



Government of **Western Australia**
School Curriculum and Standards Authority

Western Australian Certificate of Education ATAR course examination, 2016

Question/Answer booklet

MATHEMATICS SPECIALIST

Section One: Calculator-free

Place one of your candidate identification labels in this box.
Ensure the label is straight and within the lines of this box.

Student number: In figures

--	--	--	--	--	--	--	--	--

In words

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional
answer booklets used
(if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.



Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	12	12	100	97	65
Total					100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section One: Calculator-free

35% (53 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.

Working time: 50 minutes.

Question 1**(4 marks)**

Functions f and g are defined as $f(x) = \ln(x)$ and $g(x) = \frac{1}{x}$.

(a) Determine an expression for $g \circ f(x)$. (1 mark)

(b) For $g \circ f(x)$, state the:

(i) domain. (2 marks)

(ii) range. (1 mark)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 2

(7 marks)

Give exact expressions for each of the following in the form $a+bi$:

(a) $\frac{\overline{2+i}}{(1-i)^2}$. (3 marks)

(b) $(\sqrt{3}-i)^5$. (4 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 3**(5 marks)**

Consider $f(z) = z^3 + 2z^2 - 5z + 12$ where z is a complex number.

(a) Show that $(z + 4)$ is a factor of $f(z)$. (2 marks)

(b) Solve the equation $z^3 + 2z^2 - 5z + 12 = 0$. (3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 4

(6 marks)

- (a) Express $\frac{x-8}{(x+2)(x-3)}$ in the form $\frac{a}{x+2} + \frac{b}{x-3}$. (3 marks)

- (b) Hence determine $\int \frac{x-8}{(x+2)(x-3)} dx$. (3 marks)

Question 5

(7 marks)

Evaluate the following definite integrals exactly.

(a) $\int_0^{\frac{\pi}{4}} 12 \sin^4 2x \cos 2x \, dx$ Put $u = \sin 2x$ (4 marks)

(b) $\int_0^{\frac{1}{2}} \tan^2\left(\frac{\pi x}{2}\right) dx$ (3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 6

(6 marks)

- (a) Solve the system of equations.

(3 marks)

$$x + y + z = 4$$

$$3x - y + z = 8$$

$$2x - y + z = 0$$

Suppose that the third equation in part (a) is changed to $2x - y + kz = 0$. The first two equations remain unchanged.

- (b) Determine the value of the constant k so that the changed system of equations has no solution. (3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 7

(7 marks)

Points A, B have respective position vectors $\begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -2 \\ 5 \end{pmatrix}$.

- (a) Determine the vector equation for the sphere that has \overline{AB} as its diameter. (3 marks)

If point O is the origin, consider the plane that contains the vectors \overrightarrow{OA} and \overrightarrow{OB} .

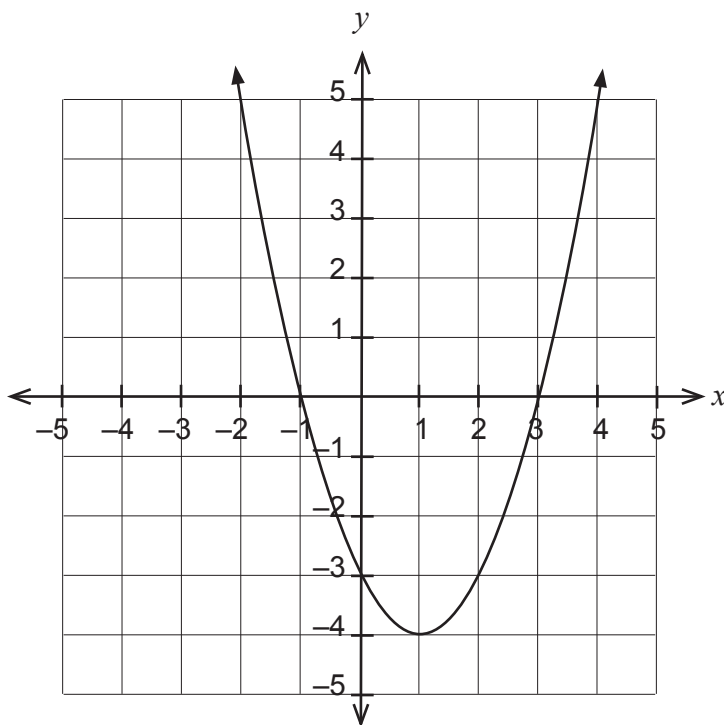
- (b) Determine the vector equation for this plane in the form $\vec{r} \cdot \vec{n} = c$. (4 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 8

