Government of Western Australia
School Curriculum and Standards Authority

Sample Assessment Tasks

Mathematics Methods
ATAR Year 12

Sample assessment task

Mathematics Methods – ATAR Year 12

Test 3 – Unit 1

Assessment type:	Response	
Conditions:		
Time for the task:	Up to 50 minutes, in class, under test conditions	
Materials required:		
Section One: Calculator-free	Standard writing equipment	
Section Two: Calculator-assumed	Calculator (to be provided by the student)	
Other materials allowed:	Drawing templates, one page of notes in Section Two	
Marks available:	**44**	
	Section One: Calculator-free	(23 marks)
	Section Two: Calculator-assumed	(21 marks)
Task weighting:	**8%**	

Section One: Calculator-free **(23 marks)**

Question 1 [3.2.16] [3.2.17] **(6 marks)**

(a) Evaluate $\frac{dy}{dx}$ given that: $y = \int_{1}^{x^3} \sqrt{1+t^2}\,dt$ (2 marks)

(b) Show that $\int_1^2 \frac{6x+4}{\sqrt{x}}\,dx = 16\sqrt{2} - 12$ (4 marks)

Question 2 [3.2.22] **(9 marks)**

A train is travelling on a straight track between two stations under the following conditions.
It starts from rest at station A and moves with acceleration $a(t) = 5\,ms^{-2}$ for $0 \le t < 4$ seconds.
It then maintains its speed for 60 seconds such that $a(t) = 0ms^{-2}$ for $4 \le t < 64$ seconds .
Finally, it slows to rest at a constant rate over 10 seconds such that
$v(t) = 148 - 2t\,ms^{-1}$ for $64 \le t \le 74$ seconds and stops in station B.

(a) Sketch the Velocity V's Time graph (5 marks)

(b) Calculate the total distance in metres between station A and station B. (4 marks)

Question 3 [3.2.19] **(3 marks)**

Explain why $\int_3^5 x(x-3)(x-5)\,dx = \int_0^2 (x+3)x(x-2)\,dx$.

Question 4 [3.3.4] **(5 marks)**

Below is the sample space for the tossing of two dice and recording the numbers on the upper face of each die.

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

One activity is to add the numbers in each pair and record how frequently these numbers came up. For example, (3,2) gives 3+2=5.

(a) Set up a discrete probability table for the possible outcomes of this activity and give the theoretical probabilities. (2 marks)

(b) Draw a relative frequency diagram from the table. (3 marks)

End of Section One

Section Two: Calculator-assumed (21 marks)

Question 5 [3.2.18] (3 marks)

A large container has developed a leak and is losing its liquid at a rate given by the equation $\frac{dv}{dt} = 3 - 3e^{-0.2t}$ in litres per hour. Given $v =$ volume in litres and $t =$ time in hours, if the leak is stopped after three hours, calculate, to the nearest millilitre, how much liquid is lost in that time.

Question 6 [3.2.20] (5 marks)

(a) Evaluate the integral $\int_{-3}^{3}(x-3)x(x+3)dx$ and explain the result. (2 marks)

(b) Evaluate the area between the graphs $y_1 = x$ and $y_2 = (x+3)x(x-3)$. (3 marks)

Question 7 [3.2.9; 3.2.21] (5 marks)

An engineer is remotely monitoring the instruments from a test car travelling in a straight line on a track. At a given instant, she noticed that the acceleration of the car was a constant $4ms^{-2}$ and 5 seconds later she recorded the car was travelling with a velocity of $50ms^{-1}$. Calculate the velocity equation of the car over this period and how far the car travelled in that time.

Question 8 [3.3.5; 3.3.6] **(3 marks)**

(a) In Q4 Section One of this test, you were asked to set up a discrete probability table for the possible outcomes of the two–dice activity and give the theoretical probabilities.

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Using the same table, calculate the Mean, and Standard deviation for the distribution: (2 marks)

(b) These terms in part (a) above are referred as parameters. Explain why. (1 mark)

Question 9 [3.3.1] **(5 marks)**

A carton contains 12 eggs, 5 of which are brown and 7 white. A chef selects 4 eggs at random, to use in an omelette.

