calculates the value of B

(b) Which State, age and gender category had the highest rate of participation in sport and physical recreation? (3 marks)

Solution
Males in the *25–34* years category in Tasmania
Specific behaviours
✓ determines the correct gender ✓ determines the correct age group ✓ determines the correct state

(c) Which State had a higher percentage of females than males participating in sport and physical recreation in the *35–44 years* category? (1 mark)

Solution
Victoria
Specific behaviours
✓ determines the correct state

(d) Compare and comment on the participation rates in the *55–64 years* category with those in the younger age groups. (2 marks)

Solution
This group had the lowest participation rates in all except males in the 45–54 years in Victoria.
Specific behaviours
✓ concludes that this age group had almost the lowest participation rates ✓ correctly states the only exception

(e) Determine the total number of females in Victoria who were surveyed. (2 marks)

Solution
$\frac{61.4}{100} \times \text{Total} = 1449.6 \Rightarrow \text{Total} = 2360.912 = 2\,360\,912$
Specific behaviours
✓ uses correct formula to calculate total ✓ correctly gives the answer in millions

Question 12 **(11 marks)**

Thomas has borrowed $16 000 from a bank at a reducible interest rate of 18% per annum with interest accrued and repayments made monthly. Standard repayments are set at $500 per month.

The table below shows the progress of the loan for the first six months. All values have been rounded to the nearest cent.

Month	Amount owing at beginning of month	Interest for the month	Repayment	Amount owing at end of month
1	16 000.00	240.00	500.00	15 740.00
2	15 740.00	236.10	500.00	15 476.10
3	15 476.10	232.14	500.00	15 208.24
4	15 208.24	228.13	500.00	14 936.37
5	14 936.37	224.04	500.00	14 660.41
6	14 660.41	$\boldsymbol{A}$	500.00	$\boldsymbol{B}$

(a) What is the monthly interest rate? (1 mark)

Solution
$18 \div 12 = 1.5\%$ per month
Specific behaviours
✓ calculates the correct monthly interest rate

(b) Determine the values of $\boldsymbol{A}$ and $\boldsymbol{B}$. (2 marks)

Solution
$\boldsymbol{A}$ = $219.91 $\boldsymbol{B}$ = $14 380.32
Specific behaviours
✓ correctly calculates the value of $\boldsymbol{A}$ ✓ correctly calculates the value of $\boldsymbol{B}$

(c) Determine the length of time it will take Thomas to pay off the loan. (1 mark)

Solution
44 months
Specific behaviours
✓ determines correct value

(d) Determine the total amount Thomas pays over the duration of the loan. (3 marks)

Solution
amount owing at the end of 43rd month = \$454.08 interest for 44th month = 0.015 × 454.08 = \$6.81 total amount paid = (43 × 500) + (454.08 + 6.81) = \$21 960.89
Specific behaviours
✓ correctly determines amount owing at end of 43rd month ✓ correctly determines interest for the 44th month ✓ correctly determines total amount paid

(e) The bank suggests that Thomas need only make repayments of \$240 per month. Describe how this would affect the length of time and total amount he pays over the duration of the loan. (2 marks)

Solution
He would never pay off the loan. Using this suggestion the repayment would = the interest per month, so he only ever pays interest and the balance never decreases.
Specific behaviours
✓ correctly states he will never pay off the loan ✓ correctly justifies why he will never pay off the loan

(f) After listening to advice, Thomas decides that he wants to pay off the loan completely in two years, making equal payments each month over that time. Determine the amount of each repayment he will need to make in order to make this happen (correct to the nearest cent). (2 marks)

Solution
Using the finance package on the calculator with N = 24, I = 18, PV = 16 000, FV = 0, P/Y = 12, C/Y = 12 gives PMT = 798.7856 The payment required per month is \$798.79
Specific behaviours
✓ correctly calculates PMT ✓ correctly rounds up to \$798.79

Question 13 **(6 marks)**

Simon has $5000 that he wants to invest for a period of time without touching it.

