



MATHEMATICS APPLICATIONS

Calculator-assumed

ATAR course examination 2022

Marking key

Marking keys are an explicit statement about what the examining panel expect of candidates when they respond to particular examination items. They help ensure a consistent interpretation of the criteria that guide the awarding of marks.

Section Two: Calculator-assumed

65% (97 Marks)

Question 8

(9 marks)

Yash is saving for a deposit on an apartment, which is about to begin construction.

- (a) Two saving plans are found in advertisements in a newspaper.
 Option 1: 3.05% per annum compounded at the end of each month.
 Option 2: 3.01% per annum compounded at the end of each day.

Which of these two options would give the better return? Justify your answer mathematically. (3 marks)

Solution
Option 1 $i = 0.0305, n = 12$ effective interest = 3.09% Option 2 $i = 0.0301, n = 365$ effective interest = 3.06% \therefore Option 1 is better
Specific behaviours
<ul style="list-style-type: none"> ✓ correctly calculates option 1 effective interest ✓ correctly calculates option 2 effective interest ✓ correct conclusion

Yash locates a better investment option online. This offers 3.5% per annum compounded at the end of each day. Yash will begin with an initial deposit of \$12 300 and a weekly deposit of \$300. The deposit required to secure the apartment in 12 months is \$30 000.

- (b) (i) Show that this option will not be sufficient to reach the total needed to secure the apartment. State how far short of the required amount this option will be. (4 marks)

Solution
$N = 52, I = 3.5, PV = -12300, PMT = -300, P/Y = 52, C/Y = 365$ $FV = 28608.96$ Short by \$1391.04
Specific behaviours
<ul style="list-style-type: none"> ✓ uses at least 4 correct values for N, I, PV, PMT, P/Y, C/Y ✓ uses all correct values for N, I, PV, PMT, P/Y, C/Y ✓ determines correct FV ✓ determines correct shortfall

- (ii) Determine what the weekly deposit needs to be for a total of \$30 000 to be achieved by the end of 12 months. (2 marks)

Solution
$N = 52, I = 3.5, PV = -12300, FV = 30000, P/Y = 52, C/Y = 365$ $PMT = -326.29$ The weekly deposit is \$326.29 (to the nearest cent)
Specific behaviours
<ul style="list-style-type: none"> ✓ uses all correct values for N, I, PV, FV, P/Y, C/Y ✓ determines correct payment

Question 9

(7 marks)

A study of a penguin colony on an island was conducted and it found the initial population size of 1200 was dropping by 14% each year due to the introduction of non-native predators.

- (a) Explain why the population after n weeks is $1200 \times (0.86)^n$ penguins. (2 marks)

Solution
Assuming population size dropping by 14% per week: The population of penguins in each week is 86% of that of the previous week, so $r = 0.86$. The initial population (P_0) is 1200.
Specific behaviours
✓ identifies that 14% decrease is equivalent to 86% of previous value ✓ identifies 1200 is initial population size

Alternate solution
Assuming population size dropping by 14% per year: The population of penguins is not $1200 \times (0.86)^n$ after n weeks. The given expression would be the population of penguins after n years, not weeks. For n years the expression would be $1200 \times \left(1 - \frac{0.14}{52}\right)^n = 1200 \times 0.99731^n$.
Specific behaviours
✓ states given expression is not the population after n weeks ✓ states valid reason

Question 9 (continued)

After eight weeks, the Parks and Wildlife Service set traps to reduce the predator numbers. This saw the penguin population increase weekly by 6%.

- (b) State the recursive formula that models the new population growth. (2 marks)

Solution
If used given expression from part (a): $P_8 = 1200 \times 0.86^8 \approx 359$ penguins Therefore, the recursive formula is $P_{n+1} = 1.06P_n, P_0 = 359$ or If used alternate solution from part (a): $P_8 = 1200 \times 0.99731^8 \approx 1174$ penguins Therefore, the recursive formula is $P_{n+1} = 1.06P_n, P_0 = 1174$
Specific behaviours
✓ states recursive formula ✓ includes correct P_0 value

- (c) How many weeks will it take to get the population back up to the initial size? (1 mark)

Solution
If used given expression from part (a): It will take 21 weeks for population to return to 1200. or If used alternate solution from part (a): The population will return to 1200 within the first week (0.37 weeks)
Specific behaviours
✓ determines correct number of weeks

Once the population returns to the initial size, it is further helped by the introduction of penguins from a breeding program at the zoo.