(a) Calculate the discrete probability distribution for x which represents the number of white eggs chosen, giving your answer in fraction form. (3 marks)

(b) Calculate the mean and standard deviation of the probability distribution. (2 marks)

x	0	1	2	3	4
Pr(X=x)					

End of Section Two

Solutions and marking key for Test 3 – Unit 1

Section One: Calculator-free **(23 marks)**

Question 1 [3.2.16] [3.2.17] **(6 marks)**

(a) Evaluate $\frac{dy}{dx}$ given that: $y = \int_1^{x^3} \sqrt{1+t^2}\,dt$ (2 marks)

$$y = \int_1^{x^3} \sqrt{1+t^2}\,dt \Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \sqrt{1+t^2} \times 3x^2$$

$$= \sqrt{1+\left(x^3\right)^2} \times 3x^2$$

Specific behaviours	Mark allocation	Item classification
Calculates the derivative of the integral correctly	1	complex
Applies the chain rule correctly	1	complex

(b) Show that $\int_1^2 \frac{6x+4}{\sqrt{x}}\,dx = 16\sqrt{2} - 12$ (4 marks)

$$\int_1^2 \frac{6x+4}{\sqrt{x}}dx = \int_1^2 \left(6x^{1/2} + 4x^{-1/2}\right)dx \quad = \left[4x^{3/2} + 8x^{1/2}\right]_1^2$$

$$= \left(8\sqrt{2} + 8\sqrt{2}\right) - \left(4+8\right) = 16\sqrt{2} - 12$$

Specific behaviours	Mark allocation	Item classification
Partitions the algebraic fraction before integrating	1	simple
Simplifies fractional indices when dividing	1	simple
Simplifies fractions accurately when integrating	1	simple
Shows adequate working with the substitution	1	simple

Question 2 [3.2.21] **(9 marks)**

A train is travelling on a straight track between two stations under the following conditions.
It starts from rest at station A and moves with acceleration $a(t) = 5\,ms^{-2}$ for $0 \le t < 4$ seconds.

It then maintains its speed for 60 seconds such that $a(t) = 0ms^{-2}$ for $4 < t < 64$ seconds

Finally it slows to rest at a constant rate over 10 seconds such that
$v(t) = 148 - 2t\,ms^{-1}$ for $64 \le t \le 74$ seconds and stops in station B.

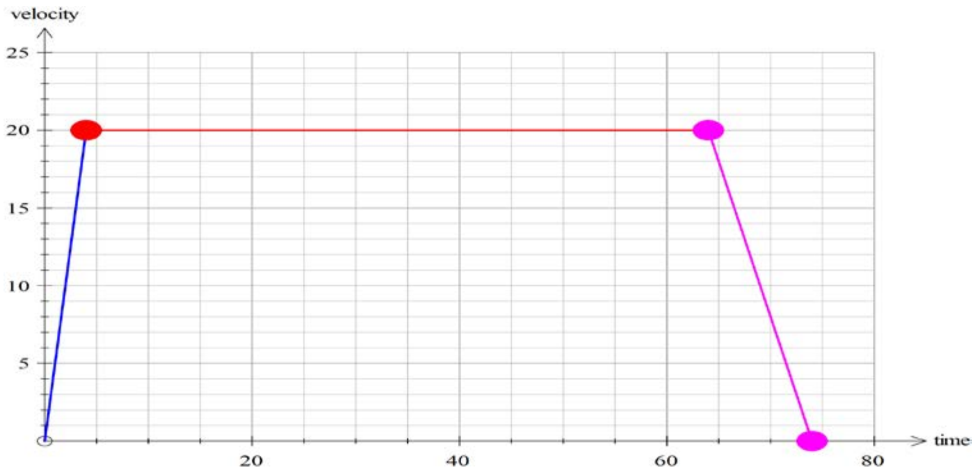
(a) Sketch the Velocity V's Time graph (5 marks)

$a(t) = 5\,ms^{-2}$ for $0 \le t < 4$ seconds $\Leftrightarrow (v)t = 5t + c$

$v(0) = 0 \Leftrightarrow c = 0 \therefore (v)t = 5t$ for $0 \le t < 4$ seconds

$t \to 4$ then $v \to 20 \therefore v = 20ms^{-1}$ for $4 \le t < 64$ seconds

Also $v(t) = 148 - 2t\,ms^{-1}$ for $64 \le t \le 74$ seconds



Specific behaviours	Mark allocation	Item classification
Determines the first two velocity functions	2	simple
Draws each section of the graph accurately	3	simple

