



**MATHEMATICS APPLICATIONS**

**Calculator-assumed**

**ATAR course examination 2021**

**Marking key**

Marking keys are an explicit statement about what the examining panel expect of candidates when they respond to particular examination items. They help ensure a consistent interpretation of the criteria that guide the awarding of marks.

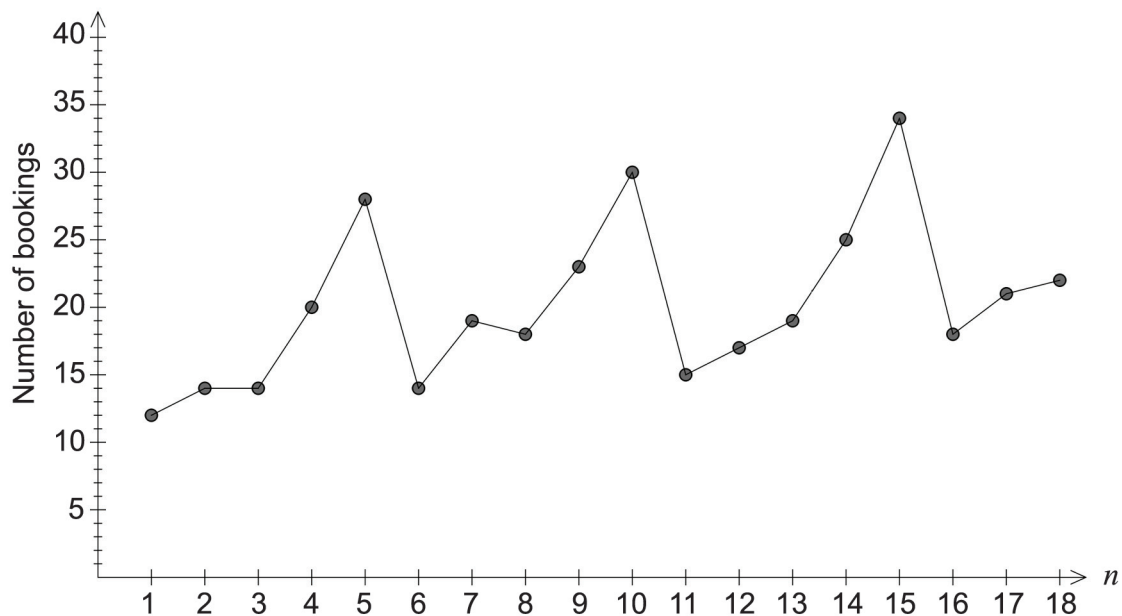
Section Two: Calculator-assumed

65% (94 Marks)

Question 8

(11 marks)

The graph below shows the number of bookings at a dog grooming salon over its first few weeks of business.



- (a) Which simple moving average would be the **most** suitable for the data displayed in this graph? (1 mark)

<b>Solution</b>
5 point moving average
<b>Specific behaviours</b>
✓ states correct moving average

A more detailed view of the same data is given in the table below.

Week	Day	<i>n</i>	Number of bookings	Seasonal mean	Number of bookings as a percentage of the seasonal mean	Seasonally adjusted figures
1	Tuesday	1	12	17.6	<b>A</b>	17.7
	Wednesday	2	14		79.55	16.9
	Thursday	3	14		79.55	16.6
	Friday	4	20		113.64	17.8
	Saturday	5	28		159.09	18.3
2	Tuesday	6	14	<b>B</b>	67.31	20.6
	Wednesday	7	19		91.35	23.0
	Thursday	8	18		86.54	21.4
	Friday	9	23		110.58	20.4
	Saturday	10	30		144.23	19.7
3	Tuesday	11	<b>C</b>	22	68.18	22.1
	Wednesday	12	17		77.27	20.6
	Thursday	13	19		86.36	22.6
	Friday	14	25		113.64	22.2
	Saturday	15	34		154.55	22.3
4	Tuesday	16	18	-	-	-
	Wednesday	17	21		-	-
	Thursday	18	22		-	-

(b) Calculate the value of **A**, **B** and **C** in the table. (3 marks)

Solution	
<b>A:</b>	$12 \div 17.6 \times 100 = 68.18$
<b>B:</b>	$(14 + 19 + 18 + 23 + 30) \div 5 = 20.8$
<b>C:</b>	$(22 \times 5) - (17 + 19 + 25 + 34) = 15$
Specific behaviours	
✓ correctly calculates the value of <b>A</b> ✓ correctly calculates the value of <b>B</b> ✓ correctly calculates the value of <b>C</b>	

