Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS APPLICATIONS

Calculator-free

ATAR course examination 2020

Marking key

Marking keys are an explicit statement about what the examining panel expect of candidates when they respond to particular examination items. They help ensure a consistent interpretation of the criteria that guide the awarding of marks.

2020/66956

Section One: Calculator-free **35% (47 Marks)**

Question 1 **(5 marks)**

The owner of a bicycle shop recorded the type of repairs he made to bicycles with different purchase prices.

		Purchase price			
		Less than $500	From $500 to $1000	Greater than $1000	Total
Type of repair	Wheels and tyres	36	6	18	60
	Gears and brakes	20	12	8	40
	Frame and suspension	15	2	3	20

(a) Identify the explanatory variable for the table above. (1 mark)

Solution
Purchase price
Specific behaviours
✓ identifies correct explanatory variable

The percentages in each row of the following table show the proportion of bicycles with different purchase prices requiring that type of repair.

(b) Complete the table. (2 marks)

		Purchase price			
		Less than $500	From $500 to $1000	Greater than $1000	Total
Type of repair	Wheels and tyres	**60**	10	**30**	100
	Gears and brakes	**50**	30	20	100
	Frame and suspension	75	**10**	**15**	100

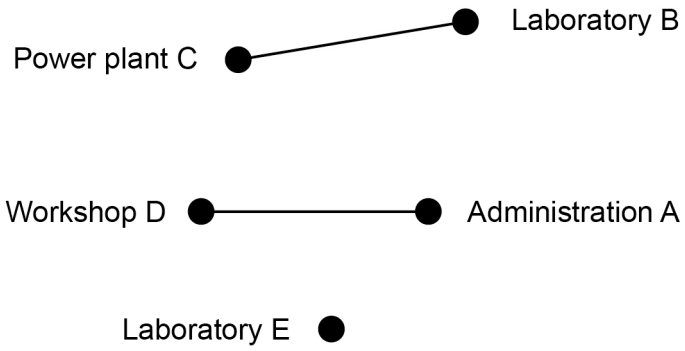
Solution
See table above
Specific behaviours
✓ calculates 3 correct values ✓ calculates all correct values

(c) Using the information from the table in part (b), describe **one** association between these variables. (2 marks)

Solution
The higher the purchase price, the lower the percentage of gears and brakes repairs that are needed.
Specific behaviours
✓ identifies an association between gears and brakes repairs and purchase price ✓ describes association in terms of differences in the percentages from the information in part (b)

Question 2 **(8 marks)**

A small research facility consists of five buildings with walkways represented by the edges in this network:



(a) Determine the smallest number of edges (walkways) to be added to ensure that the network is

(i) connected (1 mark)

Solution
2 edges
Specific behaviours
✓ states correct number

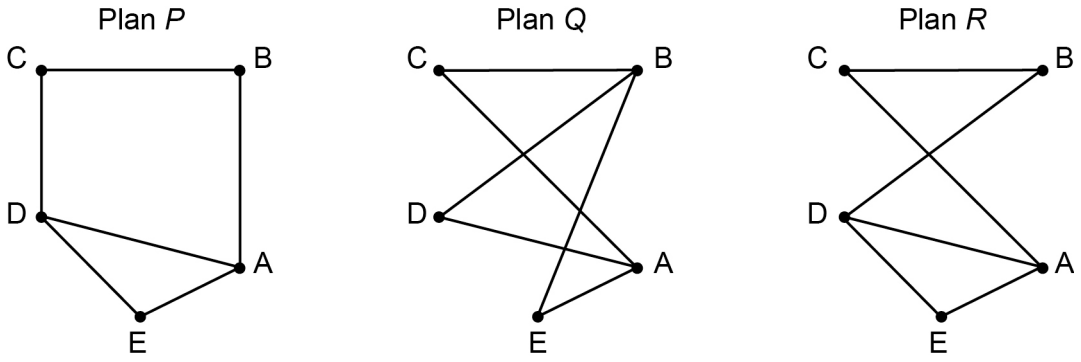
(ii) complete (1 mark)

Solution
8 edges
Specific behaviours
✓ states correct number

(iii) planar with 4 regions. (2 marks)