The new population growth model can be represented by

$$P_{n+1} = -0.25P_n + 3000, \quad P_0 = 1200.$$

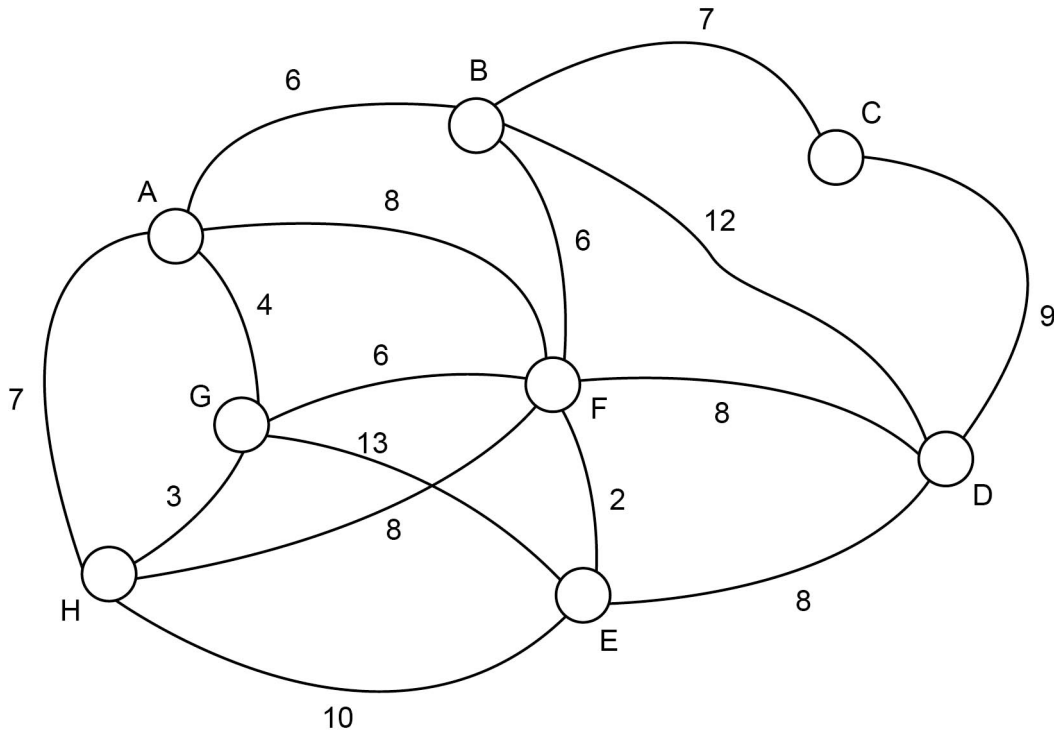
- (d) Discuss the long-term behaviour of the penguin population, now that it is being supported by the breeding program. (2 marks)

Solution
The long-term trend for the penguin population is to reach a steady state of 2400 penguins.
Specific behaviours
✓ identifies steady state solution ✓ states 2400 penguins

Question 10

(9 marks)

The network shown below represents the cycle tracks connecting the various points of interest on an island. The numbers indicate the distance between points of interest to the nearest 100 metres.



- (a) Each morning all tracks must have a safety check. Is it possible to check all individual tracks by an Eulerian trail? Justify your answer. (2 marks)

Solution
Yes, it is possible to check all individual tracks by an Eulerian trail as the network has 0 odd vertices
Specific behaviours
<ul style="list-style-type: none"> ✓ states yes it is possible ✓ identifies network has 0 odd vertices

- (b) Rubbish bins at each point of interest are emptied daily. The rubbish truck leaves from position C and must return to C after emptying all rubbish bins.