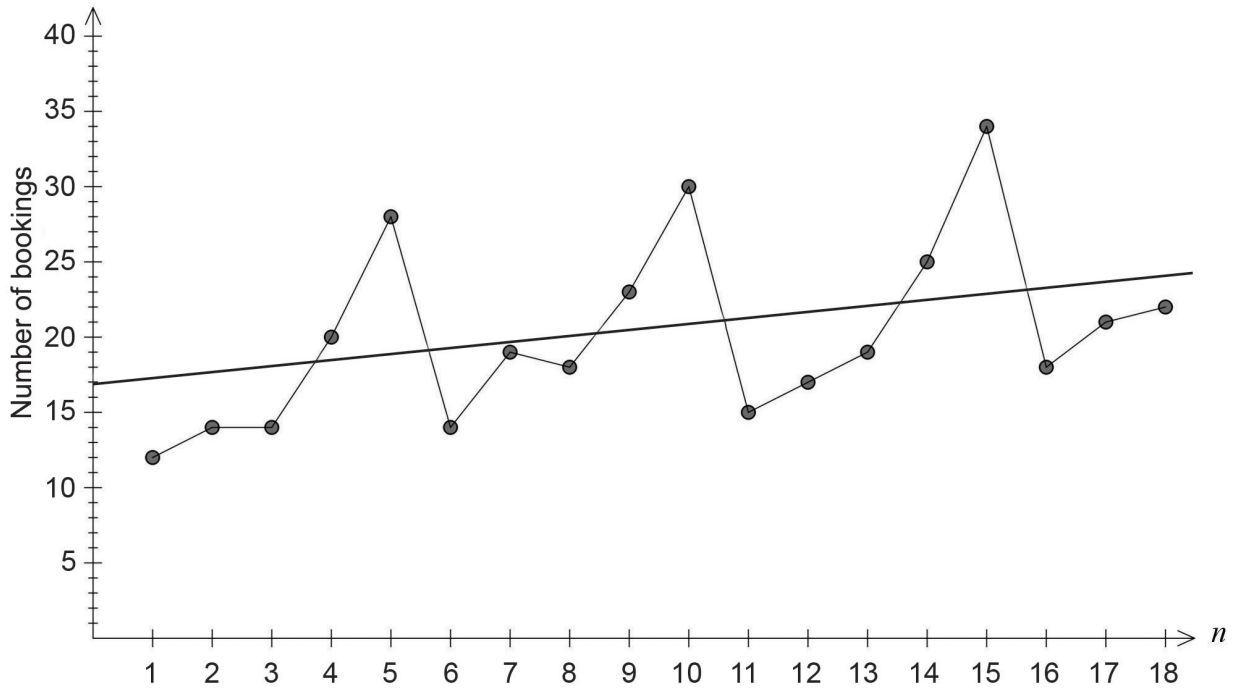
(c) Calculate the seasonal index for Saturday. (1 mark)

Solution	
SI =	1.5262 (152.62%)
Specific behaviours	
✓ correctly calculates the seasonal index	

Question 8 (continued)

- (d) The equation of the least-squares line using the seasonally adjusted figures is  $y = 0.40n + 16.94$ . Draw this line on the graph. (2 marks)

<b>Solution</b>
See graph below
<b>Specific behaviours</b>
✓ plots one correct point
✓ plots a second correct point and draws the least-squares line across the given data



- (e) (i) Use the equation of the least-squares line given in part (d) to predict the number of bookings that will be made for the Saturday of Week 5. (2 marks)

<b>Solution</b>
Saturday of Week 5: $n = 25$ $y = 0.40(25) + 16.94 = 26.94$ Therefore, predicted number of bookings is $26.94 \times 1.5262 = 41.12$ (approx. 41)
<b>Specific behaviours</b>
✓ correctly uses $n = 25$ ✓ correctly multiplies by the seasonal index to predict the number of bookings

- (ii) Comment on this prediction. (2 marks)

<b>Solution</b>
Not valid, since it is extrapolation
<b>Specific behaviours</b>
✓ correctly states it is not valid ✓ correctly states it is outside the range of the data (extrapolation)

Question 9

(15 marks)

A real estate agent is analysing data on the sale of houses over the last six months. The table shows the average sale price of houses, in thousands of dollars (\$'000), and their distance from the ocean, to the nearest kilometre.

<b>Distance from the ocean (km)</b>	1	2	3	4	5	6	7	8	9
<b>Average sale price (\$'000)</b>	1758	1909	2265	1934	1228	1641	751	967	676