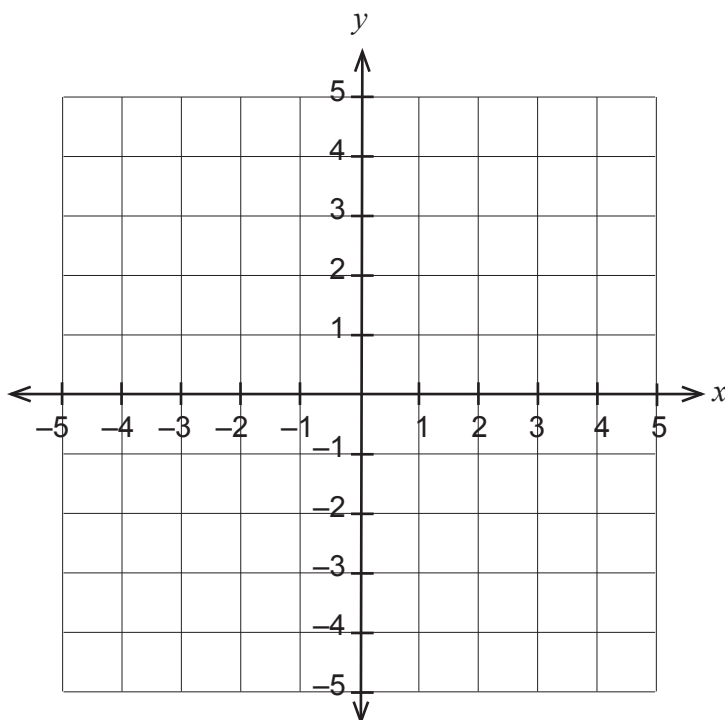
(11 marks)

The graph of $f(x) = (x - 1)^2 - 4$ is shown below.



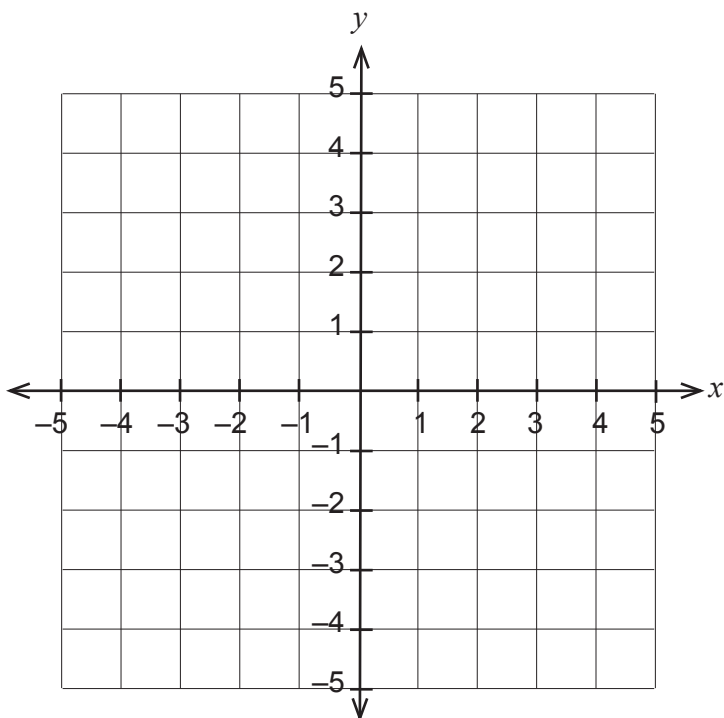
- (a) Sketch the graph of $y = \frac{1}{f(x)}$ on the coordinate axes below.

(4 marks)



See next page

- (b) Sketch the graph of $y = f(|x|)$ on the coordinate axes below. (2 marks)



- (c) The domain of function f is restricted to $x \leq k$ so that $y = f^{-1}(x)$ is a function. If this restricted domain represents the largest possible domain, state the value for the constant k . Explain. (2 marks)

- (d) Using the restriction $x \leq k$, determine the defining rule for $y = f^{-1}(x)$. Also state the domain for $y = f^{-1}(x)$. (3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Additional working space

Question number: _____

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Additional working space

Question number: _____

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

This document – apart from any third party copyright material contained in it – may be freely copied, or communicated on an intranet, for non-commercial purposes in educational institutions, provided that it is not changed and that the School Curriculum and Standards Authority is acknowledged as the copyright owner, and that the Authority's moral rights are not infringed.

Copying or communication for any other purpose can be done only within the terms of the *Copyright Act 1968* or with prior written permission of the School Curriculum and Standards Authority. Copying or communication of any third party copyright material can be done only within the terms of the *Copyright Act 1968* or with permission of the copyright owners.

Any content in this document that has been derived from the Australian Curriculum may be used under the terms of the Creative Commons Attribution-NonCommercial 3.0 Australia licence.

*Published by the School Curriculum and Standards Authority of Western Australia
303 Sevenoaks Street
CANNINGTON WA 6107*