



ATAR course examination, 2023

Question/Answer booklet

MATHEMATICS SPECIALIST

Section One: Calculator-free

Place one of your candidate identification labels in this box.
Ensure the label is straight and within the lines of this box.

WA student number: In figures

--	--	--	--	--	--	--	--

In words

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional
answer booklets used
(if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.



Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	48	35
Section Two: Calculator-assumed	11	11	100	89	65
Total					100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2023: Part II Examinations*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free

35% (48 Marks)

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

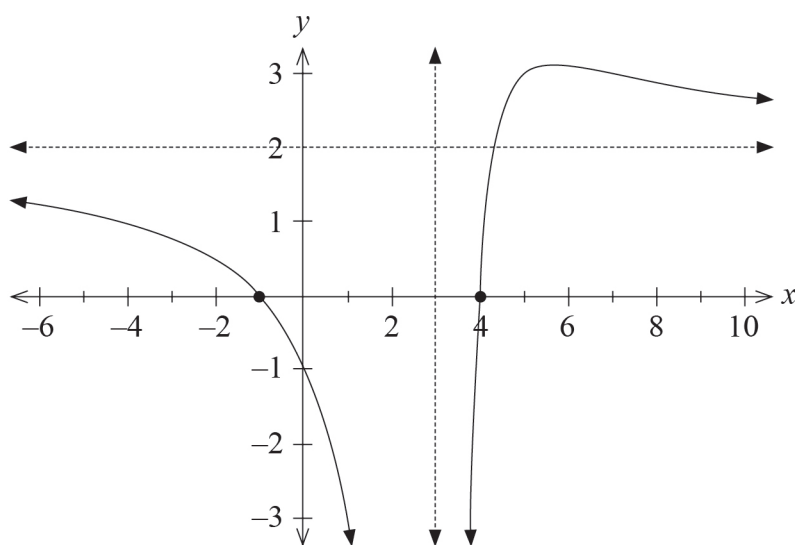
Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

Question 1

(4 marks)

The graph of the function $f(x) = \frac{k(x+a)(x-b)}{(x-c)^2}$ is shown below. The constants a, b, c and k are positive.



Complete the table below by determining the values for a, b, c and k .

a	b	c	k

Question 2

(5 marks)

$P(z) = z^5 + az^4 + bz^3 + cz^2 + dz + 14$ is a fifth order polynomial with real coefficients. It is known that $P(z) = (z - z_0)Q(z)$ where z_0 is real and $Q(z)$ is a fourth order polynomial. Two roots of $P(z)$ are $z_1 = 1 + i$ and $z_2 = 2 + \sqrt{3}i$.

(a) Determine $Q(z)$ in expanded form.

(3 marks)

(b) Determine the values of the coefficients a , b , c and d .

(2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 3

(5 marks)

Using the substitution $x = 119u + 1$, evaluate exactly $\int_1^{120} \left(2 + 4 \left(\frac{x + 118}{119} \right)^3 \right) dx$.

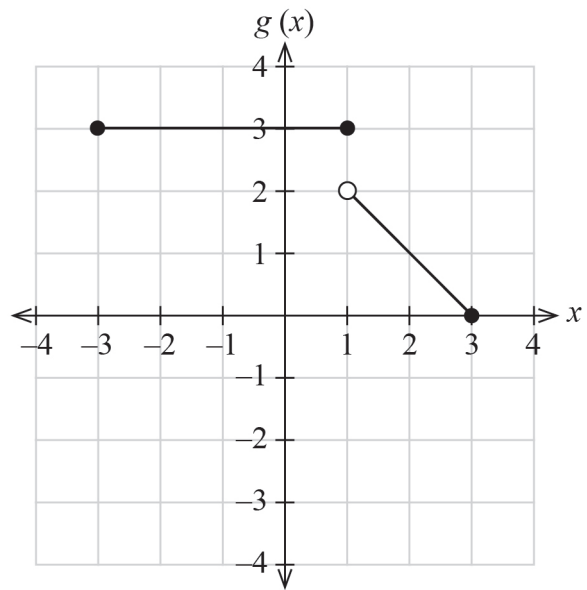
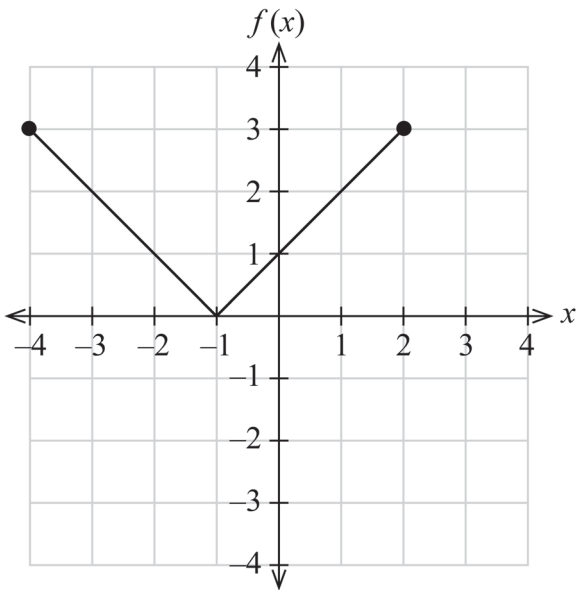
DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