Solution		
If connected, $v+f-e=2$ $5+4-e=2$ $e=7$ Therefore, 5 edges are required.	**or**	If not connected, Euler's formula does not hold. Therefore, 3 edges are required.
Specific behaviours		
✓ determines total number of edges required ✓ states correct number of additional edges required		

Three different plans for completing the network with the addition of four walkways are shown:



(b) State which, if any, of these plans has a graph that is

(i) bipartite (1 mark)

Solution
Plan *Q*
Specific behaviours
✓ states correct plan

(ii) Eulerian (1 mark)

Solution
None
Specific behaviours
✓ states correct answer

(iii) a Hamiltonian cycle. (2 marks)

Solution
Plan *P* and Plan *R*
Specific behaviours
✓ states Plan *P* ✓ states Plan *R*

Question 3 **(8 marks)**

(a) Complete the matrix below showing the information from the bipartite graph. (1 mark)

Solution
$\begin{array}{c} \\ S1 \\ S2 \\ S3 \\ S4 \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 4 & 5 & 2 & 12 \\ 10 & 4 & 8 & 2 \\ 3 & 4 & \mathbf{6} & \mathbf{14} \\ \mathbf{9} & \mathbf{3} & \mathbf{5} & \mathbf{3} \end{bmatrix} \end{array}$
Specific behaviours
✓ completes matrix correctly

(b) Show use of the Hungarian algorithm to allocate each salesperson to a specialist machine in order to maximise sales. The allocation is to be written in the table below.

(5 marks)

Solution

$$\begin{bmatrix} 10 & 9 & 12 & 2 \\ 4 & 10 & 6 & 12 \\ 11 & 10 & 8 & 0 \\ 5 & 11 & 9 & 11 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 7 & 10 & 0 \\ 0 & 6 & 2 & 8 \\ 11 & 10 & 8 & 0 \\ 0 & 6 & 4 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 1 & 8 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{8} \\ 11 & 4 & 6 & 0 \\ \cancel{0} & \cancel{0} & \cancel{2} & \cancel{6} \end{bmatrix}$$

$$\begin{bmatrix} 7 & \boxed{0} & 7 & 0 \\ 0 & 0 & \boxed{0} & 9 \\ 10 & 3 & 5 & \boxed{0} \\ \boxed{0} & 0 & 2 & 7 \end{bmatrix}$$

Salesperson	1	2	3	4
Machine	**B**	**C**	**D**	**A**

Specific behaviours

- ✓ correctly subtracts each entry from largest value of 14
- ✓ correctly subtracts the smallest value in that row from each value in that row and then correctly subtracts the smallest value in that column from each value in that column
- ✓ determines that all zeros in the matrix can be covered by just 3 horizontal/vertical lines
- ✓ correctly determines the smallest uncovered value = 1 and subtracts 1 from each uncovered value and adds 1 to each value covered by 2 intersecting lines
- ✓ determines the correct allocation of machines

Question 3 (continued)

(c) How many more sales are made by using the allocation in part (b) compared to the allocation: S1 – A, S2 – B, S3 – C and S4 – D? (2 marks)

