Government of **Western Australia**
School Curriculum and Standards Authority

scsa

ATAR course examination, 2022

Question/Answer booklet

MATHEMATICS METHODS

Section Two: Calculator-assumed

WA student number: In figures

In words

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional
answer booklets used
(if applicable):

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	6	6	50	54	35
Section Two: Calculator-assumed	9	9	100	100	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2022: Part II Examinations*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (100 Marks)

This section has **nine** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
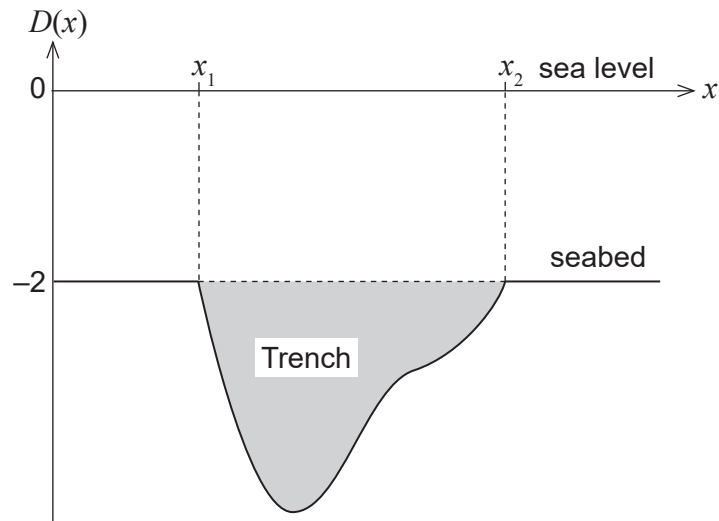
Working time: 100 minutes.

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Question 7 **(10 marks)**

A team of oceanographers surveyed the depth of the ocean in a region populated by a particular endangered fish species. They discovered a large trench extending below the otherwise flat seabed as shown in the figure below.

The displacement, in kilometres, from sea level to the ocean floor is given by

$$D(x) = \begin{cases} (x-4)^2 + \cos(2x - 3\pi) - 5, & x_1 \leq x \leq x_2 \\ -2, & \text{otherwise} \end{cases}$$

where x (measured in kilometres) is the east–west horizontal displacement relative to a reference marker at sea level.

(a) With reference to the figure above:

   (i) determine the values of x_1 and x_2. (2 marks)

   (ii) use calculus to determine the cross-sectional area of the trench shaded in the figure above. (3 marks)

See next page

(b)   Using calculus, determine the maximum distance of the trench below sea level. (5 marks)
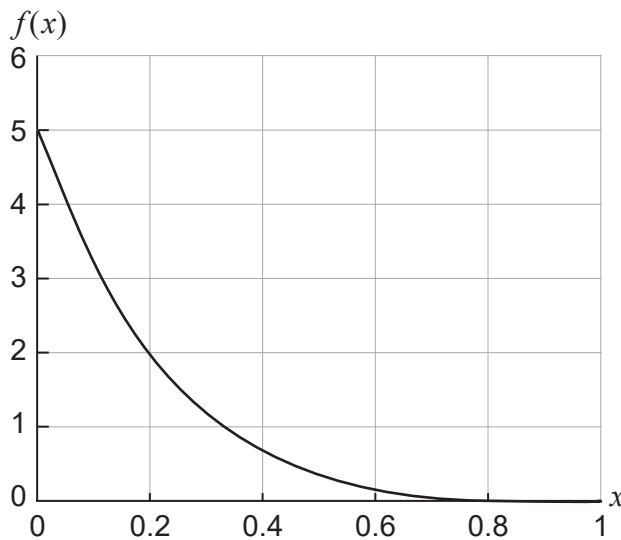
Question 8 **(10 marks)**

A small outback petrol station receives a weekly delivery of petrol. The volume of petrol sold in a week, X, (in units of 10 000 litres) is a random variable with probability density function

$$f(x) = 5(1-x)^4, \quad 0 \le x \le 1$$

as shown in the graph below.

(a) Determine, using appropriate units, the expected value and variance of the amount of fuel sold in a week. (4 marks)

(b)   What storage tank capacity will ensure that there is only a 1% chance of running out of petrol in a given week? State your answer to the nearest litre.   (3 marks)

(c)   When the petrol is delivered, it is pumped into the storage tank. The rate of change of the petrol level in the tank, $h(t)$, (measured in metres) at time t (measured in minutes) is given by

$$h'(t) = \frac{5}{2t + 3}$$

Determine the height of the storage tank if it takes 20 minutes to fill.   (3 marks)

Question 9 **(14 marks)**

Andrew plans to run a game called Lucky Cup for a school fundraising event. All profits go toward the school's fundraising efforts. The game consists of three standard dice, each placed into a red cup. The red cup is shaken, the dice rolled, and the number of sixes recorded.

Let X be a random variable denoting the number of sixes rolled in a game of Lucky Cup.