See next page

Question 4

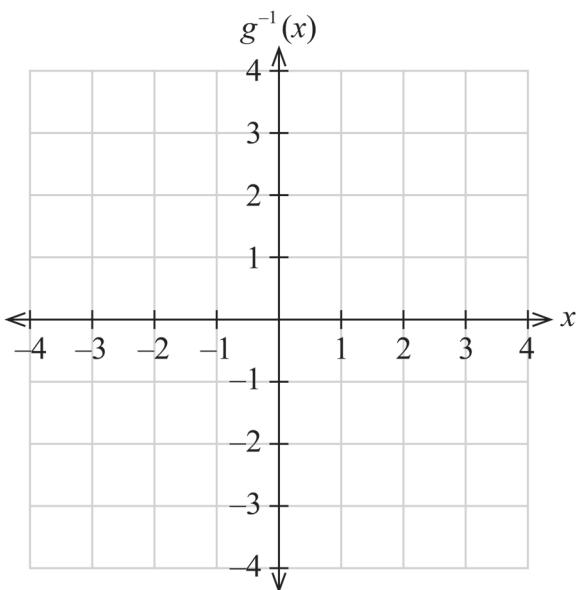
(11 marks)

The graphs of functions $f(x)$ and $g(x)$ are shown.



- (a) Sketch the graph of $y = g^{-1}(x)$ on the axes below.

(2 marks)



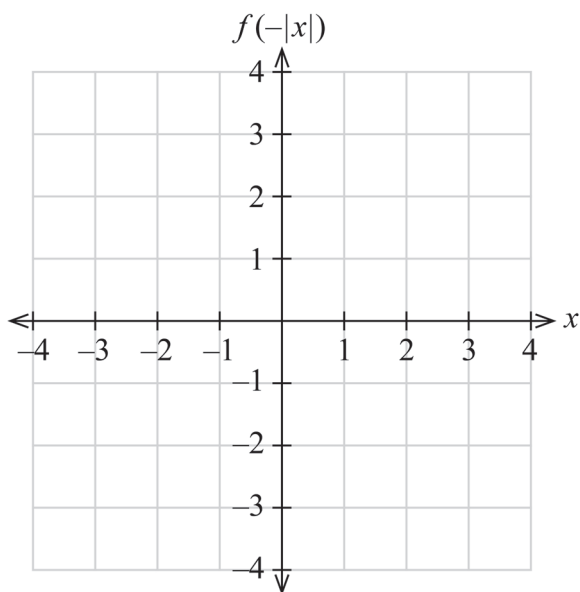
A spare grid is provided at the end of this Question/Answer booklet. If you need to use it, cross out this attempt and indicate that you have redrawn it on the spare grid.

- (b) State the value for $g(f^{-1}(0))$.

(2 marks)

(c) Determine the set of values of x such that $f(g(x))$ is defined. (2 marks)

(d) Sketch the graph of $y = f(-|x|)$ on the axes below. (2 marks)



A spare grid is provided at the end of this Question/Answer booklet. If you need to use it, cross out this attempt and indicate that you have redrawn it on the spare grid.

(e) The equation $|x + 1| = k - |x + a|$ has an infinite number of solutions, with the solution set being $-3 \leq x \leq -1$. Determine the values of the constants a and k . (3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 5**(5 marks)**

Consider two planes given by their Cartesian equations:

$$x - 3y + 3z = 9$$

$$2x + y - z = 4$$

- (a) Explain why these planes are not parallel. (1 mark)
- (b) State the geometric interpretation of the solution in the above simultaneous equations. (1 mark)
- (c) Determine the vector equation for the intersection of these two planes. (3 marks)

Question 6

(5 marks)

Solve the complex equation $z^4 = 2 - 2\sqrt{3}i$ giving solutions in the form $rcis\theta$ where $-\pi < \theta \leq \pi$.