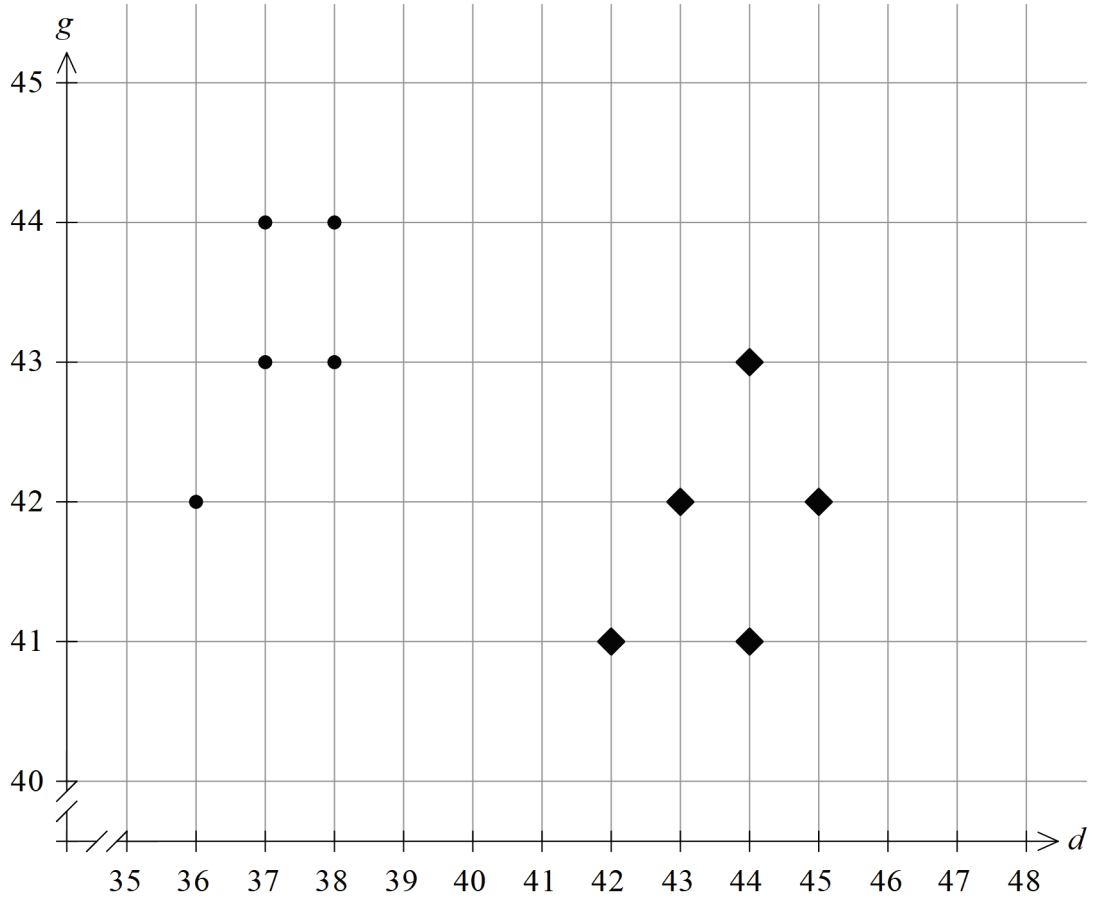
Solution
S1 – A, S2 – B, S3 – C and S4 – D gives a total of 17 sales Solution to part (b) gives a total of 36 sales. Greater by 19.
Specific behaviours
✓ determines number of sales for each allocation ✓ determines the correct difference

Question 4 **(11 marks)**

The table shows data comparing the gestation period (in days) with the birth weight (in grams) for ten Tasmanian possums.

Gestation period in days (d)	36	37	37	38	38	42	43	44	44	45
Birth weight in grams (g)	42	43	44	43	44	41	42	43	41	42

(a) Plot the last five data points on the axes below. (2 marks)



Solution
See graph above
Specific behaviours
✓ correctly plots at least three points ✓ correctly plots all points

Question 4 (continued)

The correlation coefficient for these observations is approximately – 0.6 and the least-squares line is $g = -0.17d + 49$.

(b) Describe what this correlation suggests about the general pattern of association between gestation period and birth weight. (2 marks)

Solution
Moderate negative linear relationship
Specific behaviours
✓ correctly mentions negative linear relationship ✓ correctly mentions moderate relationship

(c) Determine the coefficient of determination for these data. (1 mark)

Solution
36% (0.36)
Specific behaviours
✓ correctly calculates the coefficient of determination

(d) State the meaning of the coefficient of determination in the context of the question. (1 mark)

Solution
36% of the variation in their birth weight can be explained by the variation in the gestation period
Specific behaviours
✓ correctly states the meaning in the context of the question

(e) Use the least-squares line to predict the birth weight of a possum after 40 days gestation. (1 mark)

Solution
g = – 0.17 × 40 + 49 g = 42.2 grams
Specific behaviours
✓ correctly calculates birth weight

(f) Comment on the validity of this prediction. (2 marks)

Solution
It may not be valid (even though it is interpolation) as the correlation is moderate.
Specific behaviours
✓ correctly states prediction may not be valid ✓ correctly states correlation is moderate

