Government of **Western Australia**
School Curriculum and Standards Authority

ATAR course examination, 2017

Question/Answer booklet

MATHEMATICS APPLICATIONS

Section Two: Calculator-assumed

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

Student number: In figures | | | | | | | | | |

In words _____

_____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	53	35
Section Two: Calculator-assumed	9	9	100	94	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2017*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (94 Marks)**

This section has **nine (9)** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 8 **(6 marks)**

Ming, a former high school student and now a successful business owner, wishes to set up a perpetuity of $6000 per year to be paid to a deserving student from her school. The perpetuity is to be paid at the start of the year in one single payment.

(a) A financial institution has agreed to maintain an account for this perpetuity paying a fixed rate of 5.9% p.a. compounded monthly.

Show that an amount of $98 974, to the nearest dollar, is required to maintain this perpetuity. (3 marks)

(b) Ming allows herself five years to accumulate the required $98 974 by making regular quarterly payments into an account paying 5.4% p.a. compounded monthly.

Determine the quarterly payment needed to reach the required amount after five years if Ming starts the account with an initial deposit of $1000. (3 marks)

See next page

Question 9 **(13 marks)**

The World Health Organisation produces tables showing Child Growth Standards. The median lengths (cm) for girls at various times during the first five years of life are shown below.

Age (months)	0	3	12	21	27	42	48	60
Median length (cm)	49.1	59.8	74.0	83.7	88.3	99.0	102.7	109.4
Predicted length (cm)	58.2	61.0	69.5	77.9	***A***	97.7	***B***	114.7
Residual	–9.1	–1.2	4.5	5.8	4.7	1.3	***C***	***D***

(a) (i) Determine the equation of the least-squares line for predicting the median length from a girl's age. (1 mark)

(ii) Use the equation from (a)(i) to determine the predicted median lengths ***A*** and ***B*** in the above table.

A = _____

B = _____ (2 marks)

(iii) What increase in median length can be expected for each additional year? (1 mark)

(iv) Given that the correlation coefficient is 0.97, describe the association between age and median length in terms of its direction and strength. (2 marks)

(v) What percentage of the variation in the median length can be explained by the variation in age? (1 mark)

