2023/74305

Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS APPLICATIONS

Calculator-free

ATAR course examination 2023

Marking key

Marking keys are an explicit statement about what the examining panel expect of candidates when they respond to particular examination items. They help ensure a consistent interpretation of the criteria that guide the awarding of marks.

Section One: Calculator-free **35% (52 Marks)**

Question 1 **(8 marks)**

A survey was conducted by a film studio executive of filmgoers' favourite genres. The categories chose were action, drama and comedy. The information is displayed in the table below.

		Genre			
		Action	**Drama**	**Comedy**	**Total**
Age (years)	**1–10**	40	45	**40**	125
	11–20	17	**26**	43	86
	21–30	**70**	25	**30**	125
	Total	**127**	96	113	336

(a) Complete the two-way table above. (3 marks)

Solution
see table above
Specific behaviours
✓ determines three correct values ✓ determines four correct values ✓ determines all correct values

(b) Identify the response variable for these data. (1 mark)

Solution
genre
Specific behaviours
✓ correct response variable

(c) The incomplete two-way percentaged table is shown below.

		Genre			
		Action	**Drama**	**Comedy**	**Total**
Age (years)	**1–10**	**32**	36	**32**	**100**
	11–20	**20**	30	**50**	**100**
	21–30	56	**20**	24	**100**
	Total				

(i) Complete the table above by using either row percentages **or** column percentages, as appropriate. (3 marks)

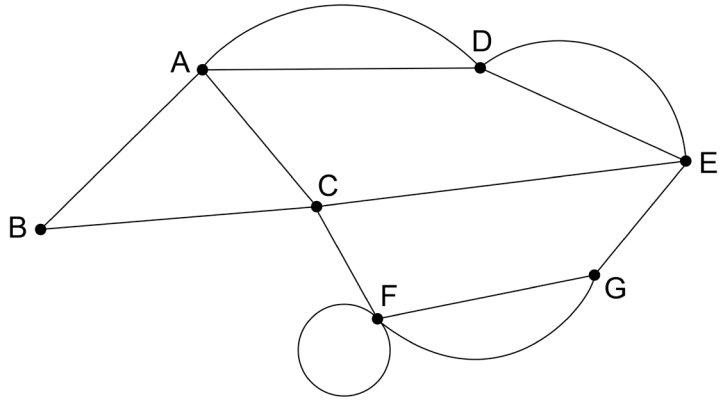
Solution
see table above
Specific behaviours
✓ determines row percentages are required ✓ determines three correct values ✓ determines all correct values

(ii) State an association that can be observed from the two-way percentaged table. (1 mark)

Solution
as age increases, the percentage of the drama genre decreases
Specific behaviours
✓ correctly states association

Question 2 **(9 marks)**

The network shown below is for walking tracks around a wildlife sanctuary. Located at each vertex is an undercover area with picnic tables and viewing stations for visitors.



(a) Complete the adjacency matrix for the network shown. (3 marks)

	A	B	C	D	E	F	G
A	0	1	1	2	0	0	0
B	1	0	1	0	0	0	0
C	1	1	0	0	1	1	0
D	2	0	0	0	2	0	0
E	0	0	1	2	0	0	1
F	0	0	1	0	0	1	2
G	0	0	0	0	1	2	0

Solution
see diagram above
Specific behaviours
✓ determines at least 12 correct entries ✓ determines at least 16 correct entries ✓ determines all correct entries

(b) Identify the feature of the adjacency matrix that indicates it is an undirected graph. (1 mark)

Solution
matrix is symmetric across leading diagonal
Specific behaviours
✓ correctly identifies feature

(c) List all possible open paths of length two starting from G. (3 marks)

Solution
GEC, GED (twice), GFC (twice)
Specific behaviours
✓ determines at least three correct paths ✓ does not list paths that include the loop ✓ determines all five correct paths

(d) Each morning before visitors are admitted, all fences along walking tracks between vertices must be checked for safety. Is it possible to carry out the safety check with an Eulerian trail, a semi-Eulerian trail or neither? Justify your answer. (2 marks)

Solution
semi-Eulerian, F and G are odd vertices
Specific behaviours
✓ states semi-Eulerian ✓ justifies semi-Eulerian

Question 3 **(9 marks)**

From January 1, 2020, a company offered its employees an income package with a starting wage of $4000 per month, paid at the end of each month. Also, as an incentive to stay with the company, there will be a monthly increase of $50 each month.