(g) Is there any statistical evidence to support the research view that a higher birth weight will cause a shorter gestation period? Justify your answer. (2 marks)

Solution
There is not enough evidence to say there is a causal relationship between birth weight and a shorter gestation period, there may be other factors involved.
Specific behaviours
✓ correctly states there is not enough supporting evidence ✓ states there may be other factors involved

Question 5 **(7 marks)**

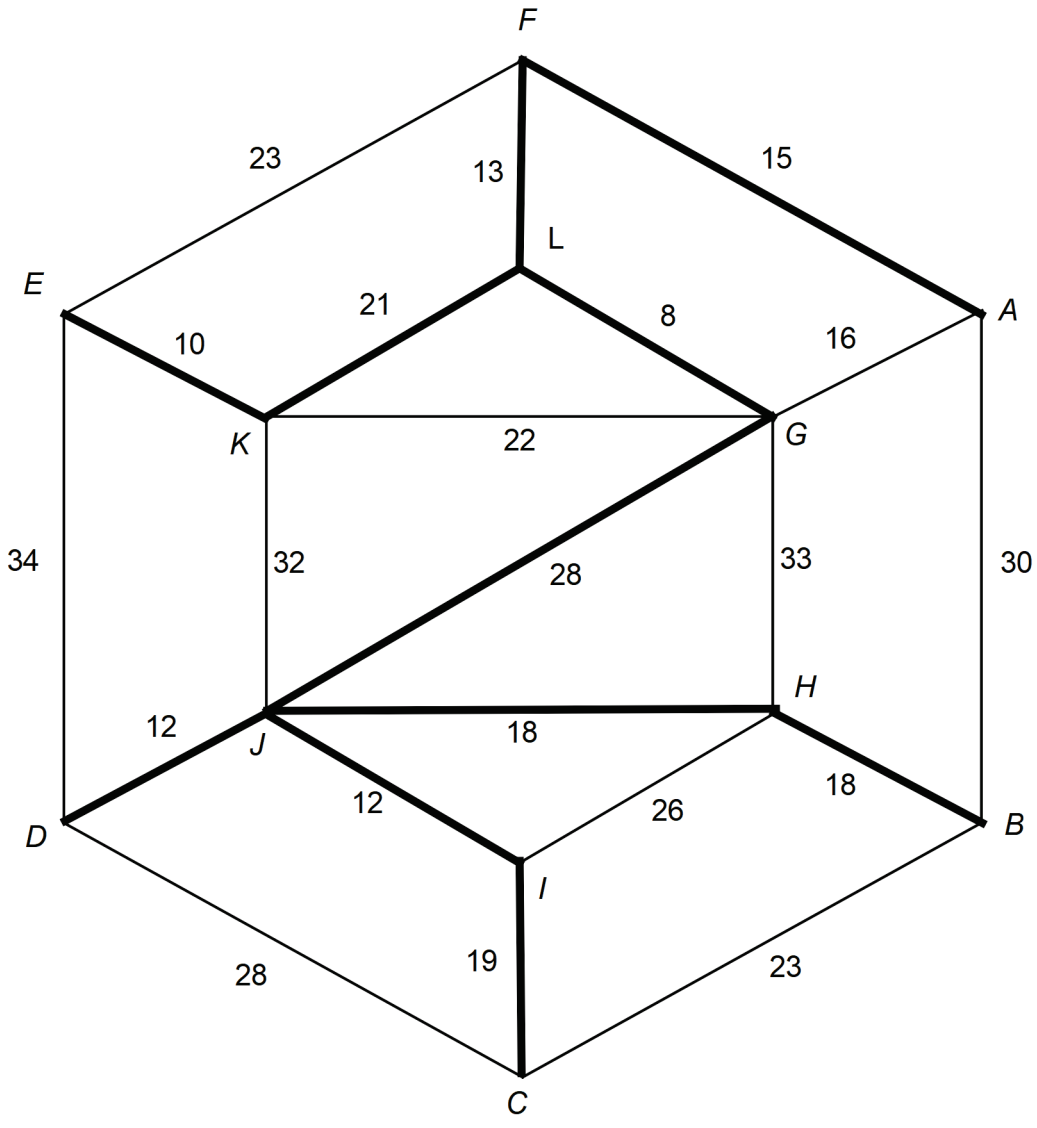
A communication wi-fi network is to be installed to service a shopping centre connecting 12 shops located at vertices *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, *I*, *J*, *K* and *L*. The only practical connections between vertices are shown on the following network.