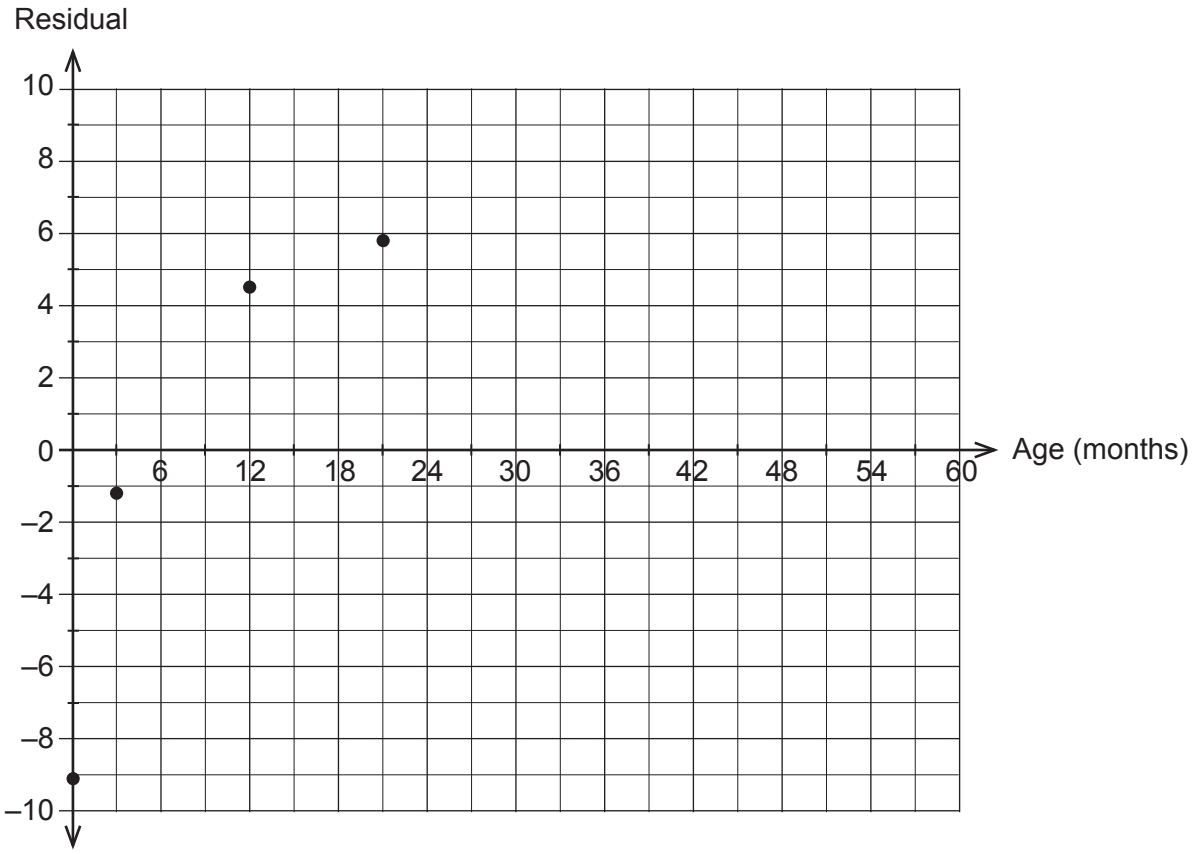
See next page

(b) (i) Determine the residuals C and D in the table.

C = _____

D = _____ (2 marks)

(ii) Hence, complete the scattergraph of the residuals against age on the axes below by plotting the last four residual values. (2 marks)



(iii) Use the residual plot to assess the appropriateness of fitting a linear model to the data. (2 marks)

Question 10 **(12 marks)**

In a laboratory experiment, the population of a particular bacteria began with 400 present. The bacteria grew at a rate of 35% each week, where P is the number of bacteria and t is the number of weeks from the start of the experiment.

(a) Four possible equations were produced to model this experiment:

$P = 400(1.35)^t$

$P = 400(0.35)^t$

$P = 540(1.35)^{t-1}$

$P = 540(1.35)^{t+1}$.

Circle the correct equation(s). (2 marks)

(b) Calculate the population of bacteria after three weeks. (1 mark)

(c) During which week did the population of bacteria first reach 1800? (2 marks)

(d) After eight weeks the growth rate slowed to 20% each week. How many weeks in total did it take for the population of bacteria to reach 15 812? (3 marks)

(e) What constant weekly growth rate would produce the same change in population from 400 to 15 812 in the same time as found in part (d)? (2 marks)

(f) Once the bacteria population reached 15 812 it began to die out at a rate of 250 each day. Approximately how many weeks did it take for the bacteria to die out completely? (2 marks)

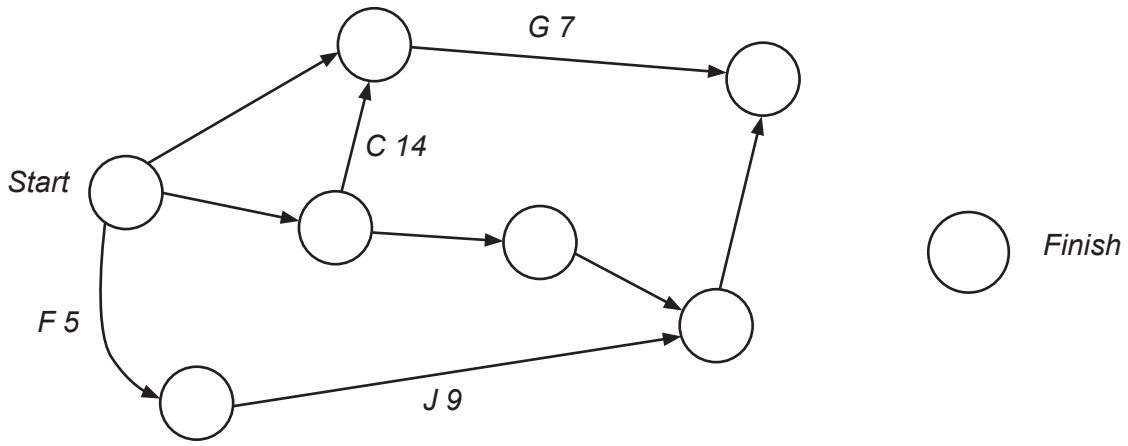
See next page

Question 11 **(11 marks)**

The following table, consisting of 11 activities, contains information for a project in a small manufacturing company.

