



ATAR course examination, 2023

Question/Answer booklet

MATHEMATICS METHODS

Section One: Calculator-free

Place one of your candidate identification labels in this box.
Ensure the label is straight and within the lines of this box.

WA student number: In figures

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In words

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional
answer booklets used
(if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.



Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	5	5	50	53	35
Section Two: Calculator-assumed	9	9	100	96	65
Total					100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2023: Part II Examinations*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free

35% (53 Marks)

This section has **five** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

Question 1

(8 marks)

(a) Consider the function $f(x) = x^3 e^{2x}$.

(i) Differentiate $f(x)$.

(2 marks)

(ii) Determine the value of x for any stationary points of $f(x)$.

(3 marks)

(b) Evaluate $\int_0^{\frac{\pi}{4}} \sin(2x + \pi) dx$.

(3 marks)

Question 2

(14 marks)

Let $p = \ln(2)$, $q = \ln(3)$ and $r = \ln(5)$.

(a) Express each of the following in terms of p , q and/or r .

(i) $\ln(6)$ (2 marks)

(ii) $\ln(6.25)$ (3 marks)

(iii) $\int_2^3 \frac{d}{dx} \ln(x) dx$ (2 marks)

(b) Evaluate e^{p+q} . (2 marks)

See next page

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(c) (i) Determine $\frac{d}{dx}(x \ln(x))$. (1 mark)

(ii) Hence show that $\int \ln(x) dx = x \ln(x) - x + c$ where c is a constant. (2 marks)

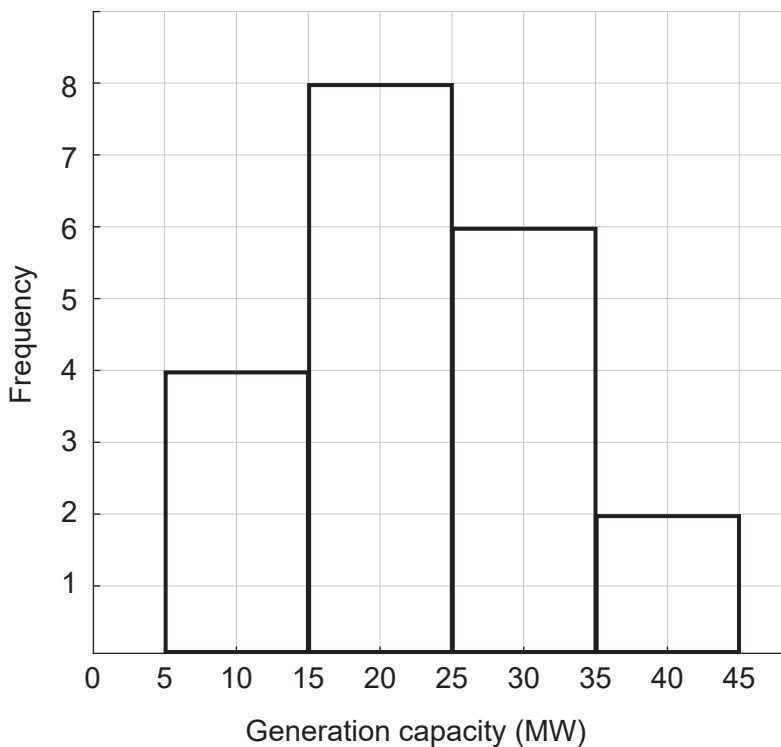
(iii) Evaluate $\int_1^3 \ln(x) dx$ in terms of p , q and/or r . (2 marks)

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Question 3

(10 marks)

Solcolwa is a green energy company that owns 20 solar farms across Western Australia. The generation capacities, in megawatts (MW), of the solar farms are displayed in the histogram below.



Suppose that one of the Solcolwa solar farms is selected at random. Let the random variable W denote the generation capacity of the randomly-selected solar farm.

- (a) Complete the following table of cumulative probabilities for W . (2 marks)

w	5	15	25	35	45
$P(W \leq w)$					

- (b) Determine $P(W \geq 35)$. (1 mark)

(c) Assuming the solar farms are uniformly distributed within each interval:

(i) estimate $P(W \geq 20)$. (2 marks)

(ii) estimate the expected value $E(W)$. (2 marks)

To increase the generation capacity of its solar farms, Solcolwa decides to upgrade all its solar panels with the latest technology. A new random variable Y denotes the generation capacity of a randomly-selected upgraded solar farm. The random variables W and Y are related by

$$Y = aW$$

for some constant $a > 0$.

