2022/55498

Government of **Western Australia**
School Curriculum and Standards Authority

ATAR course examination, 2022

Question/Answer booklet

MATHEMATICS SPECIALIST

Section Two: Calculator-assumed

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

WA student number: In figures

In words

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	48	35
Section Two: Calculator-assumed	12	12	100	86	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2022: Part II Examinations*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (86 Marks)**

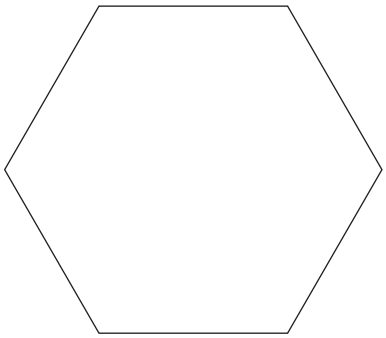
This section has **12** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 8 **(4 marks)**

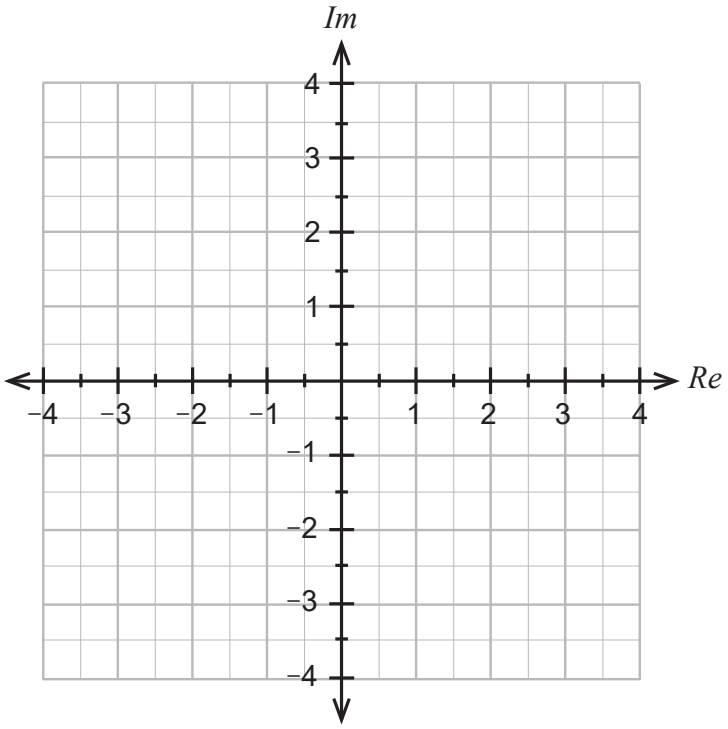
A regular hexagon expands so that the length of each side increases at a rate of 0.5 cm per second. Assuming that the polygon maintains its shape, determine the rate at which the area is increasing when the side length is 4 cm.



Question 9 **(8 marks)**

(a) Sketch the locus of a complex number z satisfying the condition:

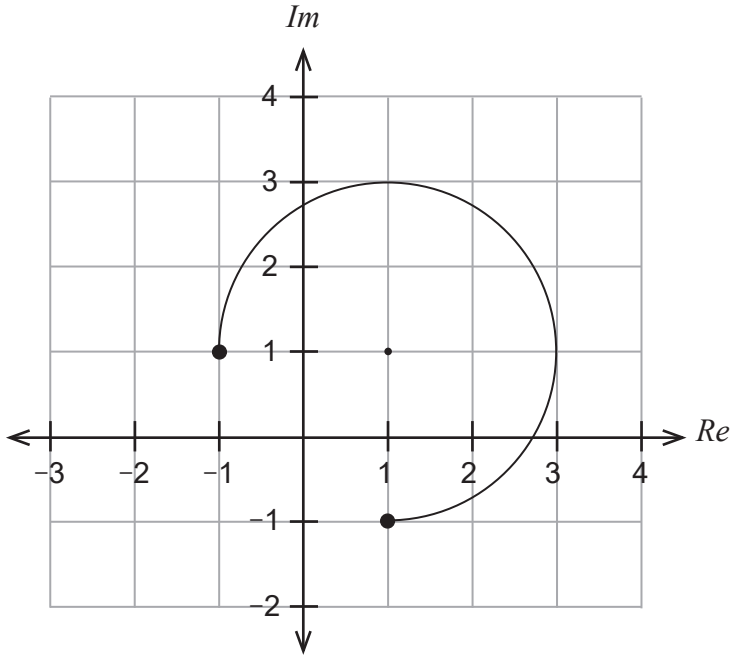
$$|z - 2i| + |z - (3 - 2i)| = 5$$ (2 marks)



A spare grid is provided at the end of this Question/Answer booklet. If you need to use it, cross out this attempt and indicate that you have redrawn it on the spare grid.