Activity	Immediate Predecessors	Time (hours)
A	–	4
B	–	5
C	A	14
D	A	7
E	–	7
F	–	5
G	B, C	7
H	D	6
J	E, F	9
K	H, J	10
L	G, K	6

(a) Complete the project network below. (3 marks)



(b) State the critical path and the minimum completion time for this network. (2 marks)

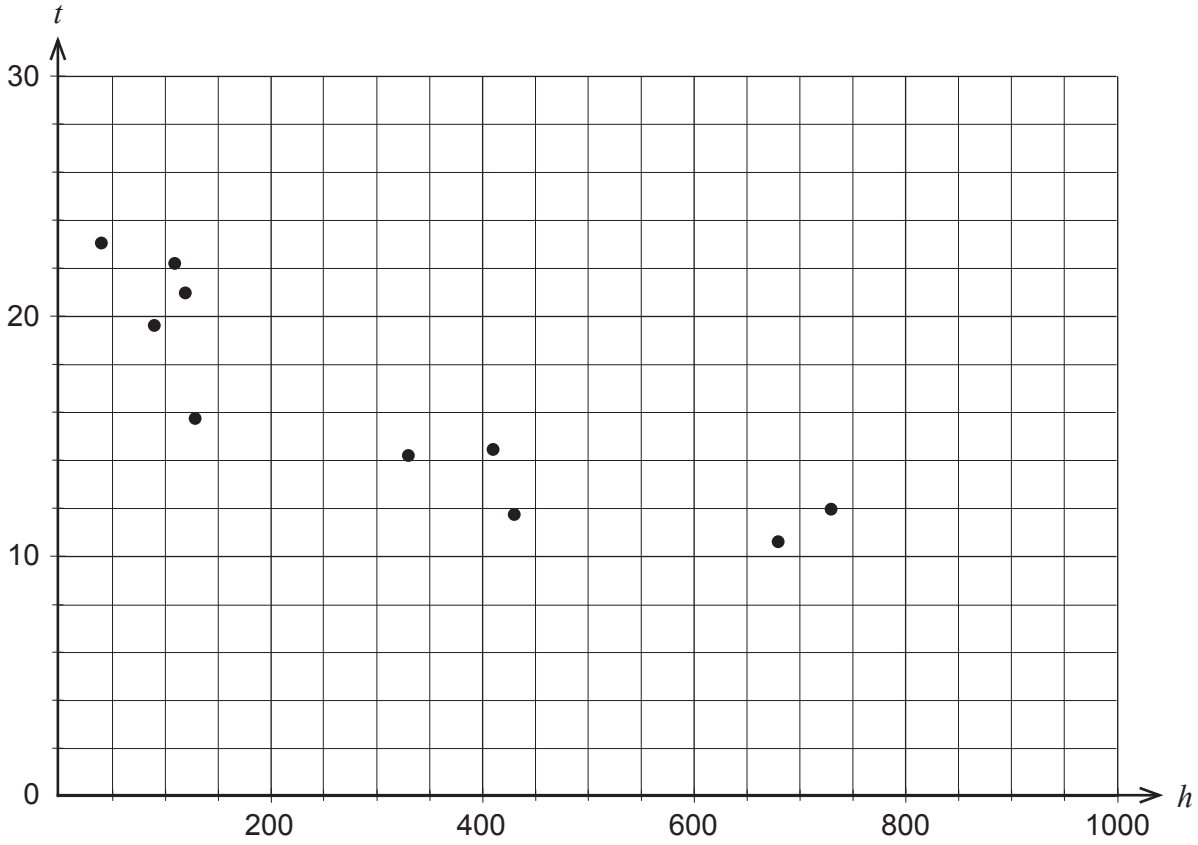
(c) Determine the float time, earliest starting time, and latest starting time for Activity G. (3 marks)