(b) Calculate the total distance in metres between station A and station B. (4 marks)

Distance travelled $= \int v(t)dt$

$$= \int_0^4 5t\,dt \quad + \int_{64}^{4} 20\,dt \quad + \int_{64}^{74} 148 - 2t\,dt$$

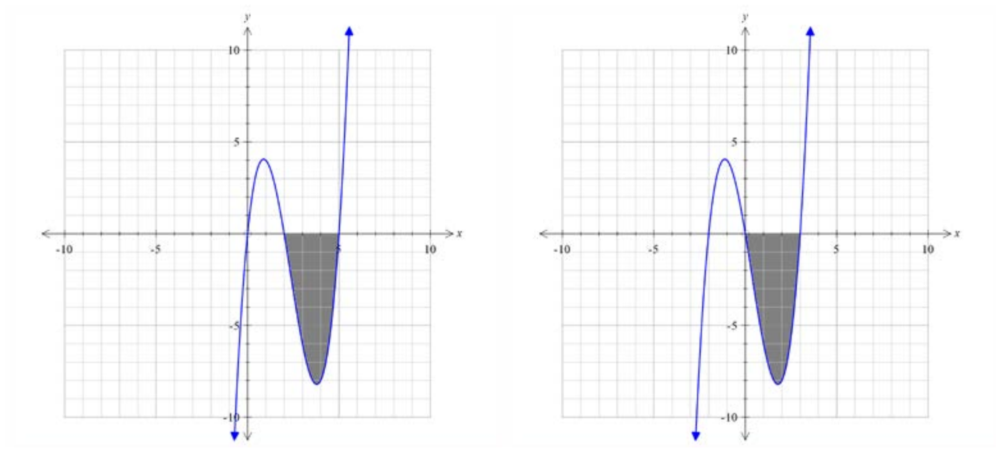
$$= 40 \quad +1200 \quad +100$$

$$= 1340\,m$$

Specific behaviours	Mark allocation	Item classification
Uses the velocity functions/graphs to calculate the distance travelled for each leg	3	simple
States the correct distance travelled	1	simple

Question 3 [3.2.19] **(3 marks)**

Explain why $\int_2^5 x(x-2)(x-5)\,dx = \int_0^3 (x+2)x(x-3)\,dx$



The original graph has been translated two units to the left and the limits for the integral have also been translated two units to the left.
Hence, the area to be calculated in both cases is the same area enclosed by the function below the *x* – axis.

Specific behaviours	Mark allocation	Item classification
States the graph has been translated to the left	1	complex
States the limits have also been translated two units left	1	complex
States the area is the same in both cases	1	complex