- (i) What name is given to the type of path taken by the rubbish truck? (2 marks)

Solution
Hamiltonian cycle
Specific behaviours
<ul style="list-style-type: none"> ✓ correctly states Hamiltonian ✓ correctly states cycle

- (ii) The rubbish truck wants to follow the path of least distance. State the path and its length. (3 marks)

Solution
CBAGHFEDC $7 + 6 + 4 + 3 + 8 + 2 + 8 + 9 = 47$ 4700 metres
Specific behaviours
✓ path starts and ends at C and visits all vertices ✓ correct length ✓ correct distance

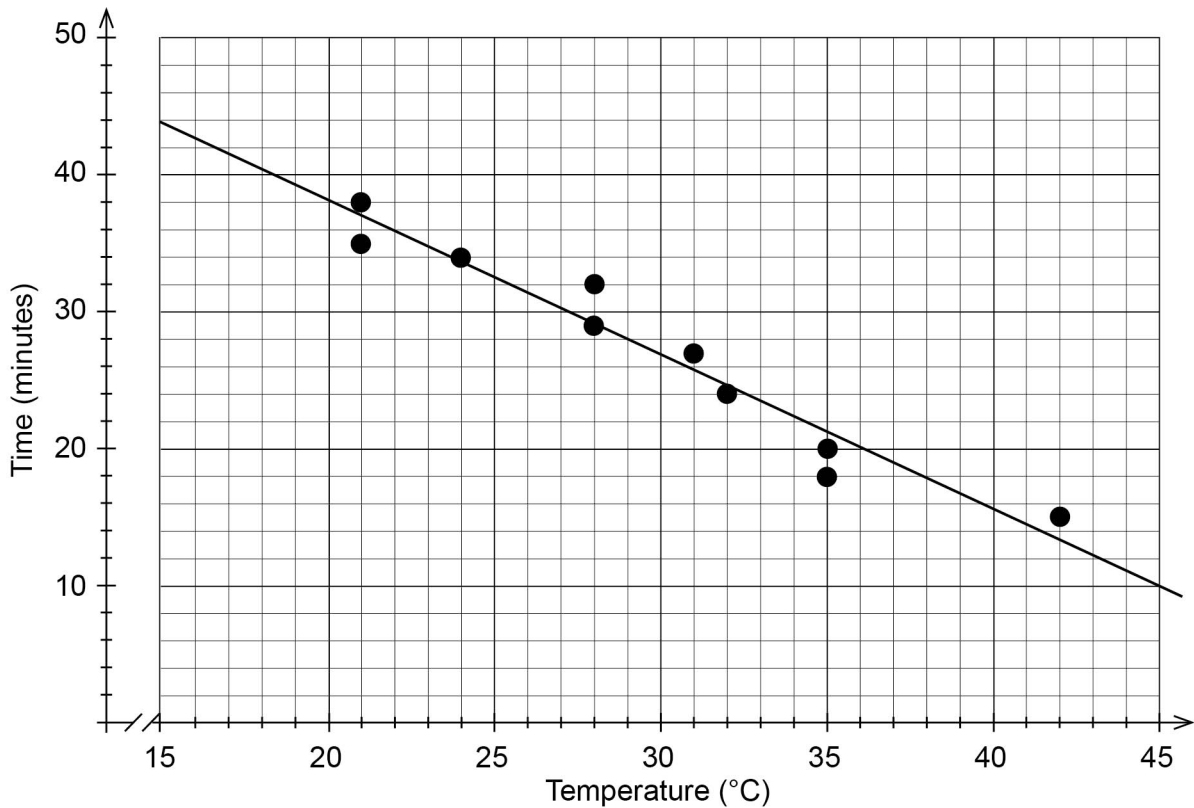
- (c) A tourist at G wishes to travel to C but path AG is closed for maintenance. Determine the shortest path possible and its length. (2 marks)

Solution
GFBC 1900 metres
Specific behaviours
✓ correct path ✓ correct length

Question 11

(13 marks)

Nullah wanted to see if there was a relationship between outside temperature and the time taken to dry his laundry. The following data was collected over a 10 day period.



Temperature (°C)	28	35	42	31	24	21	21	35	32	28
Time (minutes)	29	20	15	27	34	38	35	18	24	32

- (a) Complete the table by locating the data in the graph. (2 marks)

Solution
see table above
Specific behaviours
✓ one correct entry
✓ both correct entries

- (b) Determine the equation of the least-squares line and state the correlation coefficient. (2 marks)

Solution
$y = -1.13x + 60.74, r = -0.97$
Specific behaviours
✓ states correct equation
✓ states correct correlation coefficient

- (c) Draw the least-squares line onto the graph above. (2 marks)

Solution
see graph above
Specific behaviours
✓ line goes through approximately (28, 29) ✓ line has correct slope

- (d) Describe the association between the two variables in terms of direction and strength. (2 marks)

