



MATHEMATICS SPECIALIST ATAR COURSE

FORMULA SHEET

2017

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This document is valid for teaching and examining until 31 December 2017.

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Differentiation and integration

$\frac{d}{dx} (x^n) = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$			
$\frac{d}{dx} (e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$			
$\frac{d}{dx} (\ln x) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln x + c$			
$\frac{d}{dx} (\ln f(x)) = \frac{f'(x)}{f(x)}$	$\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c$			
$\frac{d}{dx} (\sin f(x)) = f'(x) \cos (f(x))$	$\int \sin (ax) dx = -\frac{1}{a} \cos (ax) + c$			
$\frac{d}{dx} (\cos f(x)) = -f'(x) \sin (f(x))$	$\int \cos (ax) dx = \frac{1}{a} \sin (ax) + c$			
$\frac{d}{dx} (\tan f(x)) = f'(x) \sec^2 (f(x)) = \frac{f'(x)}{\cos^2 f(x)}$	$\int \sec^2 (ax) dx = \frac{1}{a} \tan (ax) + c$			
Product rule	<table style="width: 100%; border: none;"> <tr> <td style="width: 50%; border: none;"> If $y = uv$ then $\frac{d}{dx} (uv) = u \frac{dv}{dx} + \frac{du}{dx} v$ </td> <td style="width: 10%; border: none; text-align: center;">or</td> <td style="width: 40%; border: none;"> If $y = f(x) g(x)$ then $y' = f'(x) g(x) + f(x) g'(x)$ </td> </tr> </table>	If $y = uv$ then $\frac{d}{dx} (uv) = u \frac{dv}{dx} + \frac{du}{dx} v$	or	If $y = f(x) g(x)$ then $y' = f'(x) g(x) + f(x) g'(x)$
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Quotient rule	<table style="width: 100%; border: none;"> <tr> <td style="width: 50%; border: none;"> If $y = \frac{u}{v}$ then $\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ </td> <td style="width: 10%; border: none; text-align: center;">or</td> <td style="width: 40%; border: none;"> If $y = \frac{f(x)}{g(x)}$ then $y' = \frac{f'(x) g(x) - f(x) g'(x)}{(g(x))^2}$ </td> </tr> </table>	If $y = \frac{u}{v}$ then $\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$	or	If $y = \frac{f(x)}{g(x)}$ then $y' = \frac{f'(x) g(x) - f(x) g'(x)}{(g(x))^2}$
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Chain rule	<table style="width: 100%; border: none;"> <tr> <td style="width: 50%; border: none;"> If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ </td> <td style="width: 10%; border: none; text-align: center;">or</td> <td style="width: 40%; border: none;"> If $y = f(g(x))$ then $y' = f'(g(x)) g'(x)$ </td> </tr> </table>	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$	or	If $y = f(g(x))$ then $y' = f'(g(x)) g'(x)$
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Fundamental theorem	<table style="width: 100%; border: none;"> <tr> <td style="width: 50%; border: none;"> $\frac{d}{dx} \left(\int_a^x f(t) dt \right) = f(x)$ </td> <td style="width: 10%; border: none; text-align: center;">and</td> <td style="width: 40%; border: none;"> $\int_a^b f'(x) dx = f(b) - f(a)$ </td> </tr> </table>	$\frac{d}{dx} \left(\int_a^x f(t) dt \right) = f(x)$	and	$\int_a^b f'(x) dx = f(b) - f(a)$
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Applications of calculus

Growth and decay	
Exponential equation	$\frac{dP}{dt} = kP \Leftrightarrow P = P_0 e^{kt}$
Logistic equation	$\frac{dP}{dt} = rP(k - P) \Leftrightarrow P = \frac{kP_0}{P_0 + (k - P_0)e^{-rkt}}$
Volumes of solids of revolution	
About the x -axis	$V = \pi \int_a^b [f(x)]^2 dx$
About the y -axis	$V = \pi \int_c^d [f(y)]^2 dy$
Simple harmonic motion	
<p>If $\frac{d^2x}{dt^2} = -k^2x$ then $x = A \sin(kt + \alpha)$ or $x = A \cos(kt + \beta)$</p> <p>where A is the amplitude, α and β are phase angles, v is the velocity and x is the displacement</p>	
$v^2 = k^2(A^2 - x^2)$ Period: $T = \frac{2\pi}{k}$ Frequency: $f = \frac{1}{T}$	
Incremental formula	$\delta y \approx \frac{dy}{dx} \times \delta x$
Acceleration	$\frac{dv}{dt}$ or $v \frac{dv}{dx}$ or $\frac{d}{dx} \left(\frac{1}{2} v^2 \right)$

Functions

Quadratic function	If $f(x) = ax^2 + bx + c$ and $f(x) = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Absolute value function	$ x = \begin{cases} x, & \text{for } x \geq 0 \\ -x, & \text{for } x < 0 \end{cases}$

