Government of Western Australia
School Curriculum and Standards Authority

Sample Assessment Tasks

Mathematics Methods

ATAR Year 12

Acknowledgement of Country

Kaya. The School Curriculum and Standards Authority (the Authority) acknowledges that our offices are on Whadjuk Noongar boodjar and that we deliver our services on the country of many traditional custodians and language groups throughout Western Australia. The Authority acknowledges the traditional custodians throughout Western Australia and their continuing connection to land, waters and community. We offer our respect to Elders past and present.

Sample assessment task

Mathematics Methods – ATAR Year 12

Task 1 – Unit 3

Assessment type	Investigation
Conditions	The investigation will be completed over one week, with an authentication task at the end of this period. Students will be encouraged to work independently to complete the task and may use any appropriate technology. Note: while the Authority provides sample assessment tasks for guidance, it is the expectation of the Authority that teachers will develop tasks customised to reflect their school's context and the needs of the student cohort. This resource is available on a public website and use of the resource without modification may affect the integrity of the assessment.
Task weighting	10% of the school mark for this pair of units

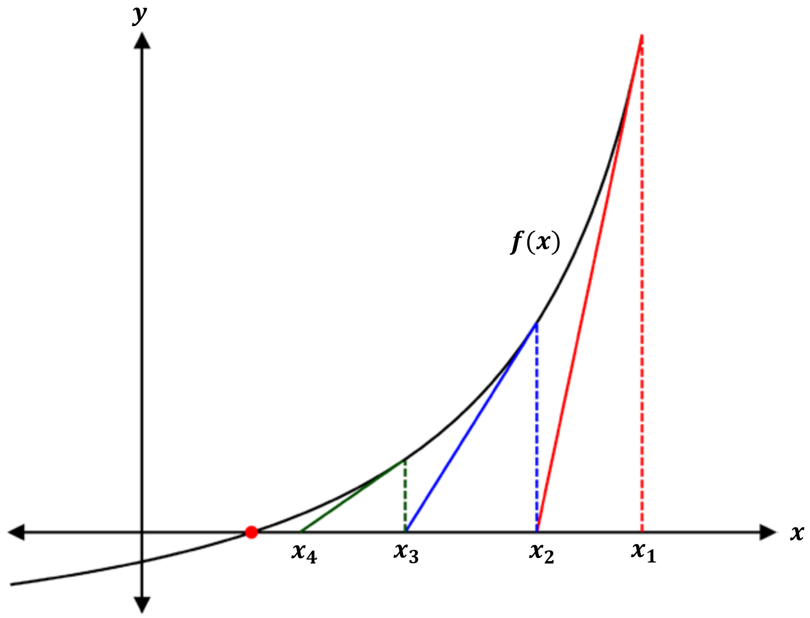
An iterative process **(50 marks)**

Background information:

You are designing calculator software that requires you to implement a process that involves determining the roots of an equation. You know that many computers and calculators use a method of approximation and refinement to do this which involves a series of calculations called iterations to zero in on the solution. You need to research this process further.

The diagram below shows how this process projects successive tangent lines onto the x-axis, beginning with the first approximation (x_1) to find a closer approximation (x_2) to the true solution. The process is repeated (x_3 and so on), until a solution is found to an acceptable level of accuracy.

The iterative process uses the output of one iteration (calculating an approximation to the solution of $f(x) = 0$) as the input for the next consecutive iteration.



You are required to write a report that clearly communicates your research findings regarding the use of this method to find the root/s of a function. Use appropriate mathematical statements, through the mathematical thinking process, to support your work.

Your report should be no more than six one-sided A4 pages long and should include the following:

- an **introduction**, that clearly defines the purpose of the task, identifies key information, any assumptions made and an outline of your research strategy (6 marks)
- **evidence of the application of mathematical model and strategies**, including calculations and results using appropriate representations (graphs, tables, formulae etc.) (19 marks)
- your research communicated in a systematic and concise manner, including **analysis and interpretation** in the context of the problem and consideration of the reasonableness and limitations of the results (14 marks)
- use of correct mathematical conventions, symbols and terminology. (4 marks)

The format of the report may be written or digital.

Consider the following pointers to help with this task.

- Explain how this method can be developed into an algorithm to determine the next approximation for the iterative process.
- You may investigate the process through a variety of functions but must include:
  - the use of this algorithm to determine a root/s using at least three different types of functions encountered in this course
  - comparison results for different starting values, for each function you choose
  - consideration of the number of iterations needed to obtain a specific level of accuracy.
- Use diagrams and provide justification throughout your investigation:
  - Provide the conditions required for the process to have the best possible chance of working.
  - Explain the importance of the first approximation and provide a list of different situations, with appropriate examples and justification, in which the choice of the first approximation for this iterative process will not result in a required true solution.
  - Is there an example of a function in which there is **no** choice for the first approximation that will result in the process converging to the required true solution?

Investigation authentication

This is an activity designed to authenticate the research students have completed regarding the iteration process they have investigated.