(d) Given that W and Y have variances $Var(W) = 81$ and $Var(Y) = 324$, determine the expected value $E(Y)$. (3 marks)

Question 4

(8 marks)

An internet search engine uses a logarithmic scale to rank the importance of internet websites. If a website has S visits each week, the site rank, R , is given by

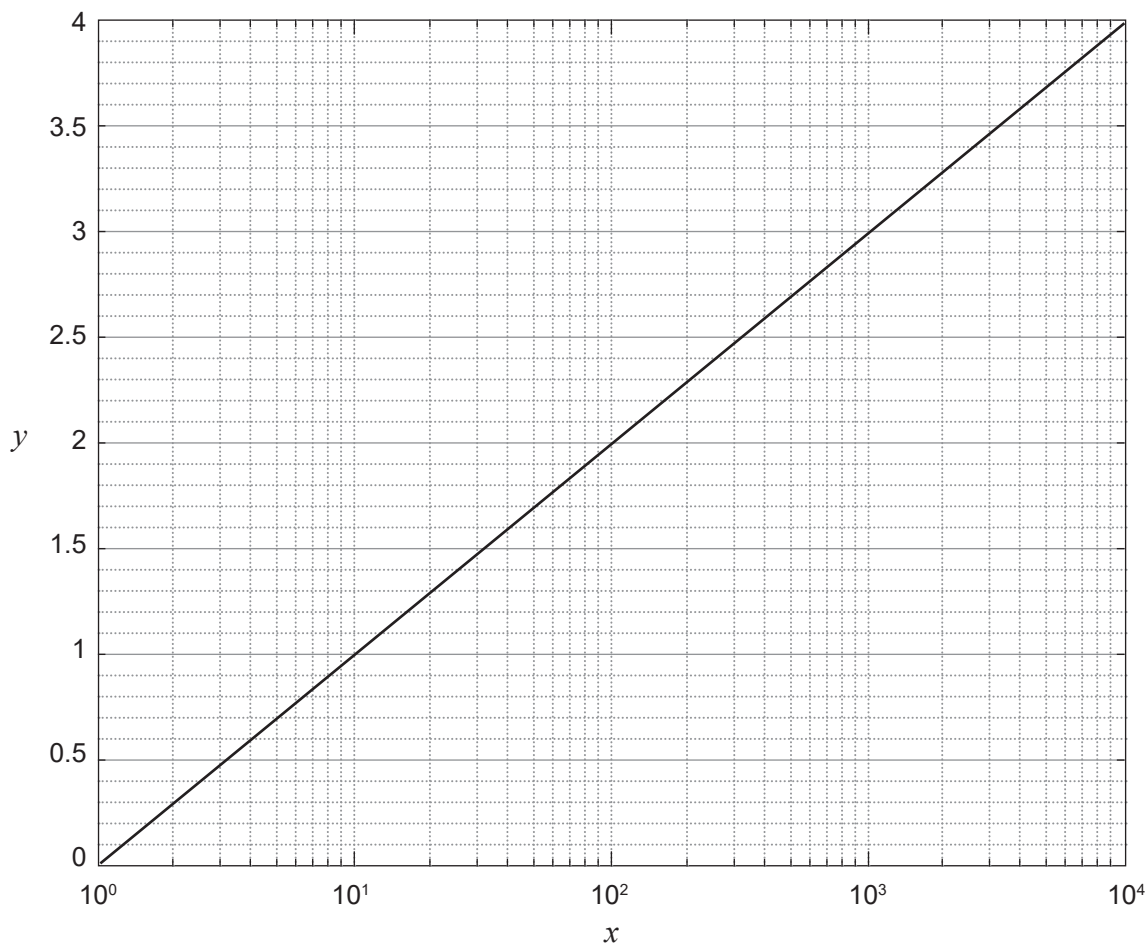
$$R = 2 \log_{10} \left(\frac{S}{S_0} \right)$$

where S_0 is the reference value (the same for all websites). The reference value is the minimum number of visits per week required for a website to register on the site rank scale.

- (a) Determine the site rank for a website whose weekly visits are one hundred times the reference value. (2 marks)

- (b) Given that a site rank of 12 is assigned to a website with 1.5 billion (1.5×10^9) visits per week, determine the value of S_0 . (3 marks)

- (c) The plot of $y = \log_{10}(x)$ is shown below. If a website has a site rank of 3.2, use the plot and your answer from part (b) to approximate the website's number of weekly visits. (3 marks)



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Question 5

(13 marks)

The table below contains values of the polynomial function $f(x)$, its first and second derivatives, and the function $F(x) = \int_0^x f(t) dt$ for $x = 0, 1, 2, 3, 4, 5, 6$.