Statistical inference

Confidence interval for the mean of the population:	$\bar{X} - z \frac{s}{\sqrt{n}} \leq \mu \leq \bar{X} + z \frac{s}{\sqrt{n}}$
Sample size:	$n = \left(\frac{z \times s}{d} \right)^2$

Mensuration

Parallelogram	$A = bh$	
Triangle	$A = \frac{1}{2}bh$ or $A = \frac{1}{2}ab \sin C$	
Trapezium	$A = \frac{1}{2}(a + b)h$	
Circle	$A = \pi r^2$ and $C = 2\pi r = \pi d$	
Prism	$V = Ah$, where A is the area of the cross section	
Pyramid	$V = \frac{1}{3}Ah$, where A is the area of the cross section	
Cylinder	$V = \pi r^2 h$	$S = 2\pi r h + \pi r^2 h$
Cone	$V = \frac{1}{3}\pi r^2 h$	$S = \pi r s + \pi r^2$, where s is the slant height
Sphere	$V = \frac{4}{3}\pi r^3$	$S = 4\pi r^2$

Vectors in 3D

Magnitude	$ (a_1, a_2, a_3) = \sqrt{a_1^2 + a_2^2 + a_3^2}$
Dot product	$\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \mathbf{b} \cos\theta = a_1b_1 + a_2b_2 + a_3b_3$
Cross product	$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$
Equation of a line	One point and direction $\mathbf{r} = \mathbf{a} + \lambda\mathbf{u}$
	Two points A and B $\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$
Equation of a plane	$\mathbf{r} = \mathbf{a} + \lambda\mathbf{u}_1 + \mu\mathbf{u}_2$ or $\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$
Equation of a sphere	$ \mathbf{r} - \mathbf{d} = r$ or $(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$
Cartesian equation of a line	$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3}$
Cartesian equation of a plane	$ax + by + cz = d$
Parametric equation of a line	$x = a_1 + \lambda b_1 \dots \dots (1)$ $y = a_2 + \lambda b_2 \dots \dots (2)$ $z = a_3 + \lambda b_3 \dots \dots (3)$

Complex numbers

Cartesian form	
$z = a + bi$	$\bar{z} = a - bi$
$\text{Mod}(z) = z = \sqrt{a^2 + b^2} = r$	$\text{Arg}(z) = \theta, \quad \tan \theta = \frac{b}{a}, \quad -\pi < \theta \leq \pi$
$ z_1 z_2 = z_1 z_2 $	$\left \frac{z_1}{z_2} \right = \frac{ z_1 }{ z_2 }$
$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$	$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$
$z\bar{z} = z ^2$	$z^{-1} = \frac{1}{z} = \frac{\bar{z}}{ z ^2}$
$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$	$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$
Polar form	
$z = a + bi = r(\cos \theta + i \sin \theta) = r \text{cis } \theta$	$\bar{z} = r \text{cis } (-\theta)$
$z_1 z_2 = r_1 r_2 \text{cis } (\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \text{cis } (\theta_1 - \theta_2)$
$\text{cis}(\theta_1 + \theta_2) = \text{cis } \theta_1 \text{cis } \theta_2$	$\text{cis}(-\theta) = \frac{1}{\text{cis } \theta}$
De Moivre's theorem	
$z^n = z ^n \text{cis } (n\theta)$	$(\text{cis } \theta)^n = \cos n\theta + i \sin n\theta$
$z^{\frac{1}{q}} = r^{\frac{1}{q}} \left(\cos \frac{\theta + 2\pi k}{q} + i \sin \frac{\theta + 2\pi k}{q} \right), \quad \text{for } k \text{ an integer}$	

Trigonometry

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	Length of arc = $r\theta$
$a^2 = b^2 + c^2 - 2bc \cos A$	Area of segment = $\frac{1}{2} r^2 (\theta - \sin \theta)$
$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$	Area of sector = $\frac{1}{2} r^2 \theta$
Identities	
$\cos^2 x + \sin^2 x = 1$	$1 + \tan^2 x = \sec^2 x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\cos 2x = \cos^2 x - \sin^2 x$ $= 2 \cos^2 x - 1$ $= 1 - 2 \sin^2 x$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\sin 2x = 2 \sin x \cos x$
$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$
$\cos A \cos B = \frac{1}{2} (\cos(A - B) + \cos(A + B))$	$\sin A \cos B = \frac{1}{2} (\sin(A + B) + \sin(A - B))$
$\sin A \sin B = \frac{1}{2} (\cos(A - B) - \cos(A + B))$	$\cos A \sin B = \frac{1}{2} (\sin(A + B) - \sin(A - B))$

Note: Any additional formulas identified by the examination panel as necessary will be included in the body of the particular question.