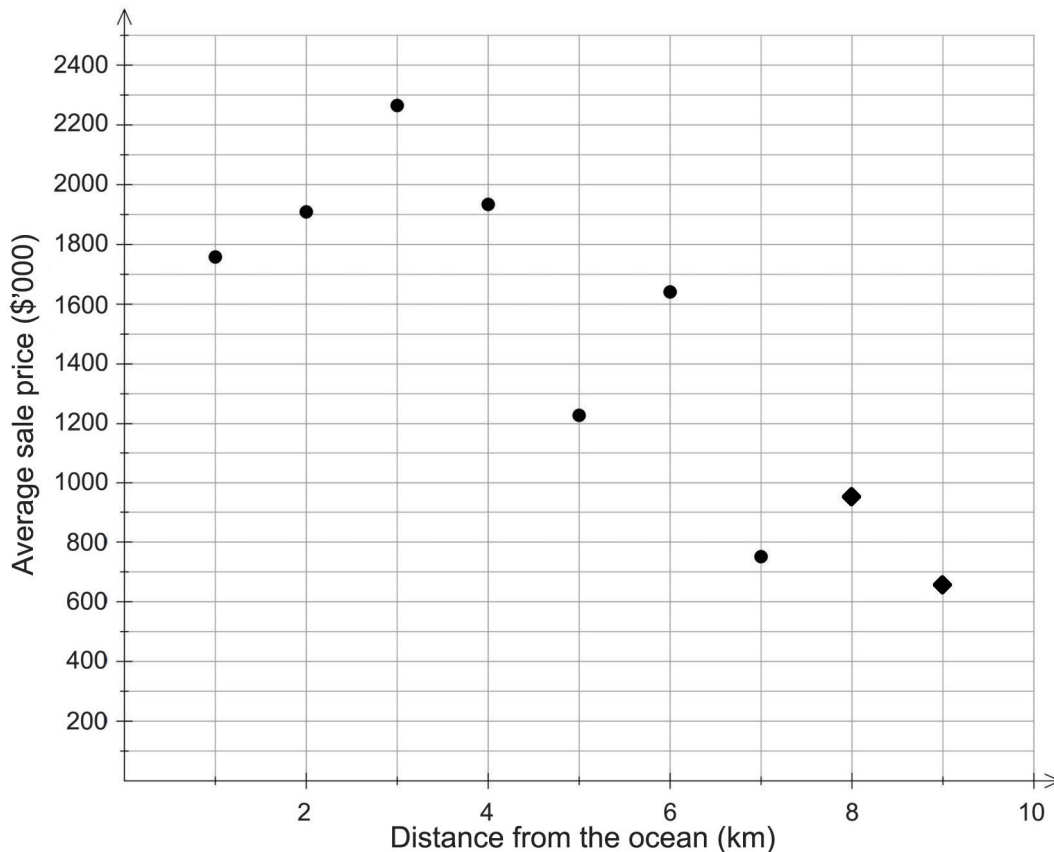
(a) State the explanatory variable.

(1 mark)

<b>Solution</b>
Distance from the ocean
<b>Specific behaviours</b>
✓ states correct variable

(b) On the scatterplot below, plot the last two data points from the table.

(1 mark)



<b>Solution</b>
see graph above
<b>Specific behaviours</b>
✓ both points are plotted correctly

Question 9 (continued)

- (c) Determine the equation of the least-squares line for these data. (1 mark)

<b>Solution</b>
$p = -174.58d + 2331.7$ ( $y = -174.58x + 2331.7$ )
<b>Specific behaviours</b>
✓ correctly determines equation

- (d) Interpret the slope of the least-squares line from part (c) in the context of this question. (2 marks)

<b>Solution</b>
For each kilometre extra from the ocean, the selling price drops by an average of \$174 580
<b>Specific behaviours</b>
✓ refers to correct variables ✓ correctly states average price drop of \$174 580

- (e) (i) State the value of the correlation coefficient for these data. (1 mark)

<b>Solution</b>
$r = -0.84$
<b>Specific behaviours</b>
✓ correct answer

- (ii) What does the correlation coefficient measure? (1 mark)

<b>Solution</b>
Strength of the linear relationship
<b>Specific behaviours</b>
✓ correct answer

- (iii) Describe the association between the variables in terms of direction and strength. (2 marks)

<b>Solution</b>
Strong, negative association
<b>Specific behaviours</b>
✓ correctly states direction ✓ correctly states strength

- (f) What percentage of the variation in average sale price can be explained by the variation in the distance from the ocean? (1 mark)

<b>Solution</b>
$r^2 = 0.7007 \approx 70.1\%$
<b>Specific behaviours</b>
✓ calculates correct percentage

- (g) In six months time, a homebuyer will have saved enough money for a deposit on a house. He would like to live about four kilometres from the ocean.
- (i) Use the equation of the least-squares line from part (c) to predict the average sale price of houses four kilometres from the ocean. (1 mark)

<b>Solution</b>
$d = 4 \Rightarrow p = 1633.38 \Rightarrow \text{price} = \$1\ 633\ 380$
<b>Specific behaviours</b>
✓ determines correct price estimate

- (ii) Explain why your prediction is different from the average sale price given in the table. (1 mark)

<b>Solution</b>
The table value is the true average value, whereas the prediction comes from the least-squares line.
<b>Specific behaviours</b>
✓ gives a valid reason

- (h) Give a reason why extrapolation in the context of this question would not make sense. (1 mark)

<b>Solution</b>
Following this model, the selling price would eventually become less than zero.
<b>Specific behaviours</b>
✓ gives a valid reason

- (i) The real estate agent was talking to some potential buyers and was heard to make the statement, "Having property closer to the ocean causes higher selling prices". Comment on this statement. (2 marks)

<b>Solution</b>
Not a valid statement. Only <b>average</b> price increases the closer to the ocean. <b>or</b> Not a valid statement. One variable does not cause the other to occur (not causally related).
<b>Specific behaviours</b>
✓ states comment not valid ✓ gives an appropriate reason

Question 10

(14 marks)

Wendy moved into an apartment and organised a loan of \$16 000 to purchase new furniture. To pay off the loan Wendy makes repayments of \$600 at the end of each month. The spreadsheet below shows the progress of her loan.