(d) Due to some unforeseen problems with Activities G and J, **one** of these activities will require an extra three hours to complete. Which of the activities should be chosen for the completion time to be at a minimum? Justify your answer. (3 marks)

See next page

Question 12 **(8 marks)**

The Bureau of Meteorology recorded data taken from several weather stations. The scatterplot below shows the height, h (m), of each weather station above sea level and the mean minimum temperature, t (°C), recorded at that station for the month of April.



The following table provides information for three more weather stations for the month of April.

Height of weather station above sea level, h (m)	250	60	930
Mean minimum temperature, t (°C)	13.1	26.2	10.6

(a) Plot this additional information on the scatterplot above. (2 marks)

(b) The equation of the least-squares line for these data is $t = -0.015h + 21.476$. Draw this line on the scatterplot above. (2 marks)

See next page

(c) The correlation coefficient (r) was determined for the collected data. Circle the value of r most likely to be the result from the list below. (1 mark)

$r = -1.2$

$r = -0.8$

$r = -0.2$

$r = 0.5$

$r = 0.9$

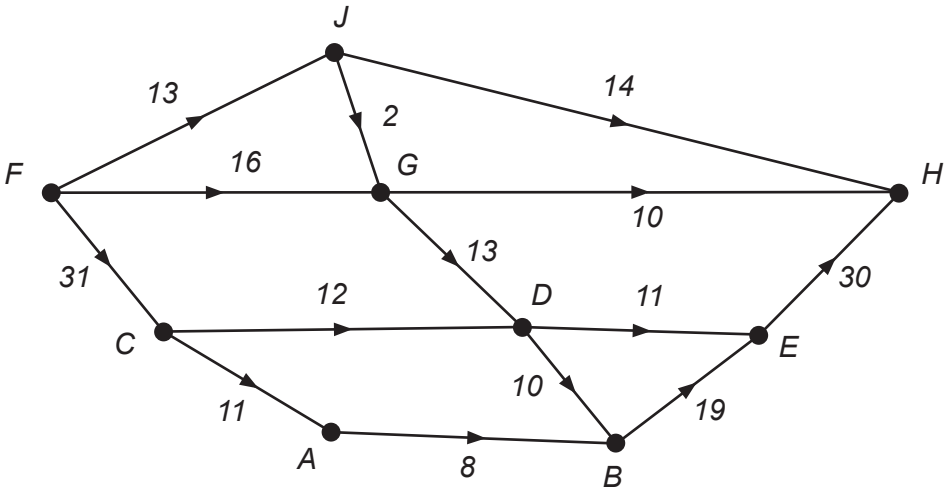
(d) Identify whether the nature of the relationship between the height of a weather station above sea level, h, and the mean minimum temperature, t (°C), is linear or non-linear. (1 mark)

(e) A spokesperson for the Bureau of Meteorology summarised the information from parts (a)–(d), saying 'It is evident that raising the height of a weather station above sea level causes the mean minimum temperature to drop'. Is this statement correct? Justify your decision. (2 marks)

See next page

Question 13 **(8 marks)**

The traffic flow (in hundreds of cars per hour) through a road network (F to H) is shown below.



(a) By listing the different paths and their flow rate, determine the maximum flow through the network. (4 marks)

(b) Verify the maximum flow obtained in part (a) by showing the minimum cut on the given network. (1 mark)

(c) (i) If **one** road is to be widened to allow for more traffic, which road should be chosen to increase the maximum flow the most? (1 mark)

(ii) How much more traffic should this road allow to flow and what would be the new maximum flow for the network? (2 marks)

See next page

Question 14 **(13 marks)**

Andrew takes out a $14 999 loan to purchase his first car after paying a $1200 deposit. The car dealer offered the loan at an introductory interest rate of 1.80% p.a. for the first year and then the rate becomes 3.24% p.a. for the remaining time of the loan. Interest is added monthly and Andrew has calculated he can afford to make monthly repayments of $420.