(a) Determine a recursive rule for the monthly wage. (2 marks)

Solution
$T_{n+1} = T_n + 50, T_1 = 4000$
Specific behaviours
✓ states correct recursive rule ✓ states correct first term

(b) Deduce a simplified rule for the n^{th} term of the monthly wage. (2 marks)

Solution
$T_n = 4000 + (n-1)(50)$ $T_n = 3950 + 50n$
Specific behaviours
✓ uses correct arithmetic formula ✓ gives correct simplified rule for the n^{th} term

(c) Determine the monthly wage for December 2020. (2 marks)

Solution
$T_{12} = 3950 + 50(12)$ $= 4550$ Therefore, the monthly wage for December 2020 is $4550
Specific behaviours
✓ correctly identifies term 12 ✓ correctly calculates the $4550

The company has decided to make the monthly increase $60 from the end of December 2023.

(d) Calculate the monthly wage for March 2024. (3 marks)

Solution
$12 \times 4 = 48$ $T_{48} = 3950 + 50(48) = 6350$ Therefore, the monthly wage for March 2024 is 6350 + 60 + 60 + 60 = $6530
Specific behaviours
✓ correctly calculates T_{48} ✓ calculates correct term for March 2024 ✓ states correct solution for wage

Question 4 **(4 marks)**

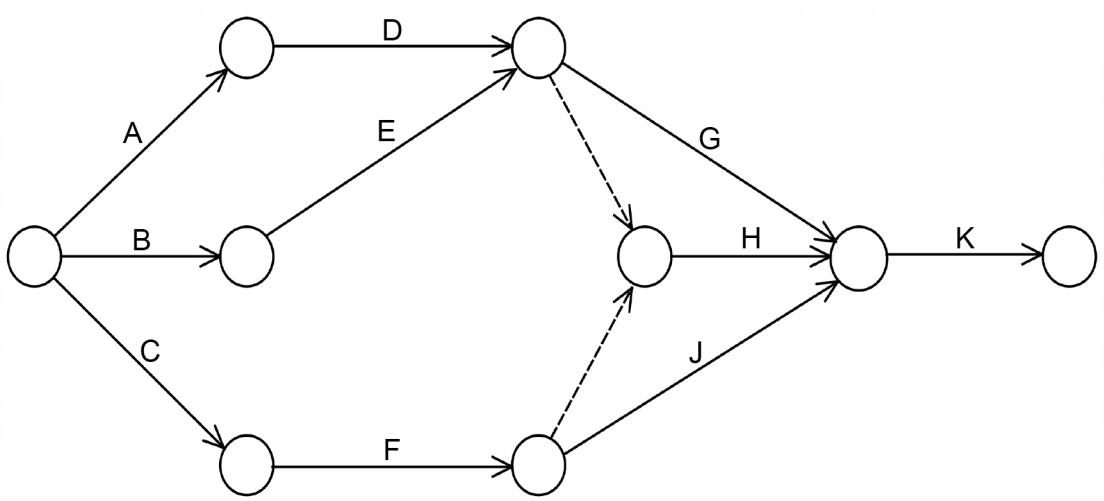
The following tables show activities and their completion requirements for a project to build a shearing shed.

Activity	Activity description
A	Design the shearing shed
B	Prepare the site
C	Purchase materials
D	Construct the frame and walls
E	Install roof and windows
F	Add ventilation and insulation
G	Build the shearing stand
H	Install electrics
J	Paint and finish the shed
K	Install shearing equipment

Requirements
A, B and C start the project together
D starts when A is completed
E starts when B is completed
F starts when C is completed
G starts when D and E are completed
J starts when F is completed
H starts when D, E and F are completed
K starts when G, H and J are completed

Draw the project network for the information given in the tables.

Solution



Specific behaviours

- ✓ all edges show task and direction
- ✓ shows at least six correct edges
- ✓ shows at least one dummy link
- ✓ shows all other correct edges