Month	Opening balance	Interest	Repayment	Closing balance
1	16 000.00	98.67	600.00	15 498.67
2	15 498.67	95.58	600.00	14 994.24
3	14 994.24	92.46	600.00	14 486.71
4				

- (a) Write a calculation to show that the yearly interest rate is approximately 7.4%. (2 marks)

Solution	
$\frac{98.67}{16\ 000} \times 100 = 0.6166875\%$	
$0.6166875 \times 12 = 7.40025\% \approx 7.4\%$	
Specific behaviours	
✓ correctly calculates monthly interest	
✓ correctly calculates yearly interest rate	

- (b) Complete the fourth row of the spreadsheet. (3 marks)

Solution					
	<b>Month</b>	<b>Opening balance</b>	<b>Interest</b>	<b>Repayment</b>	<b>Closing balance</b>
	1	16 000.00	98.67	600.00	15 498.67
	2	15 498.67	95.58	600.00	14 994.24
	3	14 994.24	92.46	600.00	14 486.71
	4	<b>14 486.71</b>	<b>89.33</b>	<b>600.00</b>	<b>13 976.04</b>
Specific behaviours					
	✓ correct Opening balance and Repayment				
	✓ correct Interest				
	✓ correct Closing balance				

- (c) Write a recursive rule to determine the closing balance of the loan at the end of each month. (2 marks)

Solution	
$T_{n+1} = \left(1 + \frac{0.074}{12}\right) T_n - 600, T_0 = 16\ 000$	
<b>or</b>	
$T_{n+1} = 1.00617 T_n - 600, T_0 = 16\ 000$	
Specific behaviours	
✓ states correct rule	
✓ states correct term zero	



- (d) Determine how many months it will take Wendy to pay off the loan. (1 mark)

<b>Solution</b>
30 months
<b>Specific behaviours</b>
✓ gives correct number of months

- (e) Calculate how much interest is paid over the duration of the loan. (3 marks)

<b>Solution</b>
$30 \times 600 - 465.56 = \$17\,534.44$ Interest = $\$17\,534.44 - 16\,000 = 1534.44$ <b>or</b> $29 \times 600 = 17\,400$ $\left(1 + \frac{0.074}{12}\right) \times 133.62 = 134.44$ $17\,400 + 134.44 = 17\,534.44$ Interest = $\$17\,534.44 - 16\,000 = 1534.44$
<b>Specific behaviours</b>
✓ 30 months of \$600 less overpayment (or second method) ✓ calculates correct total ✓ calculates correct interest

On reflection, Wendy realised she could have repaid \$800 each month.

- (f) Determine the maximum amount Wendy would have been able to borrow if all other details of the loan and repayment time remained the same. (3 marks)

<b>Solution</b>
$N = 30, I = 7.4, PMT = 800, FV = 0, P/Y = 12, C/Y = 12$ $PV = -21849.52$ $\therefore$ Wendy could borrow \$21 849.52
<b>Specific behaviours</b>
✓ uses at least 4 correct values for N, I, PMT, FV, P/Y, C/Y ✓ uses all correct values for N, I, PMT, FV, P/Y, C/Y ✓ determines correct value for PV

Question 11

(8 marks)

Judith monitors the water quality in her garden pond at the same time every day. She likes to maintain the concentration of algae at between 200 and 250 units per 100 litres (L). Her measurements show that the concentration increases daily according to the recursive rule  $C_{n+1} = 1.025C_n$ , where  $C_1 = 200$  units per 100 L (the minimum concentration).

When the concentration gets above the 250 units per 100 L limit, she treats the water to bring the concentration back to the minimum 200 units per 100 L.

(a) If Judith treated the water on Sunday, 6 December 2020, determine

(i) the concentration on Wednesday, 9 December 2020. (2 marks)

Solution	
$C_4 = 215.38$ units per 100 L	
Specific behaviours	
✓ correctly identifies Wednesday is $C_4$	
✓ correctly calculates the concentration	

(ii) the day and date when she next treated the water. (2 marks)

Solution	
$C_{10} = 249.77$ , $C_{11} = 256.02$ i.e. Wednesday 16 December (11 <sup>th</sup> term)	
Specific behaviours	
✓ calculates correct day	
✓ calculates correct date	

(b) During the first week of January 2021, Judith monitored the water and recorded the following readings.