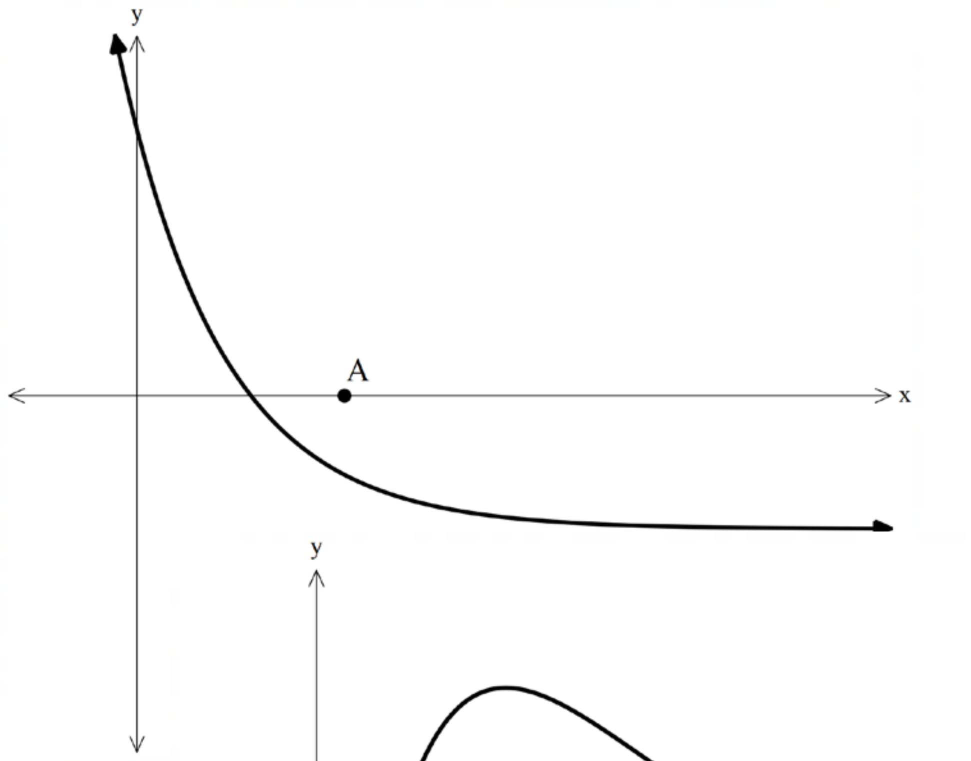
Conditions Time for the task: 15 minutes
In class, calculator permitted
No notes

