Government of **Western Australia**
School Curriculum and Standards Authority

ATAR course examination, 2018

Question/Answer booklet

MATHEMATICS METHODS

Section Two: Calculator-assumed

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

Student number: In figures | | | | | | | | | |

In words _____

_____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	11	11	100	99	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2018*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (99 Marks)**

This section has **11** questions. Answer **all** questions. Write your answers in the spaces provided.

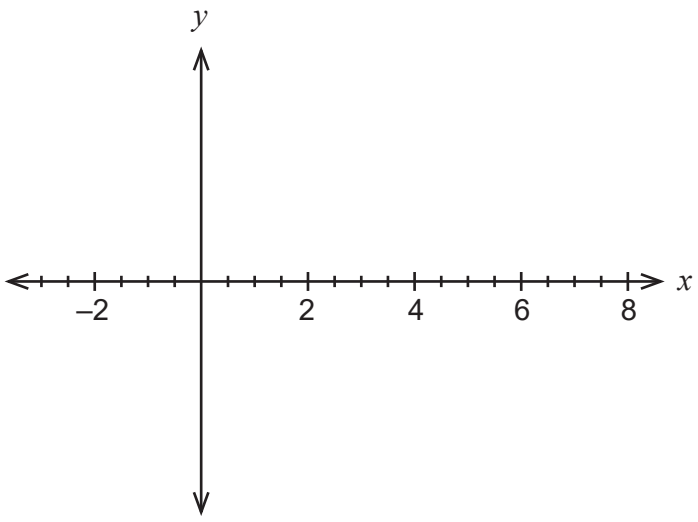
Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 8 **(8 marks)**

Consider the function $f(x) = \log_a (x - 1)$ where $a > 1$.

(a) On the axes below, sketch the graph of $f(x)$, labelling important features. (3 marks)



(b) Determine the value of m if $f(m) = 1$. (2 marks)

(c) Determine the coordinates of the x – intercept of $f(x + b) + c$, where b and c are positive real constants. (3 marks)

Question 9 **(8 marks)**

The concentration, C, of a drug in the blood of a patient t hours after the initial dose can be modelled by the equation below.

$$C = 4e^{-0.05t} \text{ mg/L}$$

Patients requiring this drug are said to be in crisis if the concentration of the drug in their blood falls below 2.5 mg/L.

A patient is given a dose of the drug at 9 am.

(a) What was the concentration in the patient's blood immediately following the initial dose? (1 mark)

(b) What is the concentration of the drug in the patient's blood at 11.30 am? (2 marks)

(c) Find the rate of change of C at 1 pm. (2 marks)

(d) What is the latest time the patient can receive another dose of the drug if they are to avoid being in crisis? (3 marks)

See next page

Question 10 **(7 marks)**

The following function is a probability density function on the given interval:

$$f(x) = \begin{cases} ax^2(x-2) & \text{for } 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of a. (3 marks)

(b) Find the probability that $x \ge 1.2$ (2 marks)

(c) Find the median of the distribution. (2 marks)

See next page

Question 11 **(8 marks)**

Ava is flying a drone in a large open space at a constant height of 5 metres above the ground. She flies the drone due north so that it passes directly over her head and then, sometime later, reverses its direction and flies the drone due south so it passes directly over her again. With $t = 0$ defined as the moment when the drone first flies directly above Ava's head, the velocity of the drone, at time t seconds, is given by:

$$v = 2 \sin\left(\frac{t}{3} + \frac{\pi}{6}\right) \text{ m/s } 0 \leq t \leq 16$$

(a) Determine $x(t)$, the displacement of the drone at t seconds, where x (0) = 0. (3 marks)

(b) Where is the drone in relation to the pilot after 16 seconds? (2 marks)

See next page

(c) At a particular time, the drone is heading due south and it is decelerating at 0.5 m/s^2. How far has the drone travelled from its initial position directly above Ava's head until this particular time? (3 marks)

See next page

Question 12 **(19 marks)**

The manager of the mail distribution centre in an organisation estimates that the weight, x (kg), of parcels that are posted is normally distributed, with mean 3 kg and standard deviation 1 kg.

(a) What percentage of parcels weigh more than 3.7 kg? (2 marks)

(b) Twenty parcels are received for posting. What is the probability that at least half of them weigh more than 3.7 kg? (3 marks)

The cost of postage, (\$) y, depends on the weight of a parcel as follows:

- a cost of \$5 for parcels 1 kg or less
- an additional variable cost of \$1.50 for every kilogram or part thereof above 1 kg to a maximum of 4 kg
- a cost of \$12 for parcels above 4 kg.

(c) Complete the probability distribution table for Y. (4 marks)

x	≤ 1	$1 < x \leq 2$	$2 < x \leq 3$	$3 < x \leq 4$	$x > 4$
y	\$5				
$P(Y = y)$					

See next page

(d) Calculate the mean cost of postage per parcel. (2 marks)

(e) Calculate the standard deviation of the cost of postage per parcel. (3 marks)

(f) If the cost of postage is increased by 20% and a surcharge of $1 is added for all parcels, what will be the mean and standard deviation of the new cost? (3 marks)

(g) Show one reason why the given normal distribution is not a good model for the weight of the parcels. (2 marks)

See next page

Question 13 **(10 marks)**

The proportion of caravans on the road being towed by vehicles that have the incorrect towing capacity is p.