Question 5 **(10 marks)**

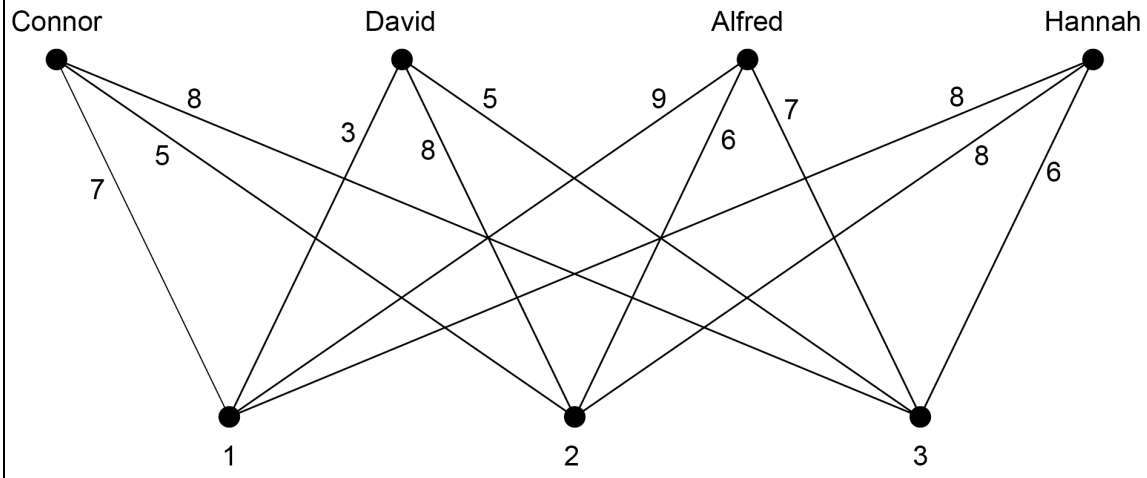
Four camp leaders, Connor, David, Alfred and Hannah, are responsible for setting up the camp site for the upcoming school camp. Today there are three tasks available. Each task will only have one camp leader assigned to it.

The number of hours each camp leader takes to complete each task is shown in the table below.

		Task		
		1	**2**	**3**
Camp leader	**Connor**	7	5	8
	David	3	8	5
	Alfred	9	6	7
	Hannah	8	8	6

(a) Draw a weighted bipartite graph showing all the possible allocations for each of the camp leaders. (2 marks)

Solution



Specific behaviours

✓ correctly allocates edges
✓ correctly allocates weights on edges

(b) Complete the 4 x 4 matrix below to represent the allocation of tasks to camp leaders. (2 marks)

Solution

$$\begin{bmatrix} 7 & \boxed{5} & \boxed{8} & \boxed{0} \\ \boxed{3} & \boxed{8} & 5 & \boxed{0} \\ 9 & \boxed{6} & \boxed{7} & \boxed{0} \\ \boxed{8} & \boxed{8} & \boxed{6} & \boxed{0} \end{bmatrix}$$

Specific behaviours

✓ correctly enters a column of zeros
✓ correct values in matrix

(c) Use the Hungarian algorithm to determine the allocation of camp leader to the task that will minimise the time taken. (4 marks)

Solution
$\begin{bmatrix} 4 & 0 & 3 & 0 \\ 0 & 3 & 0 & 0 \\ 6 & 1 & 2 & 0 \\ 5 & 3 & 1 & 0 \end{bmatrix}$ $\begin{bmatrix} \cancel{4} & \cancel{0} & \cancel{3} & \cancel{0} \\ \cancel{0} & \cancel{3} & \cancel{0} & \cancel{0} \\ 6 & 1 & 2 & \cancel{0} \\ 5 & 3 & 1 & \cancel{0} \end{bmatrix}$ $\begin{bmatrix} \cancel{4} & \cancel{\mathbf{0}} & \cancel{3} & \cancel{1} \\ \cancel{\mathbf{0}} & \cancel{3} & \cancel{0} & \cancel{1} \\ \cancel{5} & \cancel{0} & \cancel{1} & \cancel{0} \\ \cancel{4} & \cancel{2} & \cancel{\mathbf{0}} & \cancel{0} \end{bmatrix}$
Specific behaviours
✓ subtracts smallest values from every column ✓ shows that every zero can be covered by 3 horizontal/vertical lines ✓ determines that smallest uncovered value is 1 and subtracts 1 from every uncovered value and adds 1 to every value covered by 2 lines ✓ shows that every zero can be covered by 4 horizontal/vertical lines