Day	1	2	3	4	5	6	7
Concentration (C)	200	206	212.18	218.55	225.10	231.85	238.81

(i) Determine the revised recursive rule. (2 marks)

Solution	
$\frac{206}{200} = 1.03 \therefore C_{n+1} = 1.03C_n, C_1 = 200$	
Specific behaviours	
✓ calculates correct ratio	
✓ states correct initial value	

- (ii) If she treated the water on 10 January and went on holiday until 20 January, when she next treated the water, calculate the concentration of the water on her return, assuming the recursive rule from part (b)(i) is used. (2 marks)

<b>Solution</b>
20 January is the eleventh term. $C_{11} = 268.78$ units per 100 L
<b>Specific behaviours</b>
✓ determines correct term ✓ calculates correct value

Question 12

(6 marks)

Virat purchases a new motor vehicle for \$24 500. For the first two years the vehicle depreciates at a rate of 13% per year and for the third year it depreciates at a lower rate of 9.5% per year.

- (a) Calculate the value of the vehicle after one year. (1 mark)

<b>Solution</b>
$24500(1-0.13)^1 = \$21\,315$
<b>Specific behaviours</b>
✓ correctly calculates the value of the vehicle after 1 year

- (b) Calculate the value of the vehicle after the first three years. (2 marks)

<b>Solution</b>
$24500(1-0.13)^2(1-0.095)^1 = \$16\,782.37$
<b>Specific behaviours</b>
✓ correctly calculates the value of the vehicle after 2 years ✓ calculates the value of the vehicle after 3 years using the correct rate of depreciation for the third year

For the next three years the rate of depreciation is constant at  $r\%$  per year. The average rate of depreciation for the first six years is 11% per year.

- (c) Calculate the value of  $r$  as a percentage. (3 marks)

<b>Solution</b>
Value of vehicle at the end of 6 years using average rate of depreciation: $24\,500(1-0.11)^6 = \$12\,176.04$
Therefore $\$12176.04 = 16782.37\left(1-\frac{r}{100}\right)^3 \Rightarrow r = 10.14\%$
<b>Specific behaviours</b>
✓ correctly calculates the value of the vehicle after 6 years using average rate of depreciation ✓ correctly sets up an equation to solve for $r$ ✓ correctly determines the value of $r$

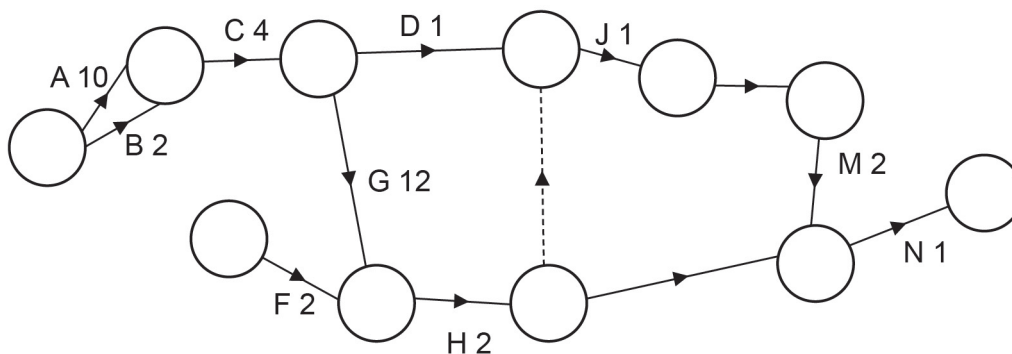
Question 13

(12 marks)

A kitchen renovation project consists of a number of tasks of different durations and completed in different orders. One such renovation has this information summarised in the table below.

Task	Task description	Duration (days)	Immediate predecessor(s)
A	Prepare plans	10	None
B	Select contractor	2	None
C	Review plans	4	A, B
D	Purchase appliances	1	C
E	Remove old appliances and benches	2	C
F	Prepare electrics and plumbing	2	E
G	Construct new cupboards and benches	12	C
H	Install cupboards and benches	2	F, G
J	Install appliances	1	see part (b)
K	Tiling and splashbacks	5	J
L	Flooring	3	H
M	Complete electrical and plumbing	2	K
N	Test and handover	1	L, M

A partially-completed project network is shown below for this table.



(a) Complete the network diagram.

(2 marks)

Solution
Specific behaviours
<ul style="list-style-type: none"> <li>✓ correctly draws edge E</li> <li>✓ writes correct weight on edges E, K, L</li> </ul>

Question 13 (continued)

- (b) Identify the immediate predecessor(s) of Task J. (1 mark)

<b>Solution</b>
Immediate predecessors of Task J are D and H.
<b>Specific behaviours</b>
✓ identifies correct immediate predecessors for task J

- (c) What does the dotted line on the network indicate? (1 mark)

<b>Solution</b>
H is the predecessor for J.
<b>Specific behaviours</b>
✓ states correct reason

- (d) Determine the critical path and the minimum completion time for the project. (2 marks)

<b>Solution</b>
ACGHJKMN. Minimum completion time is 37 days.
<b>Specific behaviours</b>
✓ states correct critical path ✓ states correct minimum completion time

- (e) Determine the float time for Task D and explain its meaning in terms of the renovation. (2 marks)

<b>Solution</b>
Float time = 13 days. This means Task D can be delayed by up to 13 days.
<b>Specific behaviours</b>
✓ states correct float time ✓ gives correct explanation in terms of the renovation

- (f) Once Task E (removal of old appliances and benches) begins, the kitchen cannot be used. What is the least amount of time the occupants of the house will be without a working kitchen? (1 mark)

<b>Solution</b>
15 days
<b>Specific behaviours</b>
✓ states correct time

- (g) If Task G was actually completed in nine days, how would this affect the critical path and minimum completion time? (2 marks)

<b>Solution</b>
Critical path stays the same. Minimum completion time is now 34 days.
<b>Specific behaviours</b>
✓ states the critical path stays the same ✓ states correct new minimum completion time

Tasks E and F are both delayed.