(a) (i) Express the loan repayment process for the first year as a recursive formula. (2 marks)

(ii) How much does Andrew still owe after one year? (1 mark)

(b) How much does Andrew owe after two years? (3 marks)

(c) How long does it take Andrew to repay the loan? (2 marks)

(d) Determine the amount of the final repayment. (2 marks)

(e) Calculate the total cost of the car. (3 marks)

See next page

Question 15 **(15 marks)**

(a) The table below shows some time series data where t represents time.

t	1	2	3	4	5	6	7	8
x	14	17	18	24	21	19	16	13

Calculate at $t = 4$

(i) the 3-point moving average. (1 mark)

(ii) the 6-point centred moving average. (2 marks)

(b) A retailer in a shopping centre sells mobile phones. The data of its quarterly sales, together with some calculations, are shown in the table below.

Year	Data number (n)	Quarter	Mobile phone sales	Quarterly mean	Percentage of quarterly mean	Deseasonalised figure (D)
2013	1	March	901	905	99.56	915
	2	June	802		88.62	914
	3	September	***A***		97.68	900
	4	December	1033		114.14	894
2014	5	March	973	984.5	98.83	988
	6	June	863		***C***	984
	7	September	964		97.92	981
	8	December	1138		115.59	985
2015	9	March	1049	1065.5	98.45	1065
	10	June	932		87.47	***E***
	11	September	1049		98.45	1068
	12	December	1232		115.63	1066
2016	13	March	1119	***B***	97.01	1136
	14	June	1006		87.21	1147
	15	September	1142		99.00	1162
	16	December	1347		116.78	1166

See next page

(i) Determine the value of A, B and C in the table in part (b) on the previous page. (3 marks)

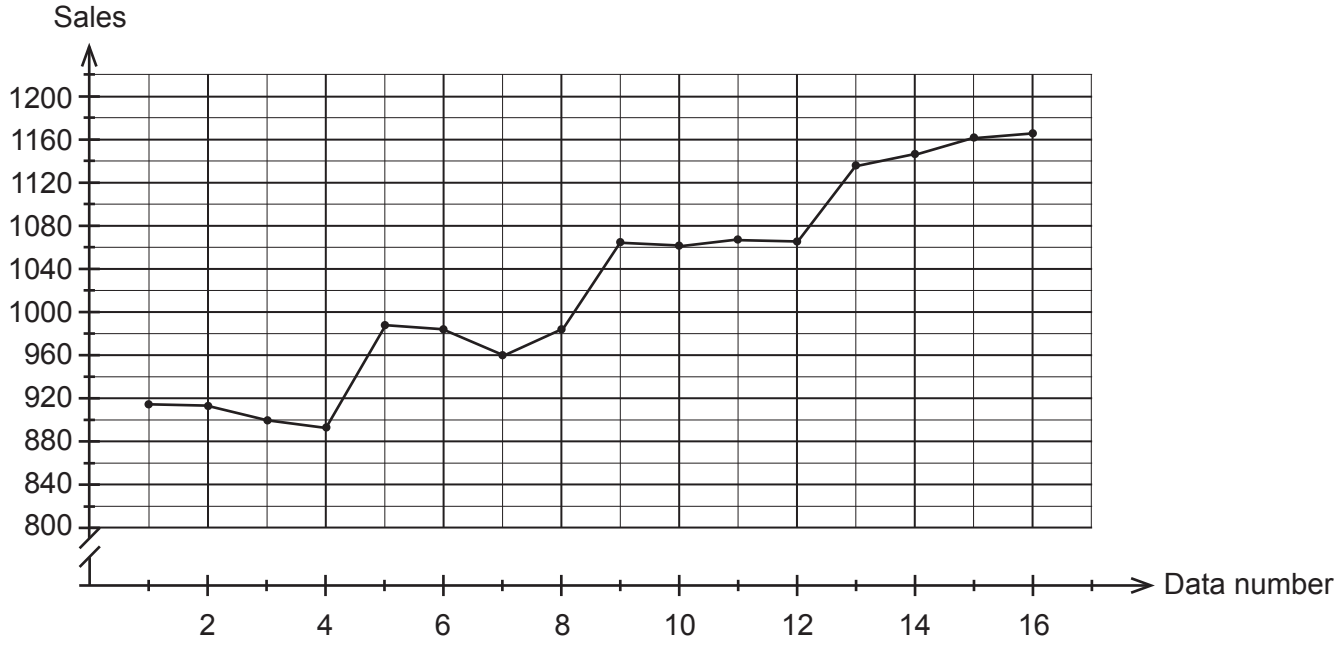
(ii) Complete the Seasonal Index table below. (1 mark)