The number on each edge is the quoted price, in hundreds of dollars, for the direct link between the vertices.

(a) A minimal spanning tree is to be used to determine the minimum cost of this installation.

(i) Show clearly on the network the minimum spanning tree solution. (3 marks)

Solution



Specific behaviours

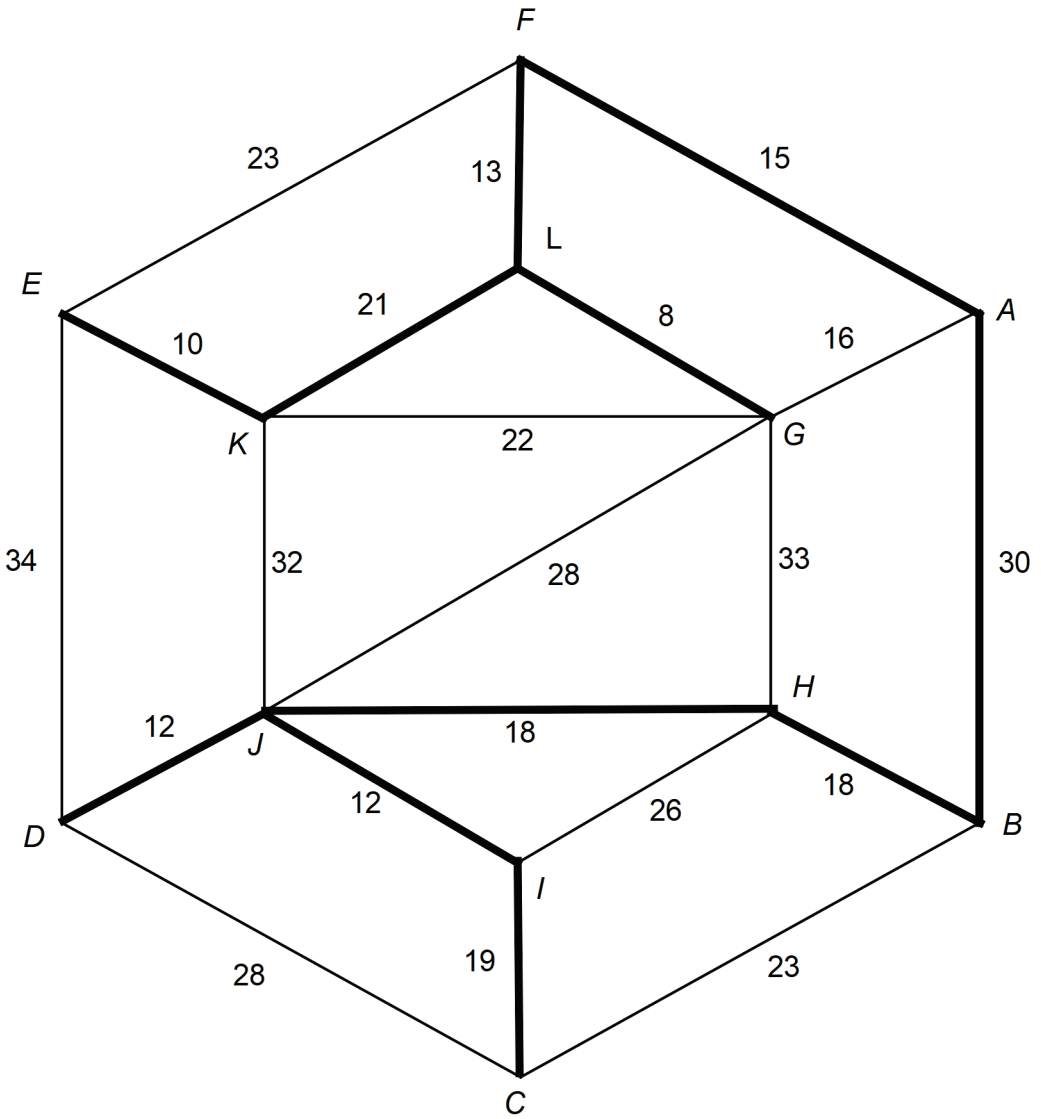
- ✓ shows at least 5 correct edges
- ✓ shows at least 8 correct edges
- ✓ shows correct minimum spanning tree