- (h) What is the maximum possible delay that does **not** affect the original minimum completion time? (1 mark)

<b>Solution</b>
8 days
<b>Specific behaviours</b>
✓ states correct time

Question 14

(11 marks)

Patrick has retired and invested his lump sum superannuation payout of \$717 850 at a rate of 5.7% per annum compounded monthly. He begins the investment strategy from 1 January.

- (a) Patrick will receive \$4500 at the end of each month for general living expenses and will also receive a further \$4000 at the end of each year for an annual holiday.

- (i) Identify this type of investment account. (1 mark)

<b>Solution</b>
Annuity
<b>Specific behaviours</b>
✓ states correct answer

- (ii) Determine the balance in the account at the end of the first year. (4 marks)

<b>Solution</b>
N = 12, I = 5.7, PV = -717 850, PMT = 4500, P/Y = 12, C/Y = 12 FV = 704 420.20 Balance at end of year 1 = 704 420.20 - 4000 = \$700 420.20
<b>Specific behaviours</b>
✓ uses at least 4 correct values for N, I, PV, PMT, P/Y, C/Y ✓ uses all correct values for N, I, PV, PMT, P/Y, C/Y ✓ determines correct value for FV ✓ determines correct end of year balance

- (iii) Determine the balance in the account at the end of the second year. (3 marks)

<b>Solution</b>
N = 12, I = 5.7, PV = -700 420.20, PMT = 4500, P/Y = 12, C/Y = 12 FV = 685 970.53 Balance at end of year 2 = \$685 970.53 - 4000 = \$681 970.53
<b>Specific behaviours</b>
✓ uses correct value for PV ✓ determines correct FV ✓ determines correct end of year 2 balance

- (b) When Patrick retired, he also considered the option of setting up a perpetuity with his superannuation payout still at 5.7% per annum compounded monthly. Calculate the quarterly payments Patrick would have received with this perpetuity in place. (3 marks)

<b>Solution</b>
N = 2 (can be any value), I = 5.7, PV = -717 850, FV = 717 850, P/Y = 4, C/Y = 12 Quarterly payments = \$10 278.03
<b>Specific behaviours</b>
✓ uses at least 4 correct values for N, I, PV, FV, P/Y, C/Y ✓ uses all correct values for N, I, PV, FV, P/Y, C/Y ✓ states correct quarterly payments



Question 15

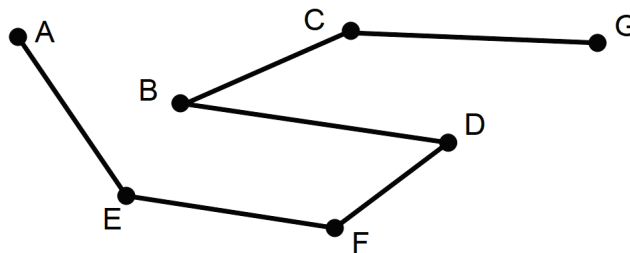
(7 marks)

A number of streets connecting locations A to G inclusive have been identified as needing a lighting upgrade with energy-efficient LED lamps. The council has decided that each location must have at least one connecting street that receives the lighting upgrade. The table below shows the cost, in dollars, of completing the upgrade in each street.

	A	B	C	D	E	F	G
A	-	7900	10 300	-	7100	-	-
B	7900	-	7600	6200	6500	-	15 400
C	10 300	7600	-	8500	-	12 200	9200
D	-	6200	8500	-	8000	4700	9800
E	7100	6500	-	8000	-	4800	-
F	-	-	12 200	4700	4800	-	10 100
G	-	15 400	9200	9800	-	10 100	-

The council has a limited budget, so it needs to complete the upgrades at minimum cost.

- (a) Demonstrate the use of Prim’s algorithm on the table above to determine the minimum spanning tree and draw it on the diagram below. (3 marks)



Solution							
	A	B	C	D	E	F	G
A	-	7900	10300	-	7100	-	-
B	7900	-	7600	6200	6500	-	15400
C	10300	7600	-	8500	-	12200	9200
D		6200	8500	-	8000	4700	9800
E	7100	6500		8000	-	4800	
F			12200	4700	4800	-	10100
G		15400	9200	9800		10100	-

See diagram above for graph of spanning tree.