Solution
The two variables have a strong, negative association
Specific behaviours
✓ correctly states strong association ✓ correctly states negative association

- (e) What percentage of the variation in drying time can be explained by the variation in outside temperature? (1 mark)

Solution
$r^2 = 0.945 = 94.5\%$ \therefore 94.5% of the variation in the drying time can be explained by the variation in outside temperature
Specific behaviours
✓ states correct percentage

- (f) Identify at least one other factor that could explain the variation in drying time. (1 mark)

Solution
Answers could include: <ul style="list-style-type: none"> • wind • type of laundry material • shade.
Specific behaviours
✓ identifies at least one valid factor
Accept other relevant answers.

Question 11 (continued)

(g) The temperature on Day 11 is predicted to be 17 °C.

- (i) Use the equation for the least-squares line from part (b) to predict the time Nullah should expect his laundry to dry on this day. (1 mark)

Solution
$y = -1.13(17) + 60.74 = 41.53$
time = 41.53 minutes
Specific behaviours
✓ calculates correct time

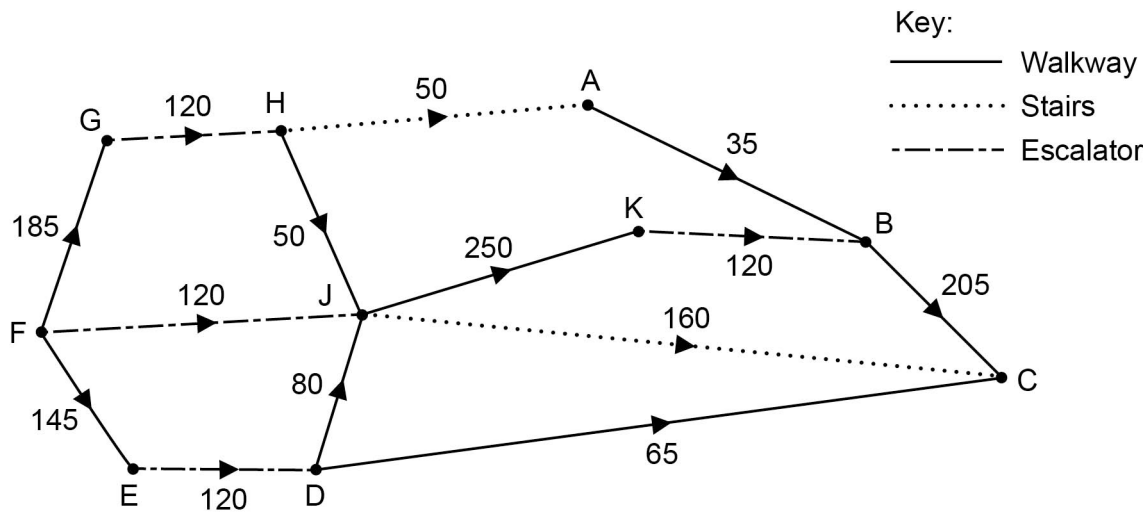
- (ii) Is this prediction reliable? Justify your answer. (2 marks)

Solution
No, it is unreliable since it is extrapolation
Specific behaviours
✓ correctly states it is unreliable
✓ correctly states extrapolation

Question 12

(8 marks)

The network below shows the different routes commuters can take, travelling from a bus stop to a designated train platform. The edges are a combination of stairs, escalators and covered walkways, with the numbers indicating the maximum number of people per minute that each route can manage.



(a) Which vertex represents

(i) the bus stop?

(1 mark)

Solution	
vertex F	
Specific behaviours	
✓ states correct vertex	

(ii) the train platform?

(1 mark)

Solution	
vertex C	
Specific behaviours	
✓ states correct vertex	

(b) Determine the maximum number of people per minute that can travel from the bus stop to the train platform, listing each path and the corresponding flow. (3 marks)

Solution	
F-G-H-A-B-C = 35	F-E-D-C = 65
F-G-H-J-K-B-C = 50	F-E-D-J-K-B-C = 55
F-J-K-B-C = 70	F-J-C = 120
F-E-D-J-C = 80	F-G-H-A-B-C = 35
F-J-C = 50	F-G-H-J-K-B-C = 50
F-E-D-C = 40	
OR	
Maximum flow = 325 people per minute	
Specific behaviours	
✓ use of a systematic approach is evident	
✓ has all possible flow paths	
✓ correctly calculates maximum flow	