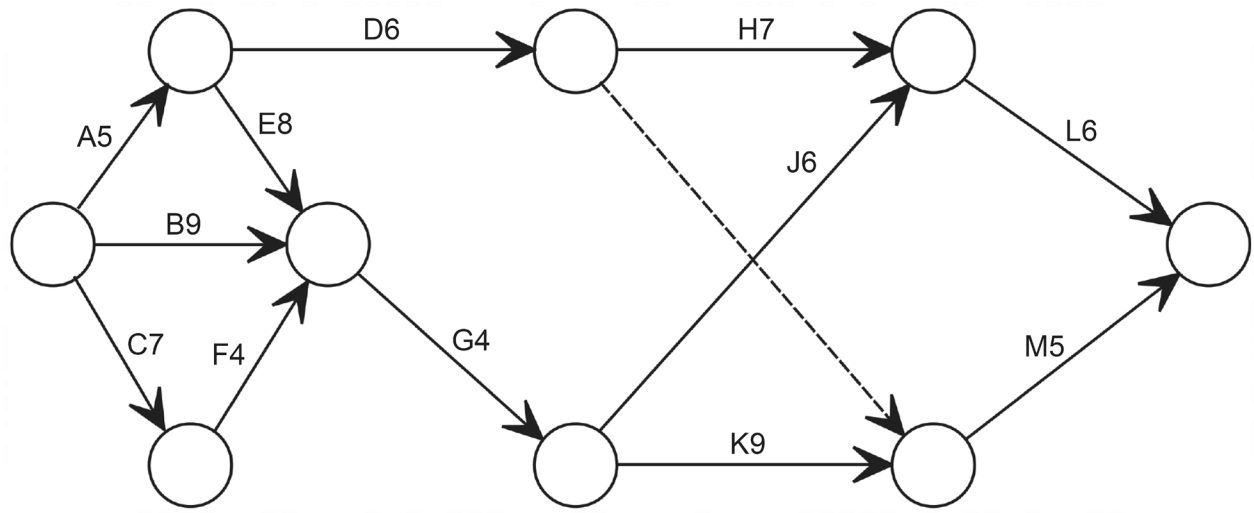
(d) Show the task allocated to each camp leader and calculate the total time taken to complete all tasks. (2 marks)

Camp leader	**Connor**	**David**	**Alfred**	**Hannah**
Task	**2**	**1**	**-**	**3**

Solution
see table above for task allocation
total time: 14 hours
Specific behaviours
✓ states correct allocation ✓ calculates correct time

Question 6 **(12 marks)**

A landscape architect has produced the following project network for the development of a community market garden. The digraph shows the order of completion of the various tasks and their expected completion time in hours.



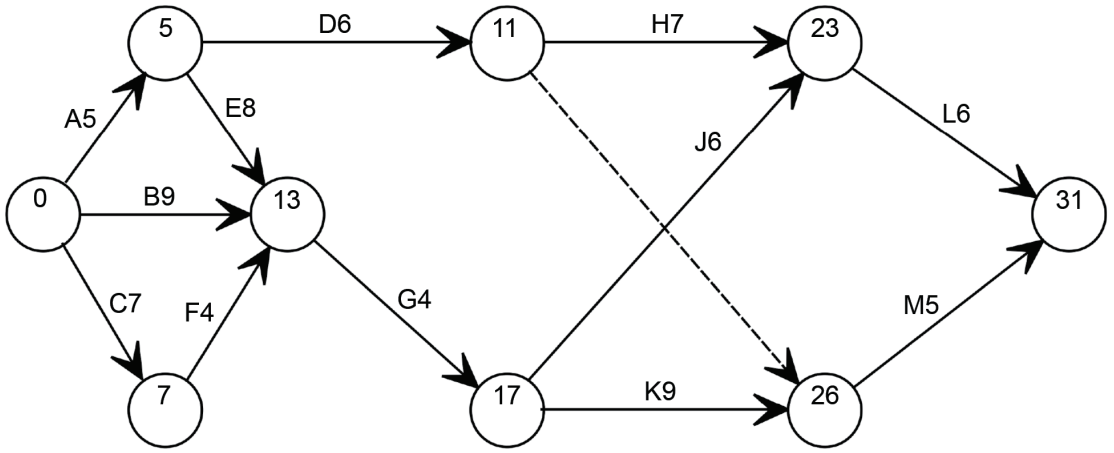
(a) Complete the immediate predecessor/s column in the table below. (3 marks)

Task	Time (hours)	Immediate Predecessor/s
A	5	**-**
B	9	**-**
C	7	**-**
D	6	**A**
E	8	**A**
F	4	**C**
G	4	**B, E, F**
H	7	**D**
J	6	**G**
K	9	**G**
L	6	**H, J**
M	5	**D, K**

Solution
see table above
Specific behaviours
✓ states correct predecessors for 10 tasks ✓ states correct predecessors for M ✓ states all correct predecessors

(b) Determine the critical path and the minimum completion time for the project. Workings must be shown to verify your answer. (3 marks)