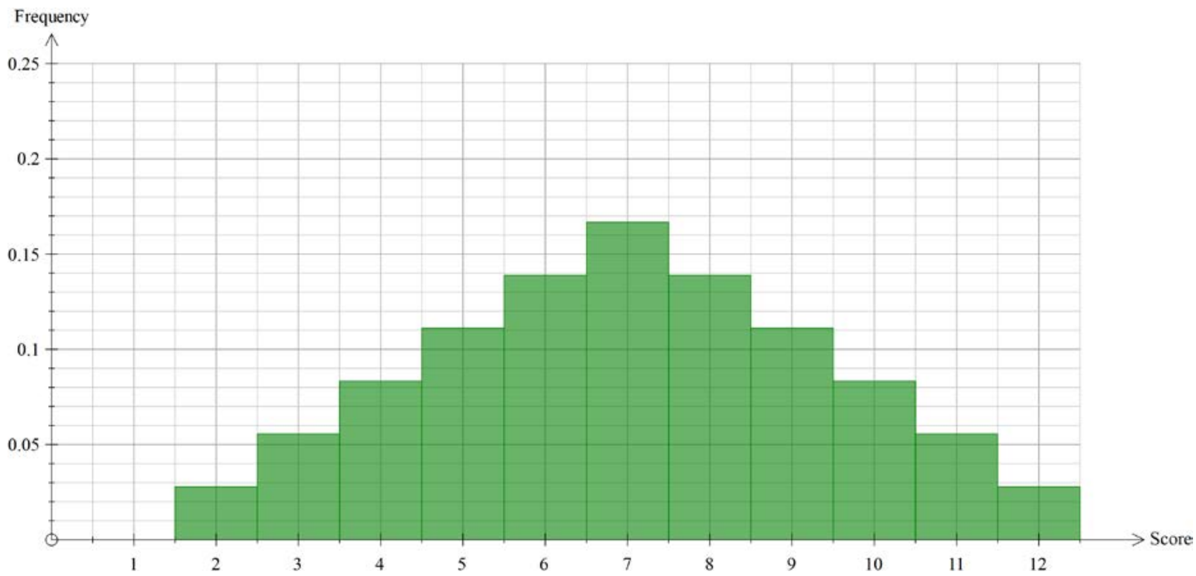
Question 4 [3.3.4] **(5 marks)**

(a) Set up a discrete probability table for the possible outcomes of this activity and give the theoretical probabilities. (2 marks)

x	2	3	4	5	6	7	8	9	10	11	12
Pr(*x*)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Specific behaviours	Mark allocation	Item classification
Defines the set of variables correctly	1	simple
Completes the probability values	1	simple

(b) Draw a relative frequency diagram from the table. (3 marks)



Specific behaviours	Mark allocation	Item classification
Centres each class on 2, 3 … etc. Sets an appropriate horizontal scale Draws a good representation of the histogram	1 1 1	simple simple simple

Solutions and marking key for Test 3 – Unit 1

Section Two: Calculator-assumed **(21 marks)**

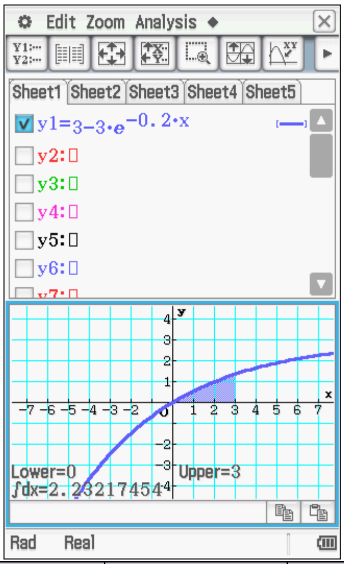
Question 5 [3.2.18] **(3 marks)**

A large container has developed a leak and is losing its liquid at a rate given by the equation $\frac{dv}{dt} = 3 - 3e^{-0.2t}$ in litres per hour. Given $v =$ volume in litres and $t =$ time in hours, if the leak is stopped after three hours, calculate, to the nearest millilitre, how much liquid is lost in that time.

If $\frac{dv}{dt} = 3 - 3e^{-0.2t}$ then liquid lost in three hours

$= \int_0^3 3 - 3e^{-0.2t}\, dt = 2.232$ litres

$= 2232\ ml$



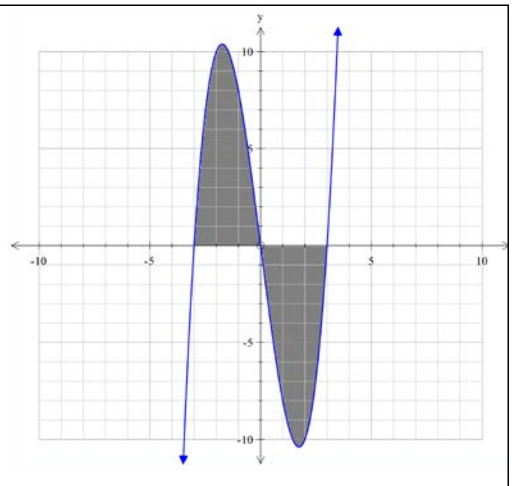
Specific behaviours	Mark allocation	Item classification
Sets up the correct integral	1	simple
Sets up the correct limits	1	simple
States the correct volume to the nearest millilitre	1	simple

Question 6 [3.2.20] **(5 marks)**

(a) Evaluate the integral $\int_{-3}^{3}(x-3)x(x+3)dx$ and explain the result. (2 marks)

$\int_{-3}^{3}(x-3)x(x+3)dx = 0$

Since the graph is symmetrical about the origin the area above the x – axis (+) equals the area below the x – axis (–). Hence these areas add to zero.