Question 12 (continued)

- (c) Draw the minimum cut that corresponds to the maximum flow on the network diagram. (1 mark)

Solution	
<p>The diagram shows a network of stations A through K. Connections are categorized as Walkway (solid line), Stairs (dotted line), or Escalator (dashed line). A minimum cut is drawn through nodes G, H, J, D, E, F, A, B, and C. The flow values on the edges are: GH (120), HA (50), HG (50), HJ (50), JH (80), JF (120), FE (145), EG (185), ED (120), DJ (80), DK (250), KB (120), BK (35), BC (205), and DC (65).</p>	<p>Key:</p> <ul style="list-style-type: none"> ———— Walkway Stairs - - - - Escalator
Specific behaviours	
<p>✓ draws correct cut</p>	

- (d) The Transport Authority has approved upgrades for one **escalator** connection to increase the capacity from 120 to 150 people per minute. Which connection should be upgraded? Justify your response. (2 marks)

Solution	
<p>Four possible escalator connections:</p> <ul style="list-style-type: none"> • GH: has not used maximum flow • FJ: has more than 30 units flowing in and out • ED: has 25 units flowing in and out • KB: has no spare flow entering from HJ or FJ 	
<p>∴ FJ would be the best connection to increase by 30 people per minute.</p>	
Specific behaviours	
<p>✓ correctly identifies FJ ✓ provides valid justification</p>	

Question 13

(15 marks)

Data have been collected for nine suburbs within a city about the number of mobile phone towers and the number of births in the last 12 months for each suburb.

	Suburb								
	1	2	3	4	5	6	7	8	9
Number of mobile phone towers (n)	4	6	7	8	6	10	5	8	7
Number of births in the last 12 months (b)	25	29	35	45	38	54	22	38	39

The data has a correlation coefficient of 0.92, and the equation of the least-squares line is $b = 5.13n + 1.31$.

- (a) Interpret the gradient of the least-squares line in the context of the question. (2 marks)

Solution
There is an average increase of approximately 5 births per year for each extra mobile phone tower
Specific behaviours
<ul style="list-style-type: none"> ✓ correctly states increase ✓ correctly states 5 births per year

- (b) Explain the significance of the correlation coefficient in the context of the question. (2 marks)

Solution
There is a strong, positive relationship between the number of mobile phone towers and the number of births in the last 12 months
Specific behaviours
<ul style="list-style-type: none"> ✓ correctly mentions both variables ✓ correctly identifies strength and/or direction

- (c) (i) Predict the number of births for a suburb in this city that has nine mobile phone towers. (1 mark)

Solution
$b = 5.13(9) + 1.31 = 47.48 \approx 48$
Specific behaviours
<ul style="list-style-type: none"> ✓ correct prediction

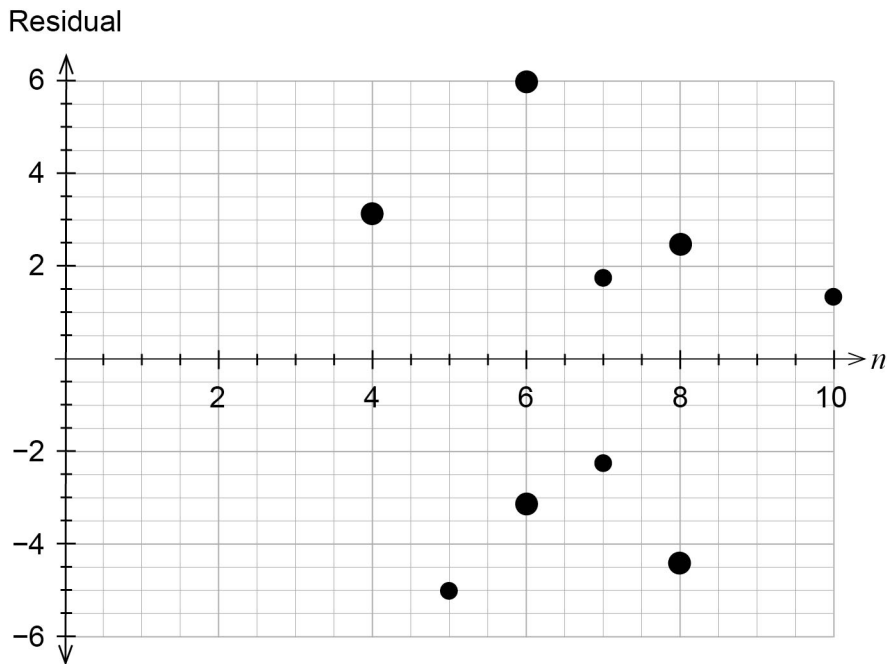
- (ii) Comment on the validity of the prediction in part (c)(i). Justify your response. (2 marks)