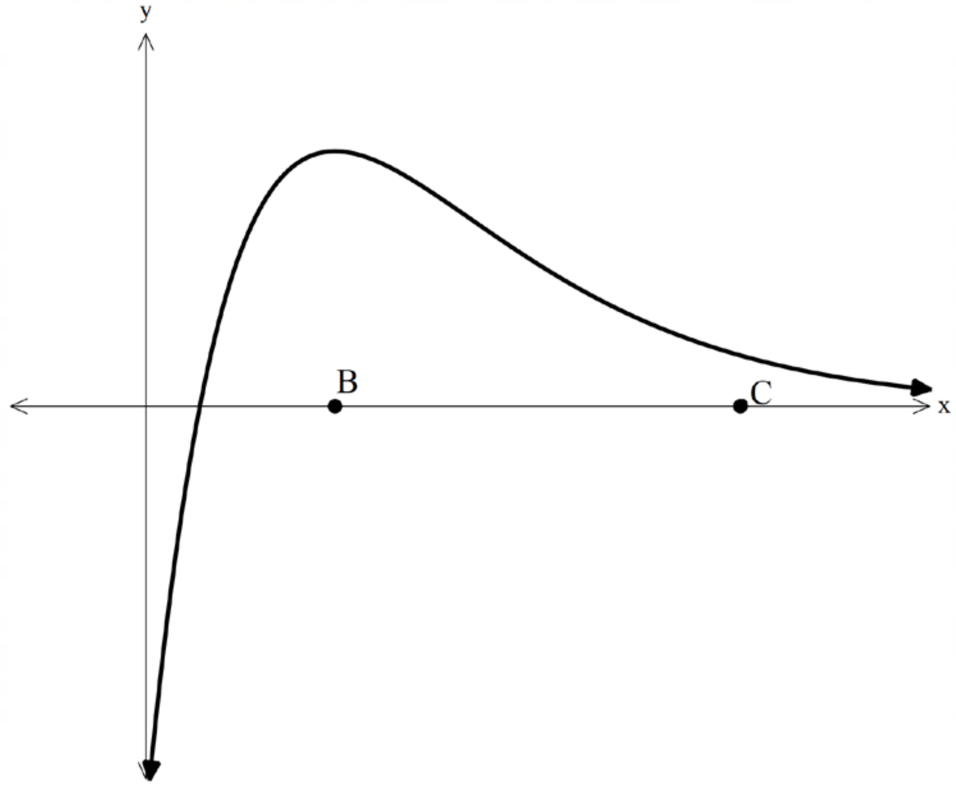
Question **(7 marks)**

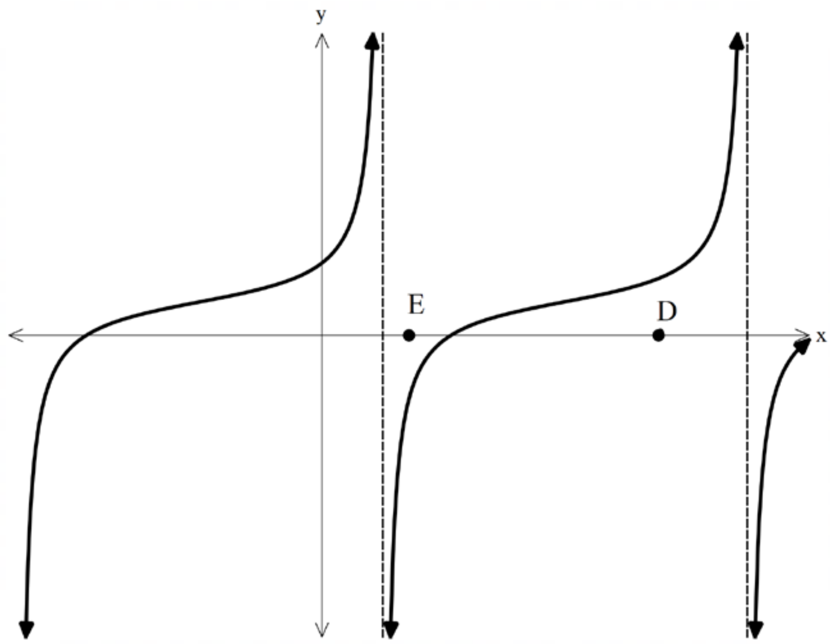
Identify whether the iterative process researched can determine the root of the functions provided when each of following labelled points are chosen as the first approximation for the root. Explain the effect in each case.

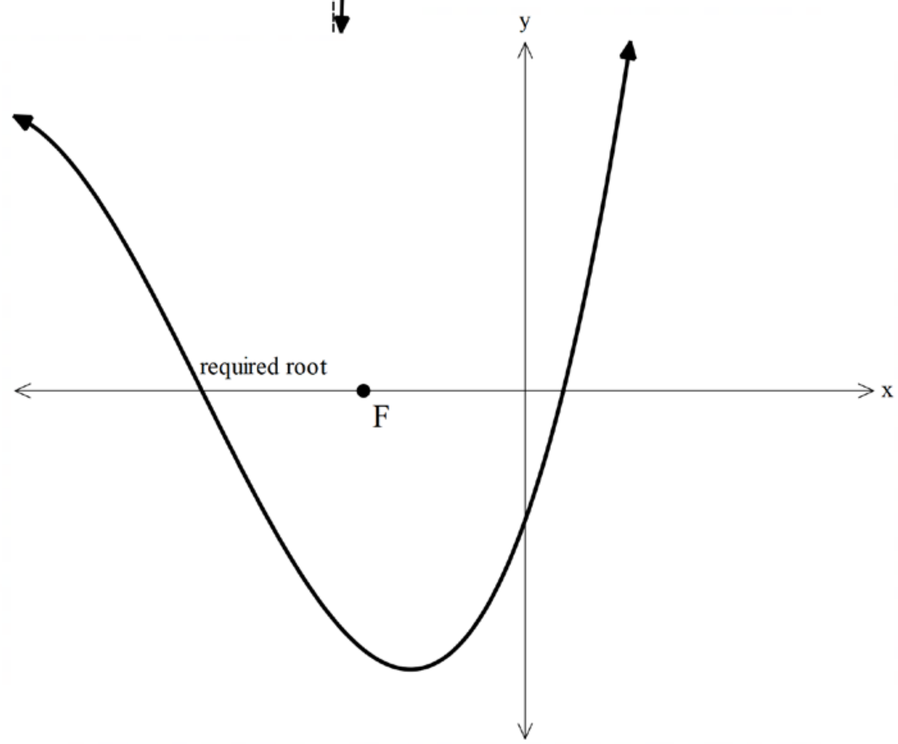
Point	Explanation of what will occur if the point is chosen as the first approximation for the root of the function
A	
B	
C	
D	
E	

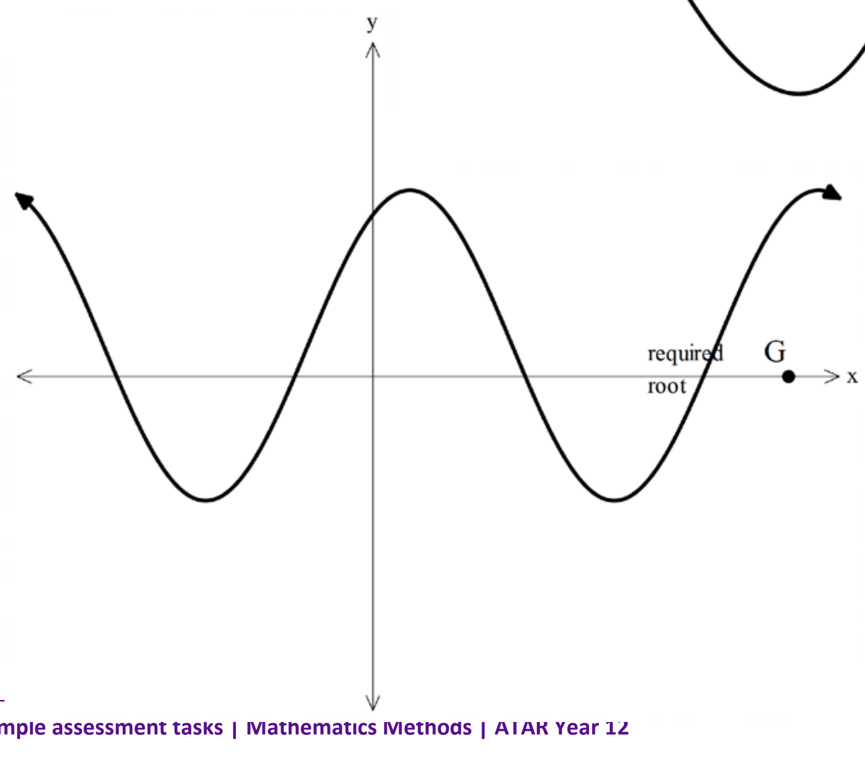
F	
G	






y
E
D
x


y
required root
F
x


y
required root
G
x

Marking key for sample assessment task 1 – Unit 3

This marking key may be adjusted based on the conditions of the task.