(ii) Determine the minimum cost. (2 marks)

Solution
10 + 21 + 13 + 15 + 8 + 28 + 18 + 18 + 12 + 12 + 19 = 174 Cost = $17 400
Specific behaviours
✓ determines sum of edges is 174 ✓ determines cost is $17 400

(b) Due to further construction at the shopping centre, edge *GJ* is now not feasible. Explain how this will change the solution for part (a). (2 marks)

Solution

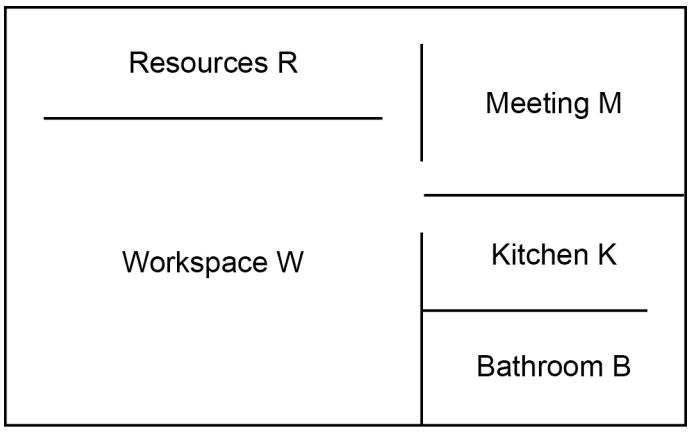


Now includes edge *AB*. Total cost = $17 600

Specific behaviours
✓ includes *AB* instead of *GJ* ✓ determines cost is $17 600 (increases by $200)

Question 6 **(8 marks)**

A small business office has five separate areas connected by doorways shown as gaps in this diagram:



This adjacency matrix below represents the number of doorways directly between each area:

$$\begin{array}{c} \\ \text{B} \\ \text{K} \\ \text{M} \\ \text{R} \\ \text{W} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{B} & \text{K} & \text{M} & \text{R} & \text{W} \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & \boldsymbol{Y} \\ 0 & \boldsymbol{X} & 1 & 2 & 0 \end{bmatrix} \end{array}$$

(a) State the meaning of the zero entries in the matrix. (1 mark)

Solution
There is no doorway between the area(s)
Specific behaviours
✓ correctly states that there is no doorway

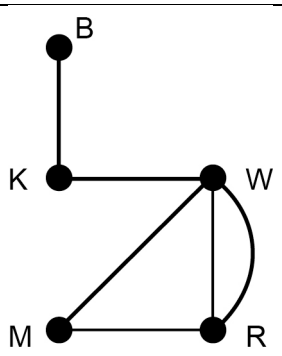
(b) Determine the value of $\boldsymbol{X}$ and $\boldsymbol{Y}$. (2 marks)

Solution
$X = 1$ $Y = 2$
Specific behaviours
✓ states correct value of X ✓ states correct value of Y

(c) Describe how the total number of doorways for each area can be found from the adjacency matrix. (1 mark)

Solution
Sum of the row or column corresponding to the specific area
Specific behaviours
✓ correctly identifies a row or column sum

(d) Complete this network with vertices corresponding to the office areas and the edges representing the doorways. (3 marks)

Solution

Specific behaviours
✓ correctly adds **B** connected to **K** ✓ correctly connects **W** to both **K** and **M** ✓ correctly connects **W** to **R** with a second edge

(e) Determine how many different routes there are between the meeting room and the workspace that pass though exactly two doorways. (1 mark)

Solution
Two (two via **R**)
Specific behaviours
✓ states correct answer

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303 Sevenoaks Street
CANNINGTON WA 6107