Solution
Prediction is valid since r is strong and it is interpolation
Specific behaviours
<ul style="list-style-type: none"> ✓ states prediction is valid ✓ states r is strong and it is interpolation

Question 13 (continued)

(d) Complete the residual plot below.

(3 marks)



Solution
(4, 3.15), (6, -3.12), (6, 5.88), (8, 2.61), (8, -4.39)
Specific behaviours
<ul style="list-style-type: none"> ✓ correctly plots one point ✓ correctly plots at least three points ✓ correctly plots all points

(e) Based on the residual plot, comment on whether the least-squares line is a suitable model for these data.

(2 marks)

Solution
Least-squares line is suitable because the residual plot shows no pattern
Specific behaviours
<ul style="list-style-type: none"> ✓ states line is suitable ✓ gives valid reason

(f) A 10th suburb has a data point (5,12) which has been verified as correct. State a practical explanation of how this could be a correct data point.

(1 mark)

Solution
New suburb with mobile towers installed and not yet fully populated
Specific behaviours
✓ correct comment

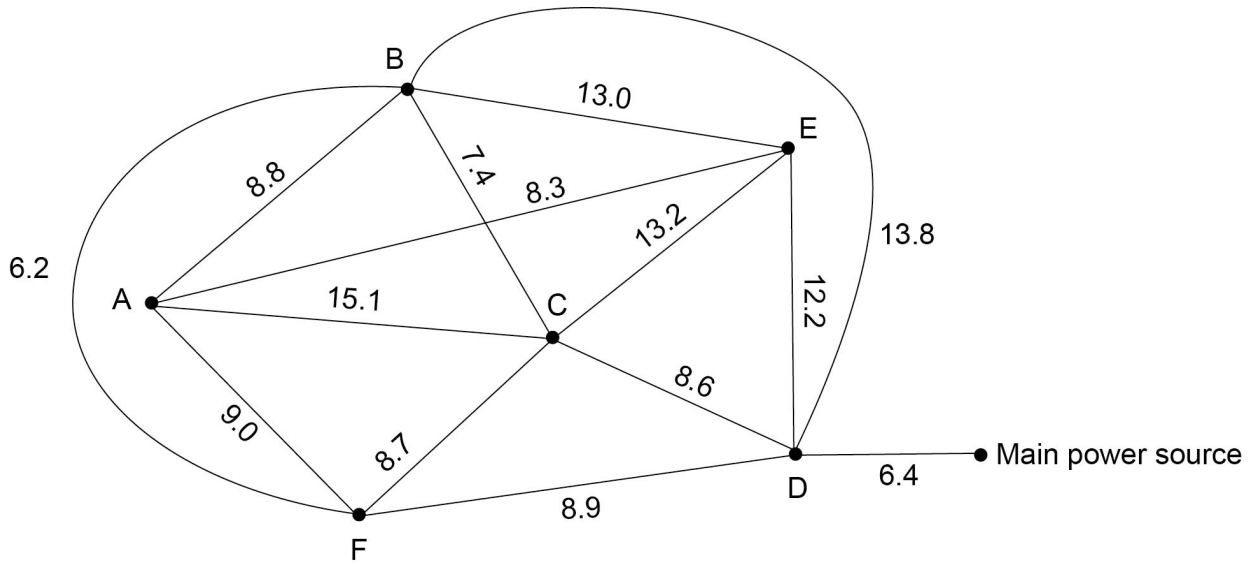
- (g) A journalist has followed the mathematics involved in working with bivariate data and is writing a report for a newspaper. What is a valid statement that could be made about the observed association between the number of mobile phone towers and the number of births in the last 12 months for suburbs within the city? (2 marks)

Solution
While the mathematics shows a strong association there is no evidence of a causal relationship. Other factors would need to be considered to establish a causal relationship.
Specific behaviours
✓ states strong mathematical relationship ✓ states causal relationship not evident

Question 14

(9 marks)

Electric wiring is being installed to six powered sites (A–F) at a new caravan park from the main power source. The distance, in metres, between each site is shown below.