Introduction **(6 marks)**

Behaviours	Marks
Succinctly writes a general introduction that accurately summarises all aspects of the investigation	1–2
Identifies and documents the iterative process	1
States three suitable functions to use	1
Identifies assumptions made, e.g. function does in fact have a root; function must be differentiable; function must be continuous	2
Subtotal	**/6**

Application of the mathematical model and strategies **(19 marks)**

Behaviours	Marks
Clearly identifies the need for the use of calculus	1
Shows research of the algorithm	1
Shows development of the algorithm using gradients and equation of a line	2
Shows accurate use of the algorithm to determine a root from each of the functions used	6
Shows different starting values and discusses the implications/differences	6
Specifies a level of accuracy and discusses/compares the number of iterations for this to be achieved	3
Subtotal	**/19**

Analysis and interpretation **(14 marks)**

Behaviours	Marks
Discusses the limitations of the process for at least four different cases by providing at least one example in which the method breaks down, such as • starting value (or a successive value) meets stationary point • starting value (or a successive value) implies divergence • starting value produces loop/cycle • specific function that regardless of starting value, will not converge to solution	8
Provides mathematical justification for each limitation as listed above	4
Discusses the reasonableness of using the method with reference to assumptions made	2
Subtotal	**/14**

Use of mathematical conventions, symbols and terminology **(4 marks)**

Behaviours	Marks
Correctly labels and displays graphs/tables appropriately (sometimes = 1 mark, consistently = 2 marks)	1–2
Uses mathematical language throughout the investigation	1
Presents investigation in a systematic and concise way	1
Subtotal	**/4**

Authentication question **(7 marks)**

Behaviours	Marks
A: Identifies that when using iteration process with start point A, convergence to required solution will occur	1
B: Identifies that when using iteration process with start point B, no root can be obtained as tangent is horizontal	1
C: Identifies that when using iteration process with start point C, divergence will occur	1
D: Identifies that when using iteration process with start point D, convergence to required solution will occur	1
E: Identifies that when using iteration process with start point E, convergence to required solution will occur	1
F: Identifies that when using iteration process for start point F, loop or cycle will occur and hence root is unable to be determined	1
G: Identifies that when using iteration process with start point G, convergence to required solution will not occur and will jump to another root	1
Subtotal	**/7**
Total	**/50**

Sample assessment task

Mathematics Methods – ATAR Year 12

Task 3

Assessment type	Response
Conditions	Total marks: 45 marks • Section One – 23 marks • Section Two – 22 marks Time for the task: up to 50 minutes In class, under test conditions
Materials required	Section One: Calculator-free, standard writing equipment Section Two: Calculator-assumed (calculator to be provided by the student)
Other materials allowed	Drawing templates, one page of notes for Section Two
Task weighting	8% of the school mark for this pair of units

Section One: Calculator-free **(23 marks)**

Question 1 **(6 marks)**

(a) Evaluate $\frac{dy}{dx}$ given that: $y = \int_{1}^{x^3} \sqrt{1+t^2}\, dt$ (2 marks)

(b) Show that $\int_{1}^{2} \frac{6x+4}{\sqrt{x}}\, dx = 16\sqrt{2} - 12$ (4 marks)

Question 2 **(9 marks)**

A train is travelling on a straight track between two stations under the following conditions.

It starts from rest at station A and moves with acceleration $a(t) = 5ms^{-2}$ for $0 \leq t < 4$ seconds.

It then maintains its speed for 60 seconds such that $a(t) = 0ms^{-2}$ for $4 \leq t < 64$ seconds.

Finally, it slows to rest at a constant rate over 10 seconds such that
$v(t) = 148 - 2tms^{-1}$ for $64 \leq t \leq 74$ seconds and stops in station B.

(a) Sketch the velocity versus time graph. (5 marks)

(b) Calculate the total distance in metres between station A and station B. (4 marks)

Question 3 **(3 marks)**

Explain why $\int_2^5 x(x-2)(x-5)\,dx = \int_0^3 (x+2)x(x-3)\,dx$.

Question 4 **(5 marks)**

Below is the sample space for the tossing of two dice and recording the numbers on the upper face of each die.

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

One activity is to add the numbers in each pair and record how frequently these numbers came up. For example, $(3, 2)$ gives $3 + 2 = 5$.

(a) Set up a discrete probability table for the possible outcomes of this activity and give the theoretical probabilities. (2 marks)

(b) Draw a relative frequency diagram from the table. (3 marks)

End of Section One

Section Two: Calculator-assumed **(22 marks)**

Question 5 **(3 marks)**

A large container has developed a leak and is losing its liquid at a rate given by the equation $\frac{dv}{dt} = 3 - 3e^{-0.2t}$ in litres per hour. Given $v =$ volume in litres and $t =$ time in hours, if the leak is stopped after three hours, calculate, to the nearest millilitre, how much liquid is lost in that time.