**Specific behaviours**

- ✓ correctly demonstrates use of Prim’s algorithm on the table
- ✓ obtains correct solution using Prim’s algorithm
- ✓ correctly draws minimum spanning tree

**Question 15** (continued)

- (b) The council has set aside \$42 000 to complete the lighting upgrades. Does it have enough in its budget to make the necessary upgrades? Justify your answer. (2 marks)

<b>Solution</b>
$\text{Cost} = 7100 + 4800 + 4700 + 6200 + 7600 + 9200$ $= \$39\,600$ <p><math>\therefore</math> sufficient funds</p>
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>✓ calculates minimum cost in dollars</li> <li>✓ states there are sufficient funds</li> </ul>

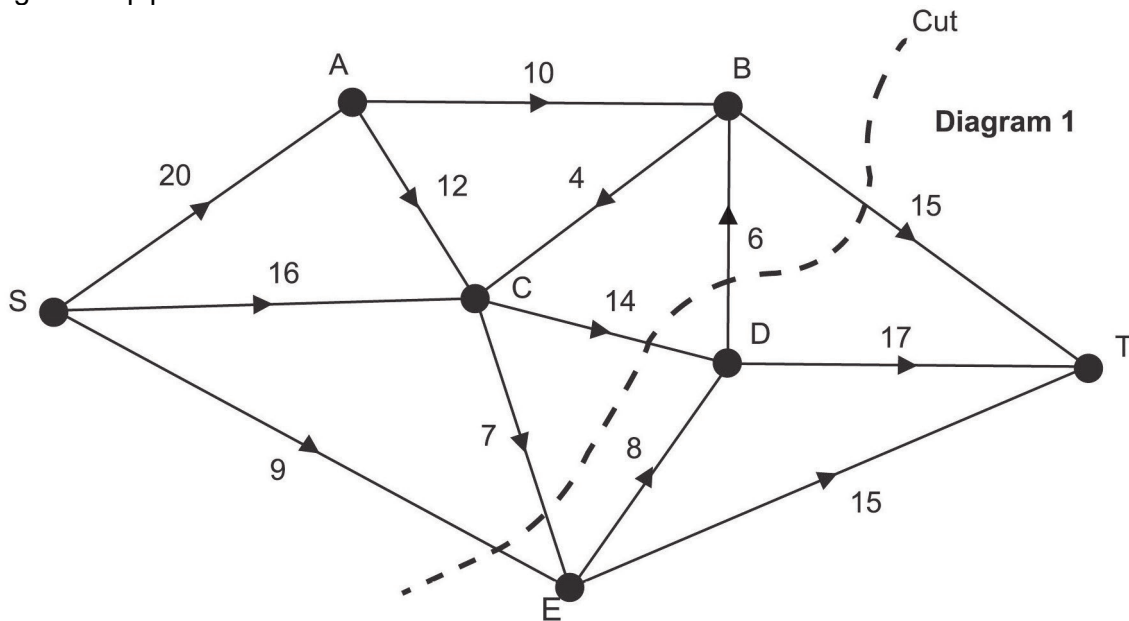
- (c) Due to the location of the police station, the upgrade from D to C must be included. What effect will this have on the minimum cost and the spanning tree? (2 marks)

<b>Solution</b>
$39\,600 - 7600 \text{ (BC)} = 32\,000. \quad 32\,000 + 8500 \text{ (DC)} = 40\,500.$ <p>This means the cost now increases by \$900 to \$40 500. The spanning tree will not include BC.</p>
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>✓ states that cost increases by \$900</li> <li>✓ correctly states that connection BC no longer exists</li> </ul>

Question 16

(10 marks)

The graph below shows a network of water pipes. The water source and main pumping station are located at S. The distribution centre is at T and the other vertices are intermediate pumping stations. The weights on the edges show the capacities in kilolitres per hour that can flow through each pipe.



- (a) (i) Determine the value of the cut shown in **Diagram 1**. (1 mark)

<b>Solution</b>
Cut = 9 + 7 + 14 + 0 + 15 = 45
<b>Specific behaviours</b>
✓ correctly calculates the value of cut

- (ii) Using your answer to part (a)(i), what can be said about the maximum flow of water through the network? (1 mark)

<b>Solution</b>
The maximum flow through the network is $\leq 45$ .
<b>Specific behaviours</b>
✓ correct answer

- (b) State the maximum possible flows along the paths SABT and SCDT. (2 marks)