(a) Complete the minimum spanning tree below.

(3 marks)

Solution	
Specific behaviours	
<ul style="list-style-type: none"> ✓ shows appropriate method for determining minimum spanning tree ✓ draws minimum spanning tree with at least 3 correct connections ✓ draws minimum spanning tree with all correct connections 	

(b) The cost of installing the electric wiring is \$14.20 per metre. If the wiring must include the connection between D and the main power source

(i) calculate the minimum length of wiring required. (2 marks)

Solution
Total = $(8.3 + 8.8 + 6.2 + 7.4 + 8.6) + 6.4$ = $39.3 + 6.4$ = 45.7 metres of wiring required
Specific behaviours
✓ calculates total on tree (39.3m) ✓ calculates total including main connection

(ii) calculate the total cost of installing the wiring. (1 mark)

Solution
Cost = 45.7×14.20 = \$648.94
Specific behaviours
✓ calculates product of wiring required and cost per metre

It is discovered that between Sites B and F there is a large section of rock. The owner can still install the wiring directly between these two sites but the cost for that section will double in price.

(c) What effect, if any, will this have on the minimum spanning tree and total cost of the installation? A copy of the **original** graph has been provided below. (3 marks)

Solution
The doubling of the price would effectively make connection B–F 12.4 instead of 6.2. The minimum spanning tree would no longer use the connection between B and F and would instead use F–C. The required wiring would increase to 48.2 metres and the total cost would increase from \$648.94 to \$684.44.
Specific behaviours
✓ states B–F connection would not be used ✓ identifies replacement connection C–F ✓ calculates new total cost

Question 15

(12 marks)

Paris, a retired hairdresser, has a superannuation balance of \$775 320.

- (a) She intends to set up an annual scholarship of \$5000 to be awarded to a worthy apprentice. Paris is able to negotiate a fixed annual interest rate of 7.2% through a share trading company. Show that \$69 444.44 (to the nearest cent) of Paris's superannuation is needed to be invested in this company to meet the requirements of the annual scholarship. (1 mark)

Solution
$\frac{5000}{0.072} = 69\,444.44$
Specific behaviours
✓ calculates correct value

After the scholarship is in place, Paris considers her future investment plans with the remaining balance. Assume a financial advisor has indicated an interest rate of 6.1% per annum, added monthly, can be locked in.

- (b) Paris decides on receiving a monthly annuity of \$7000, with the interest added before the annuity is paid, at the end of each month.
- (i) The annuity can be modelled by the recurrence relation $T_{n+1} = (1 + 0.00508)T_n - 7000, T_0 = a$. Using the information given in the question, show how to obtain the value 0.00508 and state the value of a . (2 marks)

Solution
$\frac{6.1\%}{12} = 0.00508$ and $a = 775\,320 - 69\,444.44 = 705\,875.56$
Specific behaviours
✓ correctly shows how 0.00508 was obtained ✓ correctly states the value of a

- (ii) For how many months will Paris be able to receive the annuity of \$7000? (1 mark)

Solution
141 months
Specific behaviours
✓ calculates correct number of months

- (iii) Determine the amount of the final payment. (2 marks)

Solution
$7000 - 2141.58 = 4858.42$ Therefore, the final payment is \$4858.42
Specific behaviours
✓ determines amount of over payment ✓ determines correct final payment
Note: If use $\frac{6.1}{1200}$, the final payment is \$5161.09

- (iv) Determine the total interest received on the annuity. (3 marks)

Solution
$141 \times 7000 + 4858.42 - 705\,875.56 = 285\,982.86$
Therefore, the total interest received is \$285 982.86
Specific behaviours
<ul style="list-style-type: none"> ✓ uses correct number of months of full payment ✓ forms correct calculation ✓ correct value
Note: If final payment is \$5161.09, total interest is \$286 285.53

- (c) Paris also considers receiving a perpetuity after the scholarship is in place. Determine the monthly amount she would receive. (3 marks)

Solution
$705\,875.56 \times 1.00508 - 705\,875.56 = \3585.85
Specific behaviours
<ul style="list-style-type: none"> ✓ correctly calculates monthly interest rate ✓ forms correct calculation ✓ correct payment