(a)   State the distribution of X.   (2 marks)

An incomplete probability distribution for X is shown in the table below.

x	0	1	2	3
$P(X=x)$	$\dfrac{125}{216} = 0.5787$			

(b)   Complete the table above, providing the missing probabilities.   (2 marks)

Lucky Cup costs $1 to play. If a player rolls one 6 they win $1, if they roll two 6s they win $2, and if they roll three 6s they win $3.

(c)   Determine the school's expected profit/loss for each game of Lucky Cup.   (3 marks)

(d)   Determine the probability that a player will make a profit in a game of Lucky Cup.

(2 marks)

Andrew wants to increase the attraction of the game by providing the opportunity for larger winnings. He modifies the rules of the game so that players only win money when two or more 6s are rolled, and the winnings for rolling three 6s is three times as much as the winnings for rolling two 6s. Each game still costs $1 to play. He estimates that he should be able to run 500 games and wants to make a profit of $200 for the school.

Andrew calls this game Lucky Cup II.

(e)   Determine the winnings Andrew should set for rolling three 6s in Lucky Cup II.   (3 marks)

Question 9 (continued)

Andrew decides to make the game even more dynamic and exciting. He adds a green cup with a die that he rolls at the beginning of each game. The value that Andrew rolls becomes the target value, and players must roll this target value to win. For example, if Andrew rolls a 2 from the green cup and a player rolls three 2s, the player wins the top prize.

Andrew calls this game Lucky Cup III.

(f) Explain how this change affects a player's chance of winning compared with Lucky Cup II.

(2 marks)

Question 10 **(9 marks)**

The displacement, x, of a mass on the end of a damped spring is given by

$$x(t) = 3e^{-t}\sin(t), \quad t \geq 0$$

where x is in centimetres and t is in seconds.

(a) Determine when the mass first returns to its starting position at $x = 0$. (2 marks)

(b) Determine an expression for the velocity of the mass. (2 marks)

(c) Determine the displacement of the mass when it first changes direction. (3 marks)

(d) The mass is considered to have stopped oscillating when the oscillation amplitude $A(t) = 3e^{-t}$ drops to 0.01 cm. How long does it take for the spring to stop oscillating? (2 marks)

Question 11 **(11 marks)**

The 100 m sprint is a race run on a straight section of track. During a race the velocity, v, measured in metres per second, of an athlete is given by

$$v(t) = -10e^{-0.8t} - 0.05e^{0.2t} + 10.05$$

where t is the time, in seconds, measured from the moment the athlete starts to move from the start line.

(a) Determine the acceleration of the athlete three seconds after moving from the start line.

(2 marks)

(b) Using calculus, determine the maximum velocity of the athlete during the race, and the time, t, at which it is achieved. (4 marks)

(c)   The displacement, x, of the athlete is 0 m at the start of the race. Determine an
        expression for the displacement of the athlete during the race.   (3 marks)

(d)   Determine the time, t, at which the athlete finishes the 100 m race.   (2 marks)

Question 12 **(16 marks)**

The Larje Machine Co manufactures metal rods for large industrial equipment. Their standard manufacturing process produces rods whose lengths are normally distributed with a mean of 400 cm, and a standard deviation of 5 cm. A rod is considered 'useable' if its length is between 395 cm and 405 cm.

Let X be a random variable denoting the length of a rod manufactured by the Larje Machine Co.

(a)   Determine the probability that a rod manufactured by the Larje Machine Co is useable. Round your answer to three decimal places. (3 marks)

Recently the Larje Machine Co introduced a new manufacturing process that industry experts claim will improve the percentage of useable rods produced to 80%. The quality control department decides to investigate whether this standard is being achieved and plan to collect a random sample of rods manufactured using the new process.

(b)   What condition must the sample satisfy in order to use a normal distribution to model the sample proportion of useable rods? (1 mark)

The quality control department collects a sample of 100 rods.

(c)   What is the approximate distribution of the sample proportion of useable rods? (2 marks)

Upon measuring the sample of 100 rods, it is found that 75 are useable.

(d)   Calculate a 95% confidence interval for the population proportion of useable rods.

(3 marks)

(e)   The quality control department would like to obtain a confidence interval with a smaller margin of error. State **two** methods that it could use to achieve this.   (2 marks)

(f)   The quality control department decides to select a new sample for which the maximum possible margin of error for a 95% confidence interval is 0.05. What sample size will achieve this requirement?   (3 marks)

(g)   The new sample yields the 95% confidence interval $(0.717, 0.803)$. On the basis of this sample, is the proportion of useable rods different from what was claimed by the industry experts? Justify your answer.   (2 marks)

Question 13 **(12 marks)**

According to the Association of Poultry Farmers, 35% of people living in Melbourne purchase free-range eggs.

(a)   If a random sample of 100 people living in Melbourne is surveyed, what is the probability that the sample proportion of people who purchase free-range eggs will be less than 0.28?

(3 marks)

A market research company wants to know whether the proportion p of people living in Perth who purchase free-range eggs is similar to that of Melbourne. A junior employee proposes that it gather a sample of shoppers by standing outside a particular shop between 9 am and 10 am on a Tuesday morning and asking all shoppers entering the shop if they purchase free-range eggs.