(a) If he chooses to invest this money in an account earning compound interest at the rate of 6.5% per annum, determine the:

(i) value of his investment after three years, if interest is paid annually. (1 mark)

Solution
$A = 5000(1.065)^3 = 6039.75$ Value = $6039.75
Specific behaviours
✓ correctly calculates the value of the investment after three years

(ii) time required for him to double his investment, if interest is paid monthly. (2 marks)

Solution
$10000 = 5000\left(1+\frac{0.065}{12}\right)^{12n} \Rightarrow n = 10.69266$ years Therefore time required to double investment is 10.7 years or 10 years 9 months.
Specific behaviours
✓ uses correct interest rate ✓ correctly calculates time required

(b) Simon is currently deciding between two options and wishes to compare them.

Option A: Invest the $5000 in an account earning compound interest at the rate of 5.5% per annum, with interest paid monthly.

Option B: Invest the $5000 in an account earning compound interest at the rate of 5.4% per annum, with interest paid daily.

He decides to calculate the effective annual rate of interest for each option, in order to compare the possible investments. He determines that Option A has an effective annual rate of interest of 5.64%, correct to two decimal places.

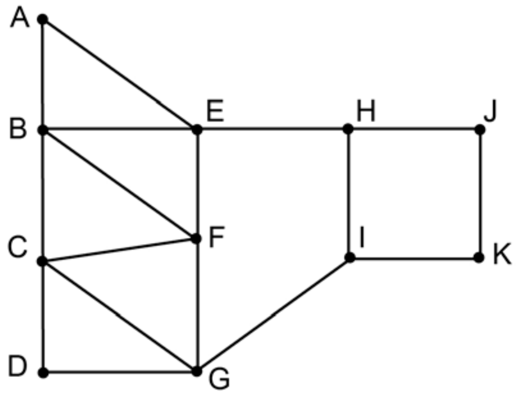
Calculate the effective annual rate of interest for Option B, correct to two decimal places, and hence decide on the better option for Simon. (3 marks)

Solution
Option B: effective interest rate = $\left(1+\frac{0.054}{365}\right)^{365} - 1 = 0.05548 = 5.55\%$ Therefore Option A is better since 5.64% is higher
Specific behaviours
✓ correctly substitutes 0.054 and 365 into formula ✓ correctly determines the effective rate of interest ✓ correctly states that Option A is better

Question 14 **(7 marks)**

Therese, a mathematics student at Trinity College, Dublin was employed as a guide for a cultural tour of Dublin. She decided to use graph theory to plan the walking tour.

Below is a network she constructed in which the:
- vertices represent the points of interest to be visited, and
- edges represent the most direct route between adjacent vertices.



(a) Use Euler's formula to verify the network is connected. (2 marks)

Solution
v = 11, e = 17, f = 8 Using Euler's formula $v+f-e=2$, 11 + 8 – 17 = 2, which verifies that the network is connected.
Specific behaviours
✓ correctly states the values of v, e and f ✓ correctly uses Euler's formula to verify that the network is connected

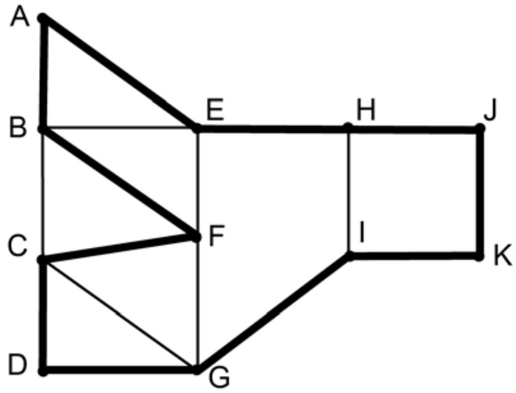
(b) Therese planned to take the group on a closed walk. Explain the meaning of a closed walk. (1 mark)

Solution
A closed walk is a walk which starts and ends at the same vertex.
Specific behaviours
✓ correctly explains the meaning of a closed walk

(c) She also stated that the walk would qualify as a Hamiltonian cycle. State the **two** properties that makes the walk a Hamiltonian cycle. (2 marks)

Solution
Hamiltonian cycle is a walk that: • passes through all vertices only once (except the start and finish vertex) • starts and finishes the walk at the same vertex (a closed path).
Specific behaviours
✓ correctly explains property one ✓ correctly explains property two

(d) Given that the walk started at G (Trinity College, Dublin), mark the Hamiltonian cycle on the network below. (2 marks)