$f(x)$ has no stationary points at non-integer values of x , and the letters a, b, c, d and e represent unspecified constants.

	$x = 0$	$x = 1$	$x = 2$	$x = 3$	$x = 4$	$x = 5$	$x = 6$
$f(x)$	a	b	4	c	0	d	e
$f'(x)$	16	0	-4	-2	0	-4	-20
$f''(x)$	-24	-9	0	3	0	-9	-24
$F(x)$	0	4.7	10.4	12.6	12.8	12.5	7.2

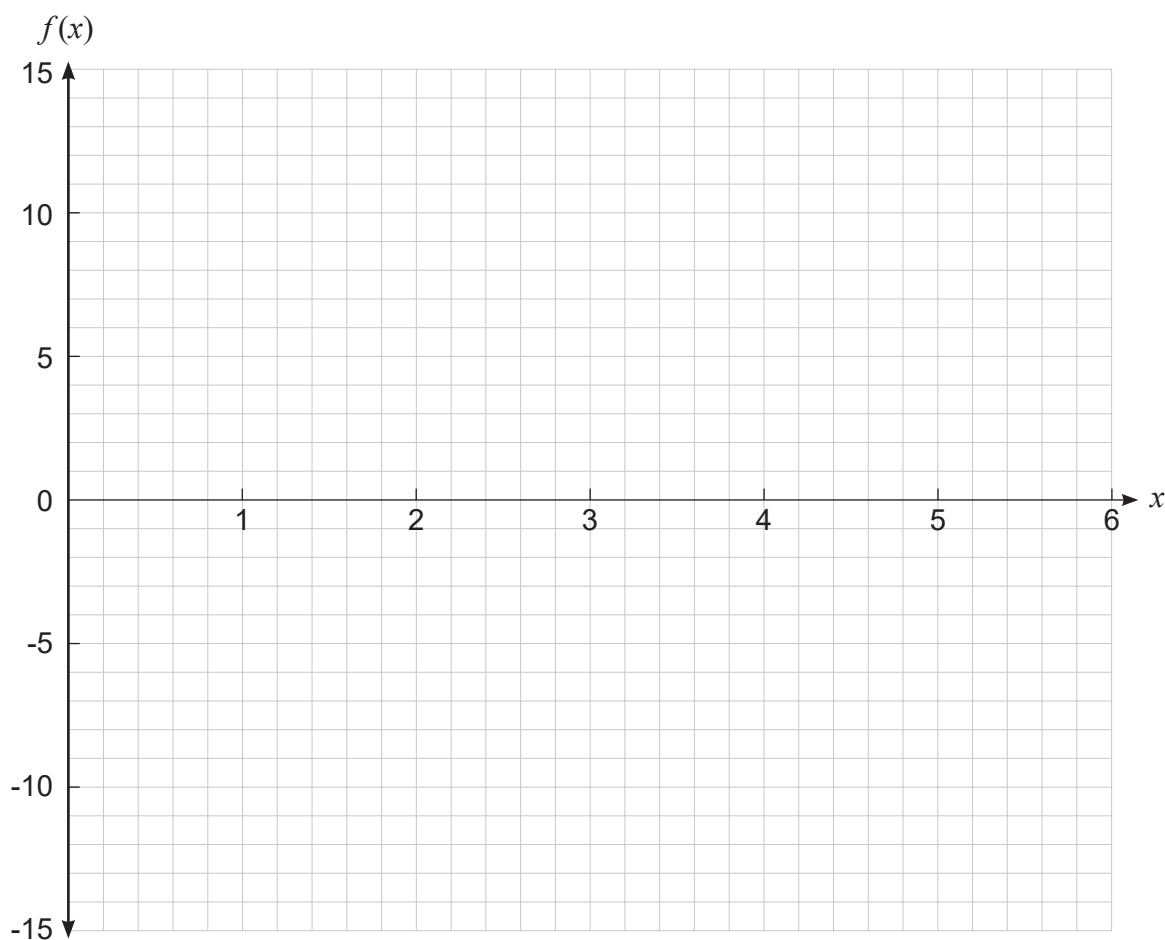
(a) Evaluate $\frac{d}{dx}(f(x)^2)$ when $x = 2$. (2 marks)

(b) Evaluate $\int_2^4 (f(x) + 2) dx$. (3 marks)

(c) Evaluate $\frac{d}{dx} \int_2^x f(t) dt$ when $x = 2$. (2 marks)

- (d) Determine the x -coordinate of any stationary points and whether they are local maxima, local minima or inflection points. Justify your answer. (3 marks)

- (e) Sketch a possible graph of $f(x)$ for $0 \leq x \leq 6$ on the axes below. (3 marks)



A spare grid is provided at the end of this Question/Answer booklet. If you need to use it, cross out this attempt and indicate that you have redrawn it on the spare grid.