Quarter	March	June	September	December
Seasonal Index	0.9846	0.8774	0.9826	

(iii) Determine the value of E in the table in part (b) on the previous page. (2 marks)

The equation of the least-squares line for deseasonalised figure against data number is $D = 19.37n + 862.4$.

(iv) The graph below shows the deseasonalised figures. Draw, on the graph, the least-squares line. (2 marks)



(v) Predict the mobile phone sales for December 2017. (2 marks)

(vi) Comment on the reliability of your prediction made in part (v). (2 marks)

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Question 16 **(8 marks)**

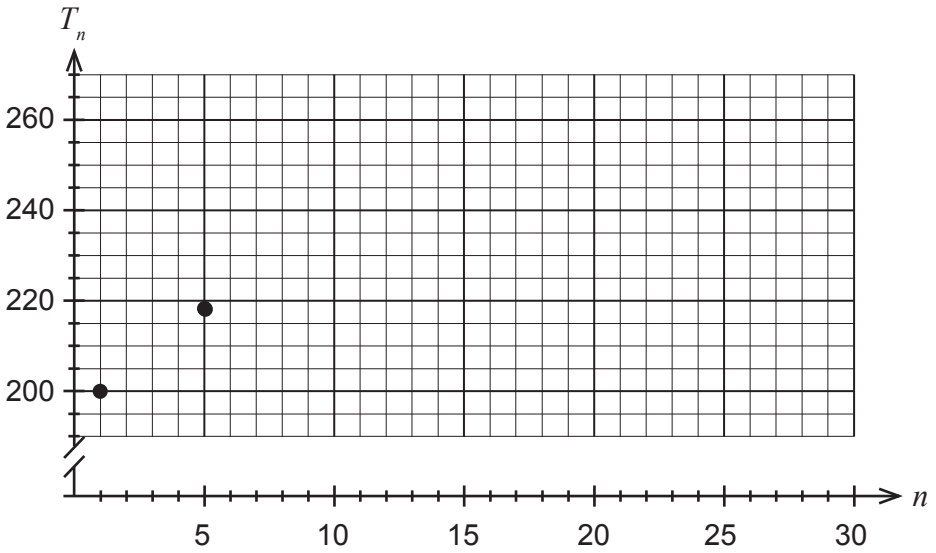
In a Northern Territory river, the crocodile population is dropping by 7.5% each year. The current population is 200. A scheme is being trialled under which 20 crocodiles are introduced to the river each year.

The population of crocodiles in the river can be modelled by the first-order linear recurrence relation $T_{n+1} = 0.925T_n + b$, $T_1 = 200$, where T_n is the number of crocodiles in the river at the beginning of the n^{th} year.

(a) (i) Interpret the coefficient 0.925 in the context of the question. (1 mark)

(ii) State the value of b. (1 mark)

(b) Graph the number of crocodiles in the river for every five year period (commencing at n = 5), up to the 30th year on the axes below. (2 marks)



(c) Using your graph, comment on how the population of crocodiles is changing over time. (2 marks)

(d) To the nearest whole number, what is the long-term effect on the crocodile population? (2 marks)

End of questions

Supplementary page

Question number: _____

ACKNOWLEDGEMENTS

Question 9 Data source: World Health Organisation (WHO). (2006). *Child growth standards: Weight for length/height* [Girls percentiles from birth to 5 years]. Retrieved June, 2017, from www.who.int/childgrowth/standards/chts_wflh_girls_p/en/

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