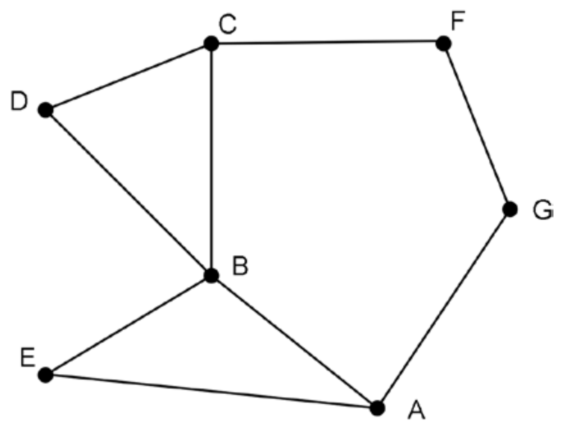
Solution
see graph above
Specific behaviours
✓ correctly draws a closed walk through G ✓ correctly visits all vertices once

Question 15 **(8 marks)**

An express bus service runs between seven adjacent shopping centres in the city. Below is an adjacency matrix of the seven shopping centres, A to G.

	A	B	C	D	E	F	G
A	0	1	0	0	1	0	1
B	1	0	1	1	1	0	0
C	0	1	0	1	0	1	0
D	0	1	1	0	0	0	0
E	1	1	0	0	0	0	0
F	0	0	1	0	0	0	1
G	1	0	0	0	0	1	0

(a) Draw the network diagram associated with the adjacency matrix, assuming the arcs are undirected. (3 marks)



Solution
see graph above
Specific behaviours
✓ correctly draws at least three edges ✓ correctly draws at least six edges ✓ correctly draws all nine edges

(b) The buses only run between adjacent shopping centres. However, a passenger can buy a multi-stage ticket at any shopping centre. A one-stage ticket means a passenger can travel from one shopping centre to an adjacent shopping centre, such as:

- A $\rightarrow$ B or A $\rightarrow$ E etc.

Similarly for a two-stage ticket:

- A $\rightarrow$ B $\rightarrow$ A which is a return journey
- A $\rightarrow$ B $\rightarrow$ C which is a one-way journey.

(i) What feature on the adjacency matrix tells us that the buses run in both directions between adjacent shopping centres? (2 marks)

Solution
The matrix is symmetrical about the leading diagonal.
Specific behaviours
✓ correctly states the symmetry ✓ correctly refers to the leading diagonal

(ii) How many different one-stage journeys are available from shopping centre B? (1 mark)

Solution
4
Specific behaviours
✓ correctly states the number of one-stage journeys

(iii) List **all** the different two-stage, one-way journeys available from shopping centre B. (2 marks)

Solution
BCF, BCD, BDC, BEA, BAE, BAG
Specific behaviours
✓ correctly gives at least three journeys ✓ correctly gives all six journeys

Question 16 **(13 marks)**

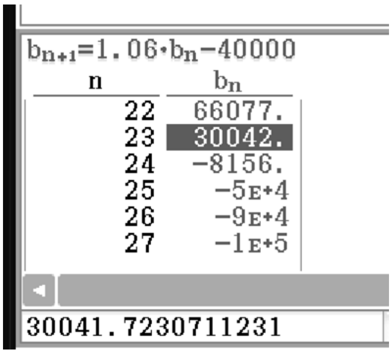
(a) Alex is about to retire and is planning to take an annuity from his pension fund. He sets up the pension fund on his 65th birthday with \$500 000 and he estimates the fund can generate a growth rate of 6% per year. He plans to start withdrawing an annuity of \$40 000 starting on his following birthday.

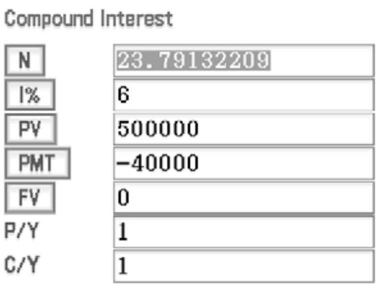
(i) Write a recurrence relation to calculate the total amount in the fund directly after each withdrawal. (3 marks)

Solution
$T_{n+1} = 1.06T_n - 40000 \quad T_0 = 500000$
Specific behaviours
✓ states the correct growth rate ✓ includes the correct annuity ✓ defines T_0 correctly

(ii) For how many years will Alex be able to receive his annuity of \$40 000? (2 marks)

Solution
Number of years is 23 since in the 24th year there would not be enough money for a withdrawal of \$40 000