Alternate solution:

Solution
$N = 1, I = 6.1, PV = -705\,875.56, FV = 705\,875.56, P/Y = 12, C/Y = 12$ $PMT = -3588.20$ Therefore, the monthly amount received is \$3588.20
Specific behaviours
<ul style="list-style-type: none"> ✓ uses at least 4 correct values for N, I, PV, FV, P/Y, C/Y ✓ uses all correct values for N, I, PV, FV, P/Y, C/Y ✓ correct PMT

Question 16

(7 marks)

After paying a deposit for his new apartment, Declan obtains a bank loan for the remaining amount of \$112 000 at 3.26% per annum compounded monthly. He can currently afford to repay \$970 per month at the end of every month.

- (a) Calculate how much he would owe after the 40th repayment. (3 marks)

Solution
Using the financial app with $N = 40, I = 3.26, PV = -112\ 000, PMT = 970, P/Y = C/Y = 12$ $FV = 83\ 910.19$ He would owe \$83 910.19 after the 40 th repayment
Specific behaviours
<ul style="list-style-type: none"> ✓ states at least 4 correct entries ✓ states all correct entries ✓ states correct answer

- (b) Declan decided to deposit a one-off extra amount of \$1600, after the 16th repayment. Calculate the new amount he would owe after the 40th repayment. (4 marks)

Solution
Step 1. Using the financial app $N = 16, I = 3.26, PV = -112\ 000, PMT = 970, P/Y = C/Y = 12$ $FV = \$101\ 128.46$ $New\ PV = \$101\ 128.46 - 1600 = \$99\ 528.46$
Step 2. Using financial app $N = 24(40 - 16), I = 3.26, PV = -99\ 528.46, PMT = 970, P/Y = C/Y = 12$ $FV = \$82\ 202.54$
New amount owing after the 40 th repayment is \$82 202.54
Specific behaviours
<ul style="list-style-type: none"> ✓ states all correct entries in step 1 ✓ correctly subtracts 1600 from FV in step 1 to give new PV ✓ states all correct entries in step 2 ✓ correctly determines new FV

Question 17

(8 marks)

Indie was in a line with 24 other people for a slide at a water park. She noticed that the approximate number of people (P) in the line for the slide increased by 1.5% every minute (m).

- (a) Write an exponential equation in the form $P = ar^m$ to represent this situation. (2 marks)

Solution
$P = 25 \times 1.015^m$
Specific behaviours
<ul style="list-style-type: none"> ✓ states correct value of a ✓ states correct value of r

- (b) Determine the approximate number of people in the line after 2 hours. (2 marks)

Solution
$P = 25 \times 1.015^{120}$ $P = 149.23$ ~149 people
Specific behaviours
<ul style="list-style-type: none"> ✓ states correct value ✓ recognises integer value required

After 3 hours, the line started to decrease by 1% per minute.

- (c) Using this new information, calculate the approximate number of people in line, 5 hours after Indie initially lined up. (4 marks)

Solution
$P = 25 \times 1.015^{180}$ $P = 364.61$ ~365 $P = 365 \times 0.99^m$ $P = 365 \times 0.99^{120}$ $P = 109.3$ ~109 people
Specific behaviours
<ul style="list-style-type: none"> ✓ calculates P for $m = 180$ ✓ states new ratio of 0.99 ✓ identifies $m = 120$ ✓ uses equation to calculate $P \sim 109$

Copyright

© School Curriculum and Standards Authority, 2022

This document – apart from any third party copyright material contained in it – may be freely copied, or communicated on an intranet, for non-commercial purposes in educational institutions, provided that it is not changed and that the School Curriculum and Standards Authority (the Authority) is acknowledged as the copyright owner, and that the Authority's moral rights are not infringed.

Copying or communication for any other purpose can be done only within the terms of the *Copyright Act 1968* or with prior written permission of the Authority. Copying or communication of any third party copyright material can be done only within the terms of the *Copyright Act 1968* or with permission of the copyright owners.

Any content in this document that has been derived from the Australian Curriculum may be used under the terms of the Creative Commons [Attribution 4.0 International \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) licence.

An *Acknowledgements variation* document is available on the Authority website.

*Published by the School Curriculum and Standards Authority of Western Australia
303 Sevenoaks Street
CANNINGTON WA 6107*