End of section

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Supplementary page

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Supplementary page

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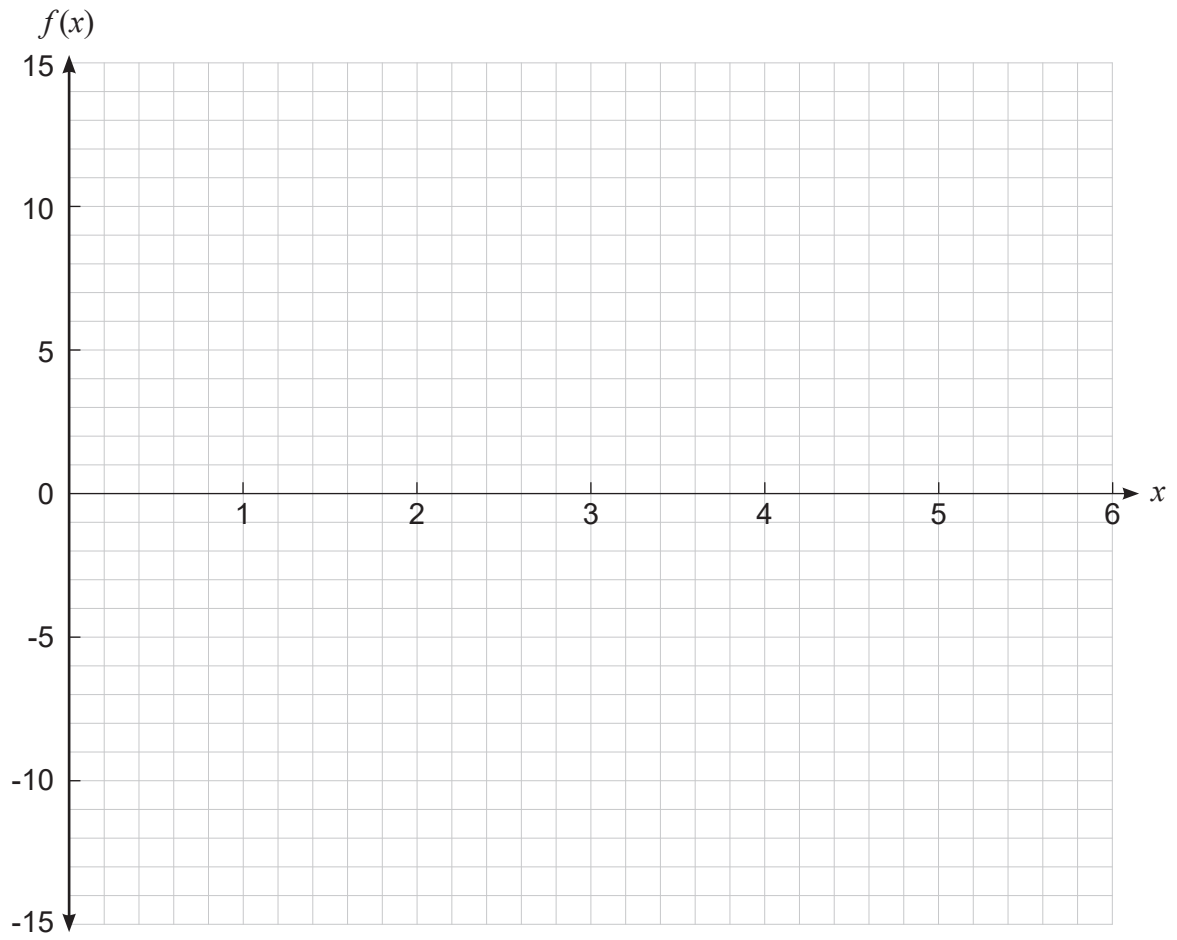
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Supplementary page

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Spare grid for Question 5(e)



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