Specific behaviours
✓ gives a value of time as a decimal ✓ states the correct number of years

(iii) Assuming that all other conditions are the same, explain what would happen if Alex decided to withdraw \$30 000 per year instead of \$40 000 per year. (2 marks)

Solution
Since 6% of \$500 000 is \$30 000 the principal would not decrease.
Specific behaviours
✓ correctly calculates 6% of \$500 000 ✓ states that the principal would not decrease

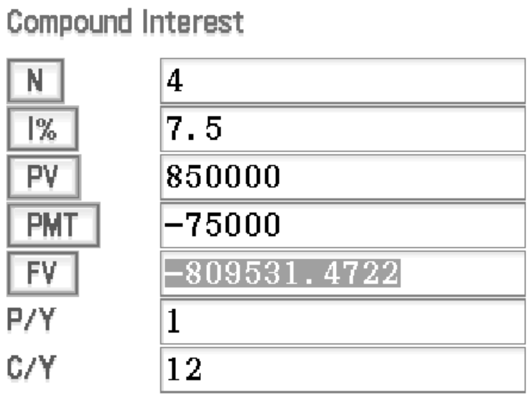
(b) Abbey sets up her pension fund on July 1 2016 with a principal of $850 000. The fund guarantees an annual growth rate of 7.5% compounded monthly and she plans to take an annuity of $75 000 each year on July 1, starting in 2017.

(i) Calculate the balance in the fund after the annuity is withdrawn in July 2020. (2 marks)

Solution

N = 4 for July 2020

Balance after withdrawal in July 2020
is $809 531.47



Specific behaviours

✓ states correct value of N
✓ correctly calculates balance in July 2020

The investment fund revised its annual interest rate to 9% compounded monthly on July 1 2020 guaranteed for the period to July 2025 and Abbey continued withdrawing $75 000 as usual.

(ii) Calculate the balance in the fund after a withdrawal is made on July 1 2025. (2 marks)

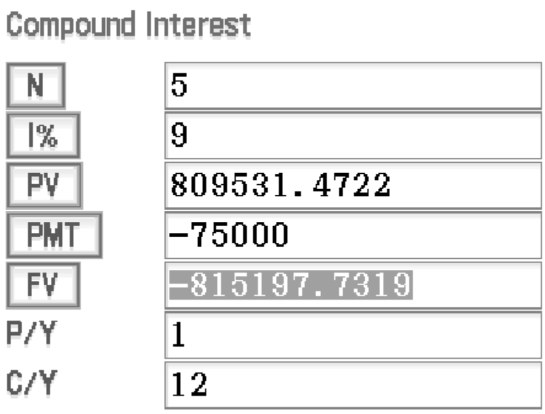
Solution

Initial balance =$809 531.47

$2021 \Rightarrow N = 1$

Balance in 2025 occurs when $N = 5$

New balance = $815 197.73 in July 2025

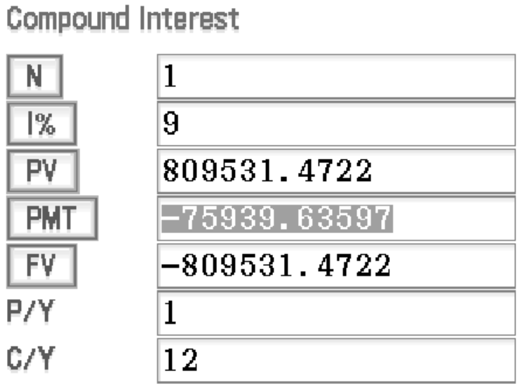


Specific behaviours

✓ states correct initial balance
✓ correctly calculates balance in July 2025

(iii) Calculate, to the nearest $100, the maximum amount Abbey could withdraw annually, starting in 2020, without decreasing her balance. (2 marks)

Solution
Initial balance = $809 531.47 Future value = $809 531.47 Abbey could withdraw $75 939.63 = $75 900 (nearest $100)



Specific behaviours
✓ correctly calculates amount Abbey could withdraw ✓ correctly rounds to nearest $100

ACKNOWLEDGEMENTS

Question 11 Data source: Australian Bureau of Statistics. (2015, February 18). *4177.0—Participation in sport and physical recreation, Australia, 2013–14*. Retrieved April, 2016, from www.abs.gov.au/AUSSTATS/abs@.nsf/DetailsPage/4177.02013-14?OpenDocument