(a) Show, using calculus, that to maximise the margin of error a value of $\hat{p} = 0.5$ should be used. Note: As z and n are constants, the standard error formula can be reduced to $E = \sqrt{\hat{p}(1-\hat{p})}$. (3 marks)

(b) A consulting firm wants to determine p within 8% with 99% confidence. How many towing vehicles should be tested at a random check? (3 marks)

(c) Six months later, the consulting firm carries out a random sampling of towing vehicles. A 99% confidence interval calculated for the proportion of vehicles with incorrect towing capacity is (0.342, 0.558). Determine the number of vehicles in the sample that have an incorrect towing capacity. (4 marks)

See next page

Question 14 **(5 marks)**

(a) The table below examines the values of $\frac{a^h - 1}{h}$ for various values of a as h approaches zero. Complete the table, rounding your values to five decimal places. (2 marks)

h	$a = 2.60$	$a = 2.70$	$a = 2.72$	$a = 2.80$
0.1	1.00265		1.05241	1.08449
0.001	0.95597	0.99375		
0.00001	0.95552			1.02962

It can be shown that $\frac{d}{dx}(a^x) = a^x \lim_{h \to 0} \left(\frac{a^h - 1}{h} \right)$.

(b) What is the exact value of a for which $\frac{d}{dx}(a^x) = a^x$? Explain how the above definition and the table in part (a) support your answer. (3 marks)

See next page

Question 15 **(5 marks)**

The population of mosquitos, P (in thousands), in an artificial lake in a housing estate is measured at the beginning of the year. The population after t months is given by the function, $P(t) = t^3 + at^2 + bt + 2$, $0 \leq t \leq 12$.

The rate of growth of the population is initially increasing. It then slows to be momentarily stationary in mid-winter (at $t = 6$), then continues to increase again in the last half of the year.

Determine the values of a and b.

See next page

Question 16 **(8 marks)**

Let $f(x)$ be a function such that $f(-2) = 4, f(-1) = 0, f(0) = -1, f(1) = 0$ and $f(3) = 2$.
Further, $f'(x) < 0$ for $-2 \leq x < 0, f'(0) = 0$ and $f'(x) > 0$ for $0 < x \leq 3$.

(a) Evaluate the following definite integrals:

(i) $\int_0^3 f'(x)\,dx.$ (2 marks)

(ii) $\int_{-2}^3 f'(x)\,dx.$ (2 marks)

(b) What is the area bounded by the graph of $f'(x)$ and the x axis between $x = -2$ and $x = 3$? Justify your answer. (4 marks)

See next page

Question 17 **(14 marks)**

Tina believes that approximately 60% of the mangoes she produces on her farm are large. She takes a random sample of 500 mangoes from a day's picking.

(a) Assuming Tina is correct and 60% of the mangoes her farm produces are large, what is the approximate probability distribution of the sample proportion of large mangoes in her sample? (3 marks)

(b) What is the probability that the sample proportion of large mangoes is less than 0.58? (2 marks)

(c) Tina decides to select the mangoes for her sample as they pass along the conveyor belt to be sorted. Describe briefly how Tina should select her sample. (2 marks)

A random sample of 500 contains 250 large mangoes.

(d) On the basis of this data, estimate the proportion of large mangoes produced on the farm. (1 mark)

See next page

(e) Calculate a 95% confidence interval for the proportion of large mangoes produced on the farm, rounded to four decimal places. (3 marks)

(f) On the basis of your calculations, how would you respond to Tina's belief that the proportion of large mangoes produced is at least 60%? Justify your response. (2 marks)

(g) What can Tina do to further test her belief? (1 mark)

See next page

Question 18 **(7 marks)**

The ear has the remarkable ability to handle an enormous range of sound levels. In order to express levels of sound meaningfully in numbers that are more manageable, a logarithmic scale is used, rather than a linear scale. This scale is the decibel (dB) scale.

The sound intensity level, L, is given by the formula below:

$L = 10\log\left(\frac{I}{I_0}\right)$ dB where I is the sound intensity and I_0 is the reference sound intensity.

I and I_0 are measured in watt/m^2.

(a) Listening to a sound intensity of 5 billion times that of the reference intensity $\left(I = 5 \times 10^9 I_0\right)$ for more than 30 minutes is considered unsafe. To what sound intensity level does this correspond? (2 marks)

(b) The reference sound intensity, I_0, has a sound intensity level of 0 dB. If a household vacuum cleaner has a sound intensity $I = 1 \times 10^{-5}$ watt/m^2 and this corresponds to a sound intensity level $L = 70$ dB, determine I_0. (2 marks)

The average sound intensity level for rainfall is 50 dB and for heavy traffic 85 dB.

(c) How many times more intense is the sound of traffic than that of rainfall? (3 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Published by the School Curriculum and Standards Authority of Western Australia
303 Sevenoaks Street
CANNINGTON WA 6107