ATAR course examination, 2017 Question/Answer booklet

# MATHEMATICS <br> METHODS 

Section One: Calculator-free


Student number: In figures


In words

## Time allowed for this section

Reading time before commencing work:
Working time:
five minutes
fifty minutes

Number of additional answer booklets used (if applicable):

## Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

## To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

## Structure of this paper

| Section | Number of <br> questions <br> available | Number of <br> questions to <br> be answered | Working <br> time <br> (minutes) | Marks <br> available | Percentage <br> of <br> examination |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Section One: <br> Calculator-free | 9 | 9 | 50 | 52 | 35 |
| Section Two: <br> Calculator-assumed | 11 | 11 | 100 | 99 | 65 |
| Total |  |  |  |  | 100 |

## Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the Year 12 Information Handbook 2017. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

This section has nine (9) questions. Answer all questions. Write your answers in the spaces provided.

Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

## Question 1

Anastasia is a university student. She records the time it takes for her to get from home to her campus each day. The histogram of relative frequencies below shows the journey times she recorded.


Use the above data to estimate the probability of her next journey from home to her university campus
(a) taking her less than 36 minutes.
(b) taking at least 35 minutes but no more than 39 minutes.

On three consecutive days, Anastasia needs to be on campus no later than 10 am .
(c) If she leaves her home at 9:22 am each day, use the above data to estimate the probability that she makes it on or before time on all three days.

## Question 2

Michelle is a soccer goalkeeper and has built a machine to help her practise. The machine will shoot a soccer ball randomly along the ground at or near a goal that is seven metres wide. The machine is equally likely to shoot the ball so that the centre of the ball crosses the goal line anywhere between point $A$ three metres left of the goal, and point $B$ five metres right of the goal, as shown in the diagram below.


Michelle sets up a trial run without anyone in the goals. Assume the goal posts are of negligible width.

Let the random variable $X$ be the distance the centre of the ball crosses the goal line to the right of point A.
(a) Complete the graphical representation of the probability density function for the random variable $X$.

(b) What is the probability that the machine shoots a ball so that its centre misses the goal to
(c) What is the probability that the machine shoots a ball so that its centre is inside the goal?
(1 mark)
(d) If the machine shoots a ball so that its centre misses the goal, what is the probability that the ball's centre misses to the right?
(2 marks)

## Question 3

Solve $4 e^{2 x}=81-5 e^{2 x}$ exactly for $x$ ．

## Question 4

Two independent samples of different sizes were taken from a population．The first sample had sample size $n_{1}$ and the second sample had sample size $n_{2}$ ．The sample proportions of males in the samples were the same．When $99 \%$ confidence intervals were calculated for each sample，it was found that the corresponding margin of error in the second sample was half that of the first sample．
What is the ratio of the two sample sizes，$\frac{n_{2}}{n_{1}}$ ？

## Question 5

(a) Consider the shaded area shown between the graph of $y=e^{x}$, the $y$ axis and the line $y=2$.

(i) Determine the coordinates of the point A .
(ii) Hence or otherwise determine the area between the graph of $y=e^{x}$, the $y$ axis and the line $y=2$.
(b) If the area between the graph of $y=e^{x}$, the $y$ axis, the $x$ axis and the line $x=k$, where $k \geq 0$, is to be equal to 2 square units, determine the exact value of $k$.
(4 marks)

## Question 6

(a) Evaluate $\int_{0}^{1} \frac{-12 x}{1+3 x^{2}} d x$.
(b) Given $f(x)=\ln \left(2-x^{3}\right)$
(i) determine $f^{\prime}(1)$.
(ii) In relation to the graph of $f(x)$, explain the meaning of your answer to (b)(i).
(1 mark)

## Question 7

Given that $\log _{10} 2=x$ and $\log _{10} 7=y$
(a) express $\log _{10} 14$ in terms of $x$ and $y$.
(b) show that $\log _{10} 17 \cdot 5=y-2 x+1$.
(c) evaluate $10^{y-x}$.

## Question 8

（a）Differentiate $2 x \sin (3 x)$ with respect to $x$ ．
（b）Hence show that $\int x \cos (3 x) d x=\frac{3 x \sin (3 x)+\cos (3 x)}{9}+c$ ．

Question 9
Consider the function $f(x)$ shown graphed below. The table gives the value of the function at the given $x$ values.

(a) By considering the areas of the rectangles shown, demonstrate and explain why
$32.5<\int_{0}^{1.5} f(x) d x<37$.

Consider the table of further values of $f(x)$ given below．

| $x$ | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 20 | 21 | 24 | 29 | 36 | 45 | 56 |

（b）Use the table values to determine the best estimate possible for $\int_{1}^{3} f(x) d x$ ．（3 marks）
（c）State two ways in which you could determine a more accurate value for $\int_{1}^{3} f(x) d x$ ．
（2 marks）

Supplementary page
Question number:

Supplementary page
Question number：

Supplementary page
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