(b)   Identify and explain **two** sources of bias in the proposed sampling method.   (4 marks)

The company does not follow the suggestion of the junior employee and instead randomly samples 243 people living in Perth and asks them whether they purchase free-range eggs. On the basis of the results of their survey, a confidence interval for p is calculated to be (0.2520, 0.3488).

(c)   Determine the number of people in the sample who purchase free-range eggs.   (2 marks)

(d)   Determine the level of confidence that was used to calculate the confidence interval.

                                                          (3 marks)

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Question 14 **(13 marks)**

The intensity of light travelling through a medium decreases due to scattering and absorption. The intensity of light, I, after travelling a distance of x centimetres through a soft tissue sample is given by

$$I = I_0 e^{-0.75x}$$

where I_0 is the initial light intensity.

(a) What percentage of the initial light intensity remains after the light has travelled 1 cm through the soft tissue? (2 marks)

(b) After how many centimetres will the light intensity have reached one quarter of its initial value? (2 marks)

(c)   Determine an expression for $\ln\left(\dfrac{I}{I_0}\right)$, and hence plot $\ln\left(\dfrac{I}{I_0}\right)$ versus x on the axes below.

(3 marks)
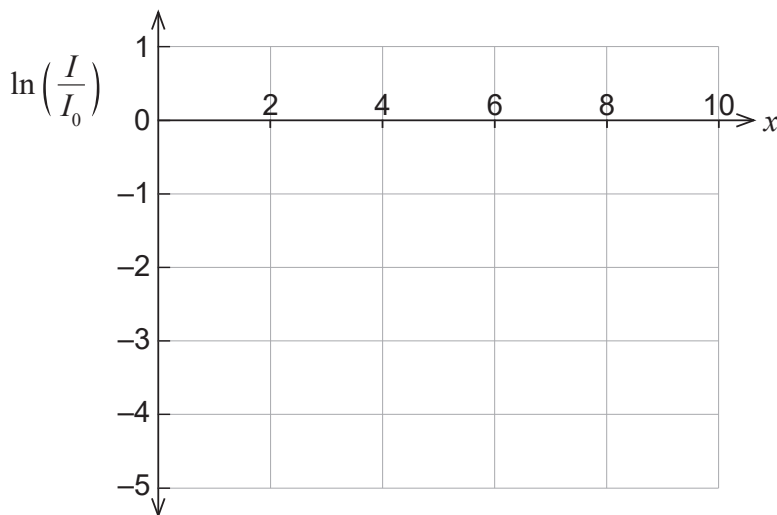
The intensity of light passing through a different type of soft tissue satisfies the equation

$$I = I_0 e^{-\mu x}$$

where μ is the attenuation constant. Light intensity measurements were made on a sample of soft tissue, and the results plotted in the graph below.
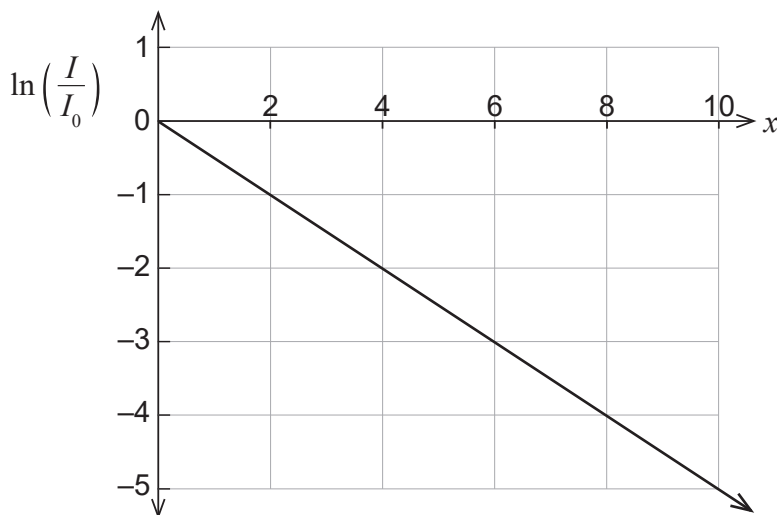
(d)   Use the graph to determine the value of the attenuation constant, μ.   (1 mark)

See next page

Question 14 (continued)

(e) (i) Express the equation $I = I_0 e^{-0.75x}$ using base 10 (in the form $I = I_0 10^{-bx}$). State the
value of b to three decimal places. (3 marks)

(ii) Describe the change in intensity over a distance of $\dfrac{1}{b}$ cm. (2 marks)

Question 15 **(5 marks)**

An object moves from the point $(0, 0)$ along the curve $y = \sqrt{3} \sin(x)$. The distance, D, travelled along the curve is given by

$$D(t) = \int_0^{\pi t} \sqrt{1 + 3 \cos^2(x)} \, dx$$

where D is measured in metres and t is measured in seconds.

(a)   Determine the speed $s = \dfrac{dD}{dt}$ of the object when $t = 1$. (3 marks)

(b)   Use the increments formula to estimate the distance travelled by the object between $t = 1$ and $t = 1.02$. (2 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____