Question 6 **(5 marks)**

(a) Evaluate the integral $\int_{-3}^{3}(x-3)x(x+3)dx$ and explain the result. (2 marks)

(b) Evaluate the area between the graphs $y_1 = x$ and $y_2 = (x+3)x(x-3)$. (3 marks)

Question 7 **(5 marks)**

An engineer is remotely monitoring the instruments from a test car travelling in a straight line on a track. At a given instant, she noticed that the acceleration of the car was a constant 4 ms^{-2} and 5 seconds later she recorded the car was travelling with a velocity of 50 ms^{-1}. Calculate the velocity equation of the car over this period and how far the car travelled in that time.

Question 8 **(4 marks)**

(a) In Section One, Question 4 of this test, you were asked to set up a discrete probability table for the possible outcomes of the two dice activity and give the theoretical probabilities.

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Using the same table, calculate the mean and standard deviation for the distribution. (2 marks)

(b) In a new activity, the totals from the two dice are tripled and five is added. Determine the new mean and standard deviation. (2 marks)

Question 9 **(5 marks)**

A carton contains 12 eggs, 5 of which are brown and 7 white. A chef selects 4 eggs at random, to use in an omelette.

(a) Calculate the discrete probability distribution for x which represents the number of white eggs chosen, giving your answer in fraction form. (3 marks)

x	0	1	2	3	4
Pr(X=x)					

(b) Calculate the mean and standard deviation of the probability distribution. (2 marks)

End of Section Two

Solutions and marking key for Task 3

Section One: Calculator-free **(23 marks)**

Question 1 **(6 marks)**

(a) Evaluate $\frac{dy}{dx}$ given that: $y=\int_1^{x^3}\sqrt{1+t^2}\,dt$ (2 marks)

$$y=\int_1^{x^3}\sqrt{1+t^2}\,dt \Rightarrow \frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}=\sqrt{1+t^2}\times 3x^2$$

$$=\sqrt{1+\left(x^3\right)^2}\times 3x^2$$

Behaviours	Marks
Calculates the derivative of the integral correctly	1
Applies the chain rule correctly	1

(b) Show that $\int_1^2 \frac{6x+4}{\sqrt{x}}\,dx=16\sqrt{2}-12$ (4 marks)

$$\int_1^2 \frac{6x+4}{\sqrt{x}}dx=\int_1^2\left(6x^{1/2}+4x^{-1/2}\right)dx \;=\left[4x^{3/2}+8x^{1/2}\right]_1^2$$

$$=\left(8\sqrt{2}+8\sqrt{2}\right)-\left(4+8\right)=16\sqrt{2}-12$$

Behaviours	Marks
Partitions the algebraic fraction before integrating	1
Simplifies fractional indices when dividing	1
Simplifies fractions accurately when integrating	1
Shows adequate working with the substitution	1

Question 2 **(9 marks)**

A train is travelling on a straight track between two stations under the following conditions.

It starts from rest at station A and moves with acceleration $a(t) = 5ms^{-2}$ for $0 \leq t < 4$ seconds.

It then maintains its speed for 60 seconds such that $a(t) = 0ms^{-2}$ for $4 \leq t < 64$ seconds.

Finally, it slows to rest at a constant rate over 10 seconds such that $v(t) = 148 - 2tms^{-1}$ for $64 \leq t \leq 74$ seconds and stops in station B.

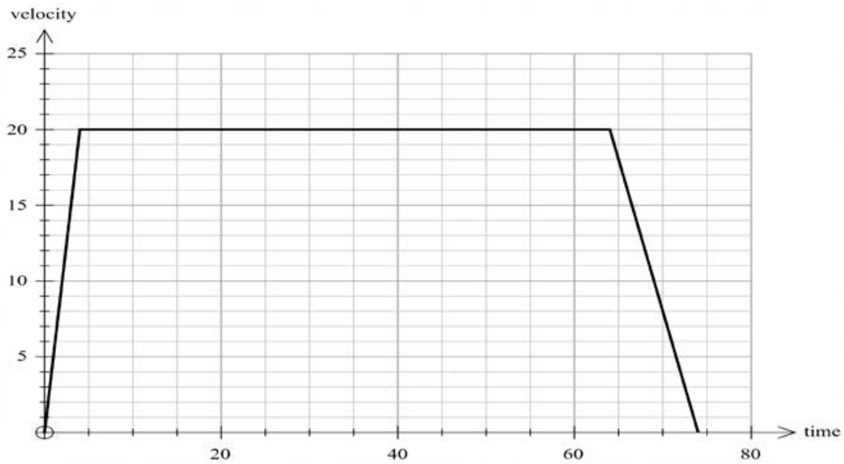
(a) Sketch the velocity versus time graph. (5 marks)

$a(t) = 5\text{ms}^{-2}$ for $0 \leq t < 4$ seconds $\Leftrightarrow (v)t = 5t + c$

$v(0) = 0 \Leftrightarrow c = 0 \therefore (v)t = 5t$ for $0 \leq t < 4$ seconds

$t \to 4$ then $v \to 20 \therefore v = 20\text{ ms}^{-1}$ for $4 \leq t < 64$ seconds

Also $v(t) = 148 - 2t\text{ms}^{-1}$ for $64 \leq t \leq 74$ seconds



Behaviours	Marks
Determines the first two velocity functions	2
Draws each section of the graph accurately	3

(b) Calculate the total distance in metres between station A and station B. (4 marks)