Solution



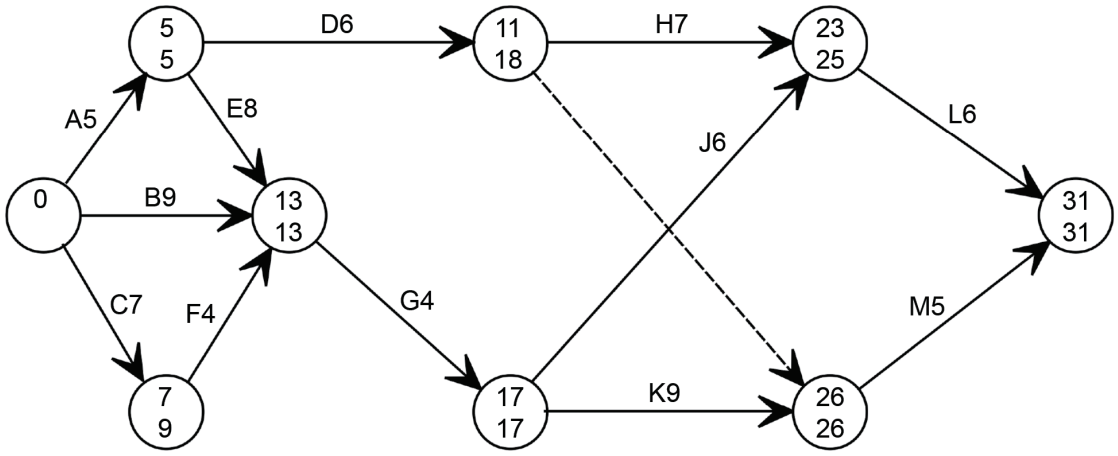
AEGKM, 31 hours

Specific behaviours

✓ shows systematic working
✓ gives correct path
✓ gives correct length

(c) Determine which task/s have a float time of exactly 2 hours. (2 marks)

Solution



Task F, C, L and J have a float time of exactly 2 hours.

Specific behaviours

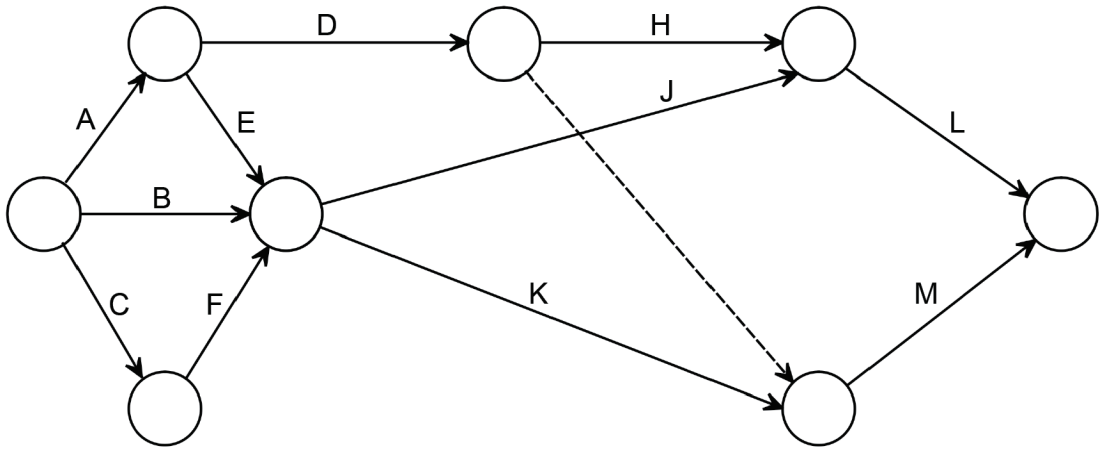
✓ identifies two correct tasks
✓ identifies all correct tasks

(d) Describe why Task D can be delayed by 6 hours and not affect the minimum completion time. (2 marks)

Solution
float time for H is 7 hours, hence, D can be delayed by 6 hours
Specific behaviours
✓ identifies the float time for H is 7 hours ✓ gives correct explanation why D can be delayed by 6 hours

(e) Due to the release of a new piece of technology for reticulation control, Task G is no longer required. Redraw the network showing how the removal of Task G will change the configuration of the network. Task times are **not** required to be shown. (2 marks)

Solution



Specific behaviours
✓ shows task G removed ✓ shows tasks J and K with immediate predecessors E, B, and F

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303 Sevenoaks Street
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