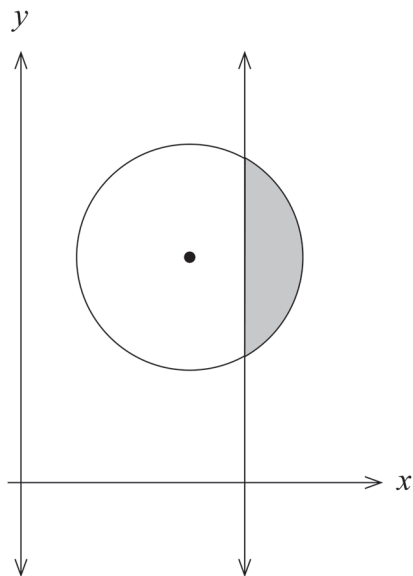
DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

See next page

Question 7

(9 marks)

The shaded region is bounded by the curve $(x - 3)^2 + (y - 4)^2 = 4$ and the line $x = 4$.



- (a) Show that the area of this region is given by the definite integral $\int_4^a 2\sqrt{4 - (x - 3)^2} dx$.
State the value for a . (3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

- (b) By using the substitution $x - 3 = 2\sin \theta$, determine the exact value for the area of the shaded region. (6 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 8

(4 marks)

In the following simultaneous equations, a and b are real numbers.

$$a^3 = 3ab^2 + 14$$

$$b^3 = 3a^2b + 2\sqrt{5}$$

In order to determine the value of $a^2 + b^2$ from these equations, it is useful to consider the complex expansion for $(a + bi)^3$. Hence, or otherwise, determine the exact value of $a^2 + b^2$.

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

End of section

Supplementary page

Question number: _____

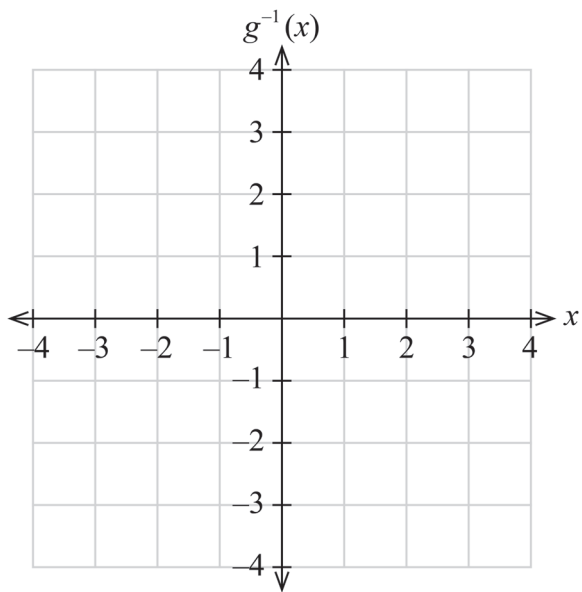
DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Supplementary page

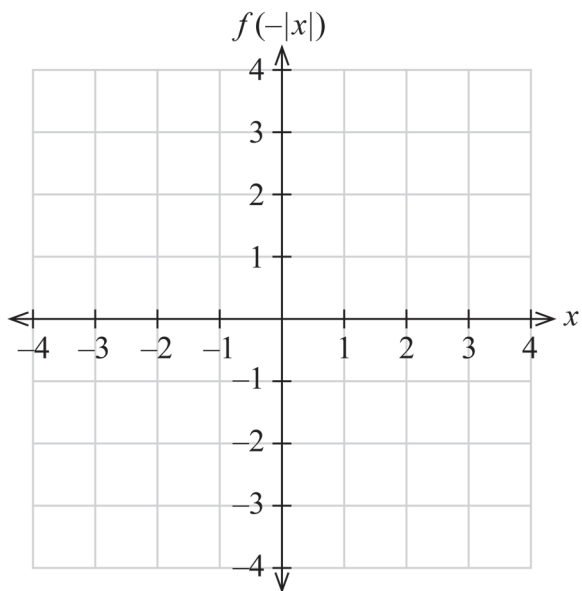
Question number: _____

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Spare grid for Question 4(a).



Spare grid for Question 4(d).



DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

This document – apart from any third party copyright material contained in it – may be freely copied, or communicated on an intranet, for non-commercial purposes in educational institutions, provided that it is not changed and that the School Curriculum and Standards Authority (the Authority) is acknowledged as the copyright owner, and that the Authority's moral rights are not infringed.

Copying or communication for any other purpose can be done only within the terms of the *Copyright Act 1968* or with prior written permission of the Authority. Copying or communication of any third party copyright material can be done only within the terms of the *Copyright Act 1968* or with permission of the copyright owners.

Any content in this document that has been derived from the Australian Curriculum may be used under the terms of the Creative Commons [Attribution 4.0 International \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) licence.

An *Acknowledgements variation* document is available on the Authority website.

<p><i>Published by the School Curriculum and Standards Authority of Western Australia</i> 303 Sevenoaks Street CANNINGTON WA 6107</p>