Distance travelled $= \int v(t)dt$

$$= \int_0^4 5t\,dt + \int_{64}^4 20\,dt + \int_{64}^{74} 148 - 2t\,dt$$

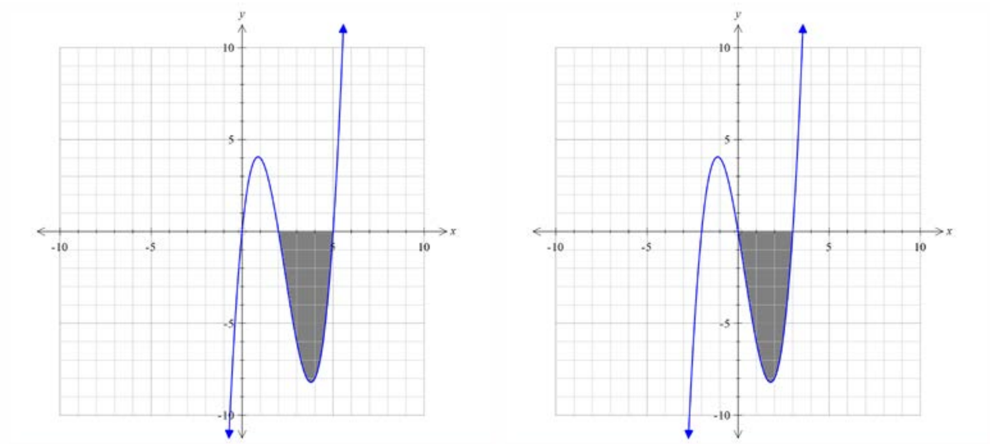
$$= 40 + 1200 + 100$$

$$= 1340\text{m}$$

Behaviours	Marks
Uses the velocity functions/graphs to calculate the distance travelled for each leg	3
States the correct distance travelled	1

Question 3 **(3 marks)**

Explain why $\int_2^5 x(x-2)(x-5)dx = \int_0^3 (x+2)x(x-3)dx$.



The original graph has been translated two units to the left and the limits for the integral have also been translated two units to the left.

Hence, the area to be calculated in both cases is the same area enclosed by the function below the x-axis.

Behaviours	Marks
States the graph has been translated to the left	1
States the limits have also been translated two units left	1
States the area is the same in both cases	1

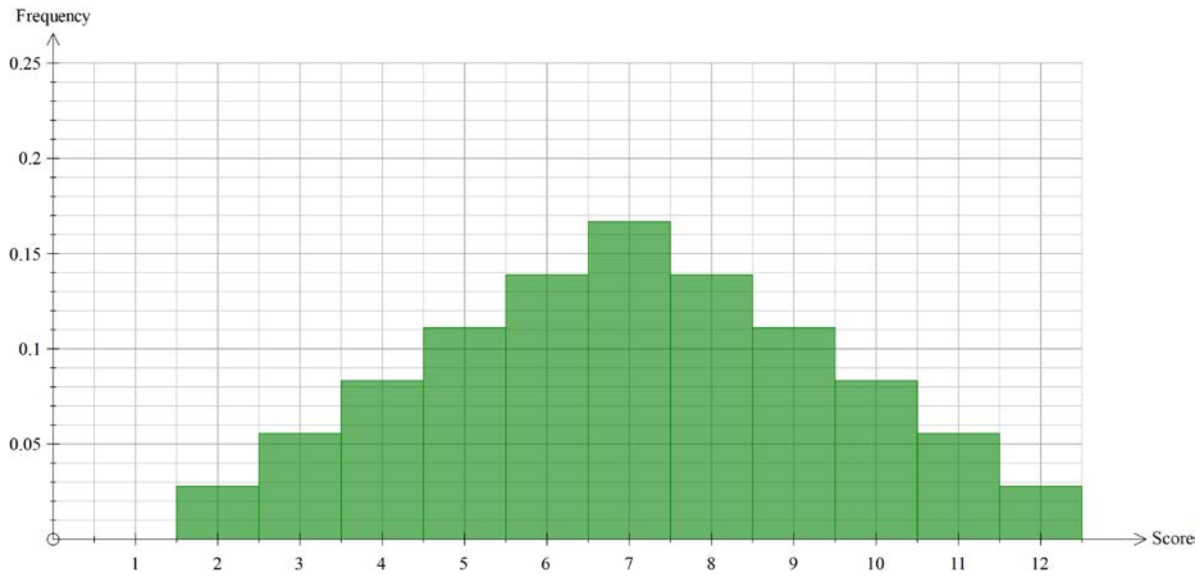
Question 4 [3.3.4] **(5 marks)**

(a) Set up a discrete probability table for the possible outcomes of this activity and give the theoretical probabilities. (2 marks)

x	2	3	4	5	6	7	8	9	10	11	12
P(x)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Behaviours	Marks
Defines the set of variables correctly	1
Completes the probability values	1

(b) Draw a relative frequency diagram from the table. (3 marks)



Behaviours	Marks
Centres each class on 2, 3 … etc.	1
Sets an appropriate vertical and horizontal scale	1
Draws a good representation of the histogram	1

Solutions and marking key for Task 3

Section Two: Calculator-assumed **(22 marks)**

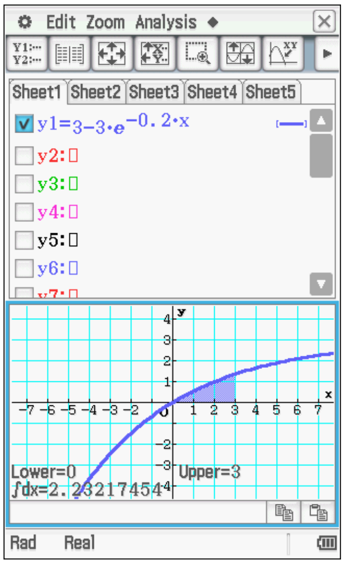
Question 5 [3.2.18] **(3 marks)**

A large container has developed a leak and is losing its liquid at a rate given by the equation $\frac{dv}{dt} = 3 - 3e^{-0.2t}$ in litres per hour. Given v = volume in litres and t = time in hours, if the leak is stopped after three hours, calculate, to the nearest millilitre, how much liquid is lost in that time.