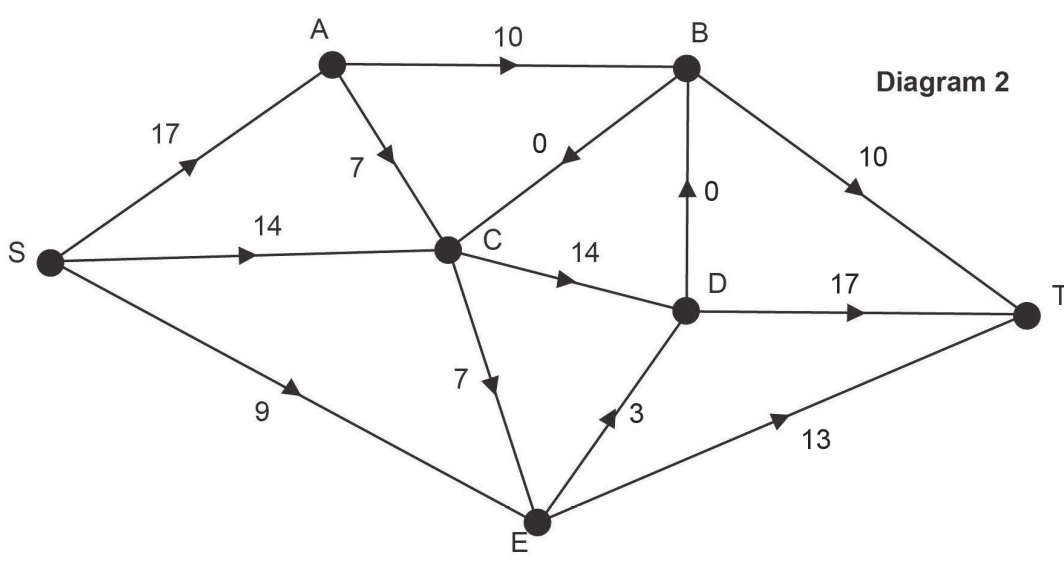
<b>Solution</b>
SABT = 10 SCDT = 14
<b>Specific behaviours</b>
✓ correctly states maximum flow for SABT ✓ correctly states maximum flow for SCDT

**Question 16** (continued)

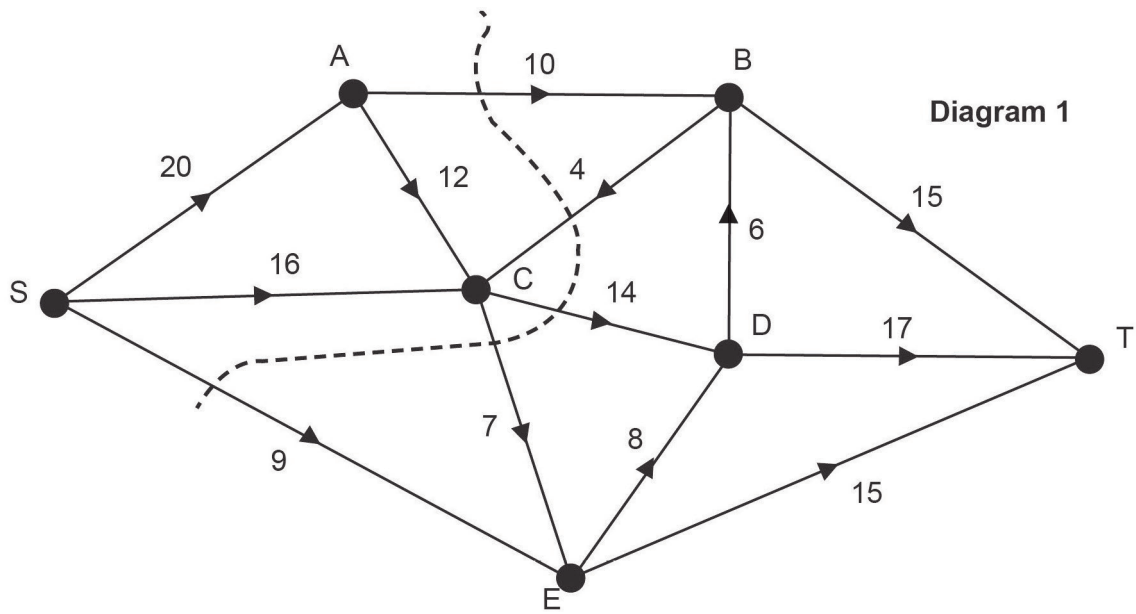
- (c) Determine the maximum flow from S to T, listing each path and the corresponding flow. (3 marks)

<b>Solution</b>
One possibility, Using answers from part (b): SABT = 10, SCDT = 14, SACEDT = 3, SAET = 4, SET = 9 $\therefore$ maximum flow = 40 kilolitres per hour <b>or</b> Another possibility: SABT = 10, SACDBT = 5, SACDT = 5, SCDT = 4, SCEDT = 7, SEDT = 1, SET = 8 $\therefore$ maximum flow = 40 kilolitres per hour
<b>Specific behaviours</b>
✓ correctly gives at least one other path ✓ correctly gives 2 correct paths ✓ states correct maximum flow

- (d) Using **Diagram 2** below, indicate a possible flow along each pipe corresponding to the maximum flow calculated in part (c). (2 marks)

<b>Solution</b>
Using the first possibility: 
<b>Specific behaviours</b>
✓ correctly shows flow from S = flow into T ✓ correctly labels diagram

- (e) Determine the minimum cut that corresponds to the maximum flow and illustrate this on the copy of **Diagram 1** below. (1 mark)



<b>Solution</b>
See above diagram for cut
<b>Specific behaviours</b>
✓ correctly draws the minimum cut

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