Specific behaviours	Mark allocation	Item classification
Uses symmetry to explain that the two areas are equal	1	simple
States the areas are additive opposite in value	1	simple

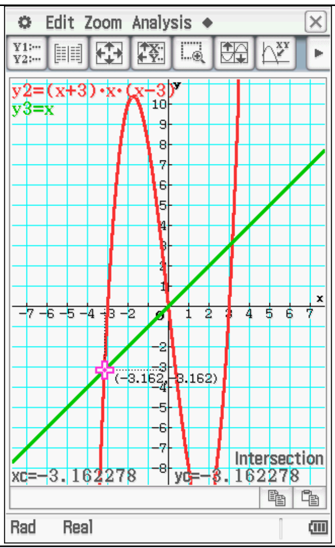
(b) Evaluate the area between the graphs $y_1 = x$ and $y_2 = (x+3)x(x-3)$ (3 marks)

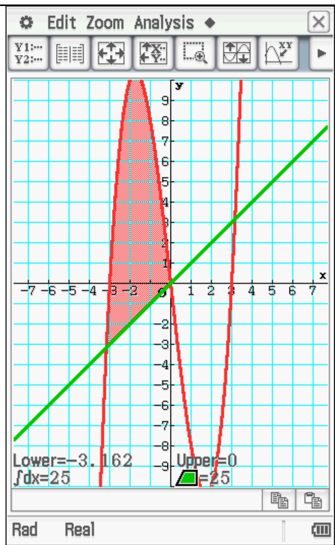
Required area

$$=\int_{-3.162}^{3.162}|y_2 - y_1|dx$$

$$=2\times\int_{-3.162}^{0} x(x-3)(x+3)-x\,dx$$

$$=50 \text{ sq units}$$





Specific behaviours	Mark allocation	Item classification
Uses the points of intersection to define integral limits	1	simple
Uses the correct integrand	1	simple
Gives the correct area	1	simple

Question 7 [3.2.9; 3.2.21] **(5 marks)**

An engineer is remotely monitoring the instruments from a test car travelling in a straight line on a track. At a given instant she noticed that the acceleration of the car was a constant 4 ms^{-2} and 5 seconds later she recorded the car was travelling with a velocity of 50 ms^{-1}. Calculate the velocity equation of the car over this period and how far the car travelled in that time.

Given $a = 4\text{ms}^{-2}$ and $v = \int a\,dt$ where a is a constant

$v = 4t + c$ since $v = 50$ when $t = 5$

$50 = 20 + c \Rightarrow c = 30$

$v(t) = 4t + 30$

$\therefore$ Distance travelled = $\int_0^5 4t + 30\,dt = 200\,m$

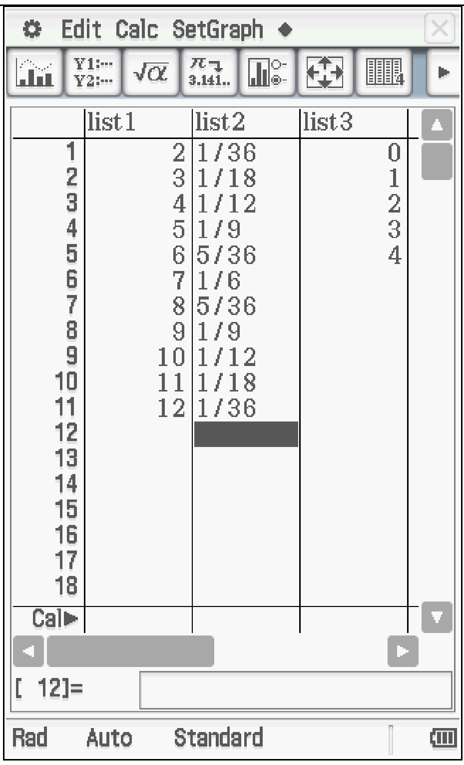
Specific behaviours	Mark allocation	Item classification
Calculates the correct constant of integration	1	simple
Gives the correct velocity equation	1	simple
Uses the integral of the velocity equation to calculate the distance travelled	1	simple
Uses the correct limits	1	simple
Calculates the correct distance	1	simple