If $\frac{dv}{dt} = 3 - 3e^{-0.2t}$ then liquid lost in three hours

$= \int_0^3 3 - 3e^{-0.2t}\,dt = 2.232$ litres

$= 2232$ ml



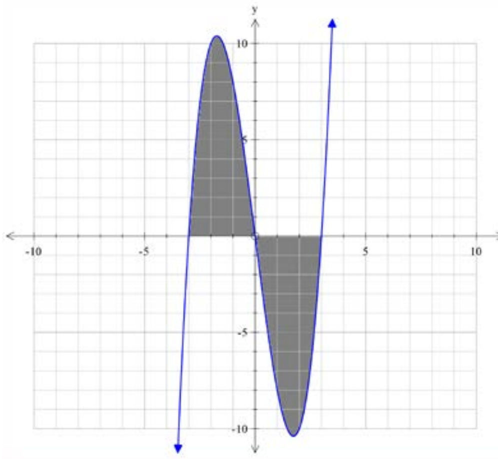
Behaviours	Marks
Sets up the correct integral	1
Sets up the correct limits	1
States the correct volume to the nearest millilitre	1

Question 6 [3.2.20] **(5 marks)**

(a) Evaluate the integral $\int_{-3}^{3}(x-3)x(x+3)dx$ and explain the result. (2 marks)

$\int_{-3}^{3}(x-3)x(x+3)dx = 0$

Since the graph is symmetrical about the origin, the area above the x-axis (+) equals the area below the x-axis (–). Hence, these areas add to zero.



Behaviours	Marks
Uses symmetry to explain that the two areas are equal	1
States the areas are additive opposite in value	1

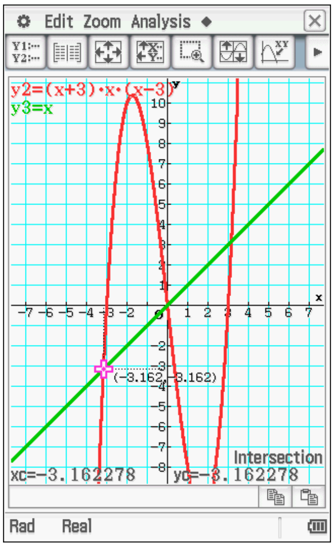
(b) Evaluate the area between the graphs $y_1 = x$ and $y_2 = (x+3)x(x-3)$.. (3 marks)

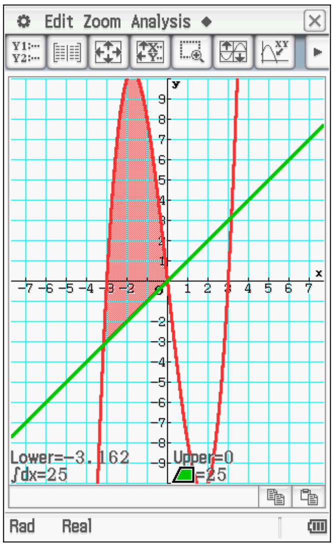
Required area

$= \int_{-3.162}^{3.162} |y_2 - y_1| dx$

$= 2 \times \int_{-3.162}^{0} x(x-3)(x+3) - x\, dx$

$= 50$ sq units





Behaviours	Marks
Uses the points of intersection to define integral limits	1
Uses the correct integrand	1
Gives the correct area	1

Question 7 [3.2.9; 3.2.21] **(5 marks)**

An engineer is remotely monitoring the instruments from a test car travelling in a straight line on a track. At a given instant, she noticed that the acceleration of the car was a constant 4 ms^{-2} and 5 seconds later she recorded the car was travelling with a velocity of 50 ms^{-1}. Calculate the velocity equation of the car over this period and how far the car travelled in that time.

Given $a = 4\text{ms}^{-2}$ and $v = \int a\,dt$ where a is a constant

$v = 4t + c$ since $v = 50$ when $t = 5$

$50 = 20 + c \Rightarrow c = 30$

$v(t) = 4t + 30$

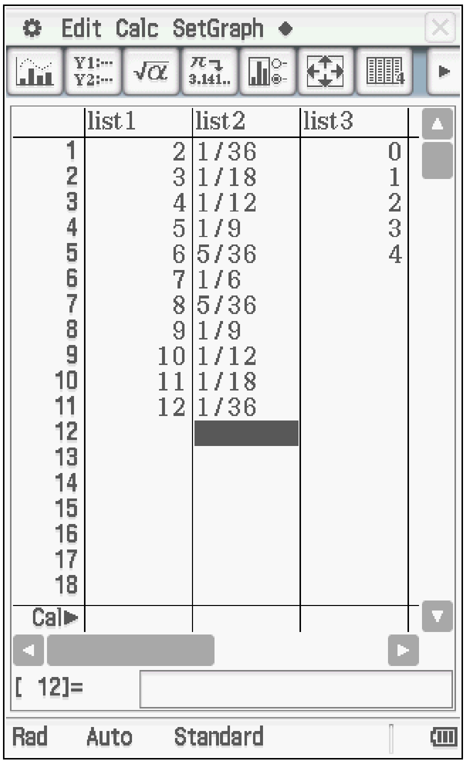
$\therefore$ Distance travelled = $\int_0^5 4t + 30\,dt = 200\,m$

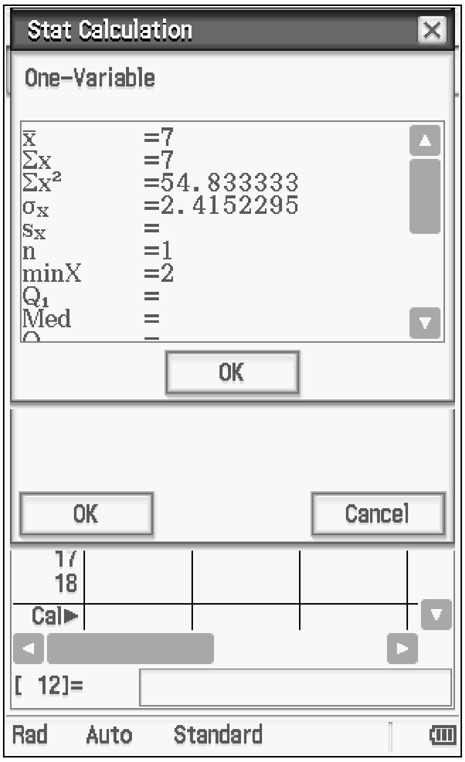
Behaviours	Marks
Calculates the correct constant of integration	1
Gives the correct velocity equation	1
Uses the integral of the velocity equation to calculate the distance travelled	1
Uses the correct limits	1
Calculates the correct distance	1

Question 8 [3.3.5] [3.3.6] [3.3.7] **(4 marks)**

(a) In Section One, Question 4 of this test, you were asked to set up a discrete probability table for the possible outcomes of the two-dice activity and give the theoretical probabilities.

Using the same table, calculate the mean and standard deviation for the distribution. (2 marks)





Mean = 7

Standard deviation = 2.4152

Behaviours	Marks
Calculates the mean correctly	1
Calculates the standard deviation correctly	1

(b) In a new activity, the totals from the two dice are tripled and five is added. Determine the new mean and standard deviation. (2 marks)

New mean: $7 \times 3 + 5 = 26$

New standard deviation: $2.4152 \times 3 = 7.2456$

Behaviours	Marks
Applies change of scale and origin to the mean to correctly determine a new value	1
Applies change of scale only to the standard deviation to correctly determine a new value	1

Question 9 [3.3.1] **(5 marks)**

(a) A carton contains 12 eggs, 5 of which are brown and 7 white. A chef selects 4 eggs at random, to use in an omelette. Calculate the discrete probability distribution for x which represents the number of white eggs chosen, giving your answer in fraction form. (3 marks)

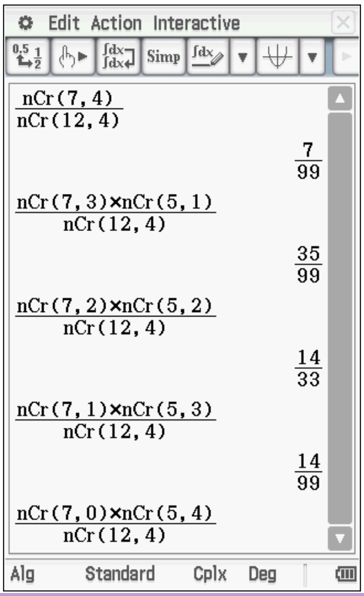
$$P(4\text{ white}) = \frac{\binom{7}{4}}{\binom{12}{4}}$$

$$P(1\text{ white}) = \frac{\binom{7}{1}\binom{5}{3}}{\binom{12}{4}}$$

$$P(3\text{ white}) = \frac{\binom{7}{3}\binom{5}{1}}{\binom{12}{4}}$$

$$P(\text{no white}) = \frac{\binom{5}{4}}{\binom{12}{4}}$$

$$P(2\text{ white}) = \frac{\binom{7}{2}\binom{5}{2}}{\binom{12}{4}}$$



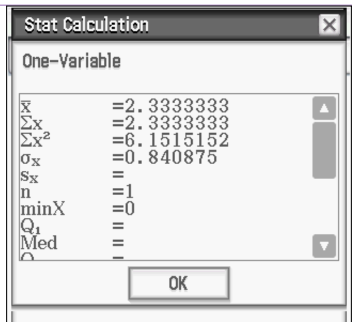
x	0	1	2	3	4
P(X = x)	$\frac{1}{99}$	$\frac{14}{99}$	$\frac{42}{99}$	$\frac{35}{99}$	$\frac{7}{99}$

Behaviours	**Marks**
Shows appropriate working for at least one value	1
Calculates the five values accurately	2

(b) Calculate the mean and standard deviation of the probability distribution. (2 marks)

Mean = 2.3333

Standard deviation = 0.8409



Behaviours	**Marks**
Gives the correct mean	1
Gives the correct standard deviation	1

End of solutions