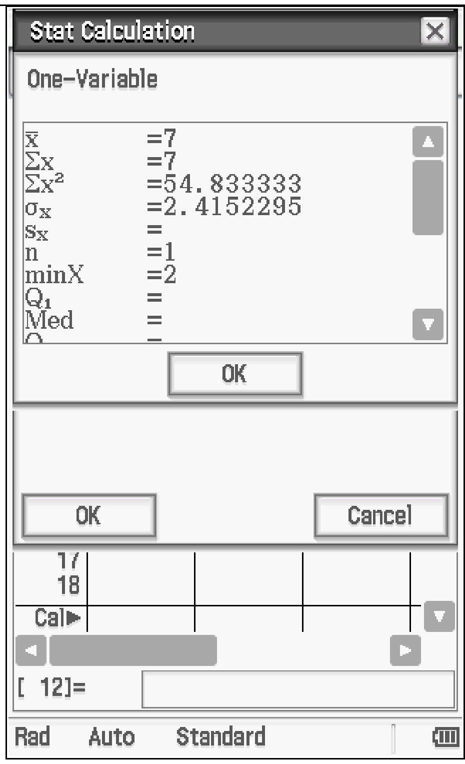
Question 8 [3.3.5; 3.3.6] **(3 marks)**

(a) In Q4 Section One, you were asked set up a discrete probability table for the possible outcomes of the two-dice activity and give the theoretical probabilities.

Using the same table, calculate the Mean, and Standard deviation for the distribution:

(2 marks)





Mean = 7
Standard deviation = 2.4152

Specific behaviours	Mark allocation	Item classification
Calculates the Mean correctly	1	simple
Calculates the Standard deviation correctly	1	simple

(b) These terms in part (a) above are referred to as parameters. Explain why. (1 mark)

Parameters refer to the measures of a population or a theoretical probability distribution.

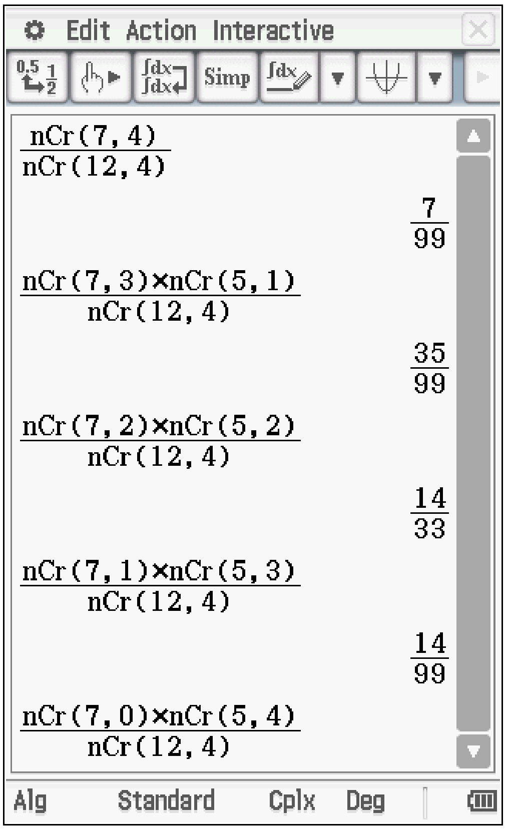
Specific behaviours	Mark allocation	Item classification
Refers to parameters as a measure of population	1	simple

Question 9 [3.3.1] **(5 marks)**

A carton contains 12 eggs, 5 of which are brown and 7 white. A chef selects 4 eggs at random, to use in an omelette.

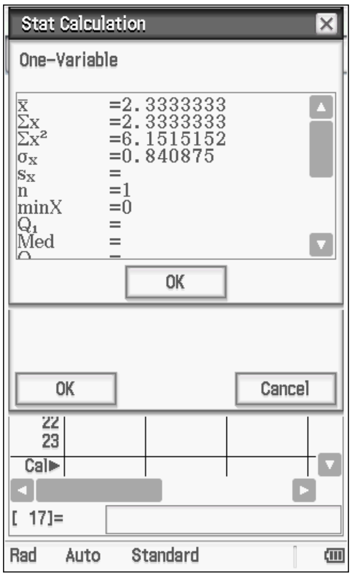
(a) Calculate the discrete probability distribution for x which represents the number of white eggs chosen, giving your answer in fraction form. (3 marks)

(b) Calculate the mean and standard deviation of the probability distribution. (2 marks)



Mean = 2.3333

Standard deviation = 0.8409



x	0	1	2	3	4
Pr(X=x)	$\frac{1}{99}$	$\frac{14}{99}$	$\frac{42}{99}$	$\frac{35}{99}$	$\frac{7}{99}$

Specific behaviours	Mark allocation	Item classification
Shows appropriate working for at least one value	1	simple
Calculates the five values accurately	2	simple
Gives the correct mean	1	simple
Gives the correct standard deviation	1	simple

End of solutions