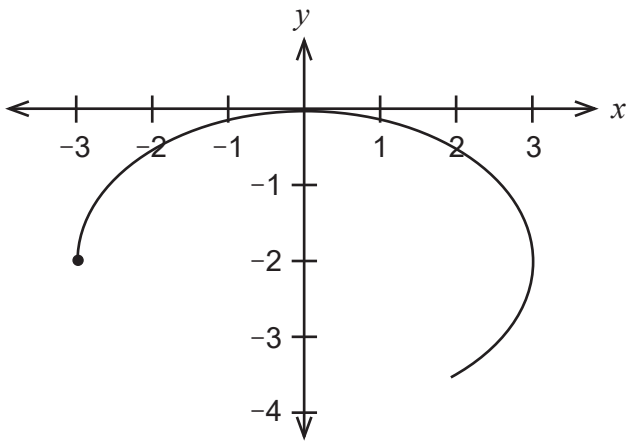
(b) Describe the locus of the equation $(z + i)(\overline{z + i}) = 2$. (3 marks)

(c) The sketch of the locus of a complex number z has been shown below. Write equations or inequalities in terms of z (without using $x = Re(z)$ or $y = Im(z)$) for the indicated locus. (3 marks)



Question 10 **(9 marks)**

The velocity of a particle is given by $\underset{\sim}{v}(t) = \begin{pmatrix} 3\sin t \\ 2\cos t \end{pmatrix}$ where $t \geq 0$. The particle's initial position vector $\underset{\sim}{r}(0) = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$. The path of the particle is shown for the first 4 seconds.



(a) State what the following definite integrals measure about the motion of the particle:

(i) $\int_0^1 \underset{\sim}{v}(t)\, dt$ (2 marks)

(ii) $\int_0^{2\pi} |\underset{\sim}{v}(t)|\, dt$ (2 marks)

(b) Determine $\underset{\sim}{r}(t)$. (3 marks)

(c) Determine the Cartesian equation for the path of the particle. (2 marks)

See next page

Question 11 **(8 marks)**

A Formula One (F1) racing car has an initial displacement of 192 metres with an initial velocity of 24 metres per second. It accelerates for a period of 11 seconds in a straight line so that its velocity v metres per second and displacement x metres are related by the equation:

$$v(x) = \frac{x}{8}$$

(a) Determine the acceleration a as a function of displacement x i.e. determine $a(x)$. (2 marks)

(b) Determine the displacement x as a function of time t. (2 marks)

(c) Calculate the top speed reached and the distance travelled during the 11 second period of acceleration. (4 marks)

See next page

Question 12 **(9 marks)**

The inner diameter of a cylinder in a motor car engine is critical to its performance. Let μ mm denote the population mean cylinder diameter produced by a manufacturing process. A random sample, R_1, of 100 cylinder diameters is taken and the standard deviation for this sample was found to be 1 mm.

Let $\bar{X} =$ the sample mean cylinder diameter for sample R_1.

(a) State the distribution for $\bar{X}$ and its parameters. (3 marks)

(b) What is the probability that $\bar{X}$ differs from μ by more than 0.2 mm. Give your answer correct to 0.001. (2 marks)

From random sample R_1, a 95% confidence interval for μ is formed.

(c) Calculate the width of this confidence interval, correct to 0.001. (2 marks)

See next page

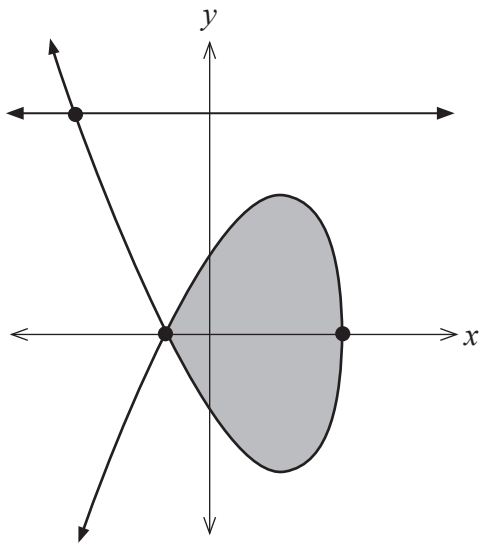
Lilian, the production manager, wishes to decrease the width of the confidence interval. She suggests:

"We can form sample R_2 by using the data from sample R_1 and then combining this data with itself to form a sample with 200 observations. Using $n = 200$ will decrease the width of the confidence interval."

(d) State **two** major problems with using this idea. (2 marks)

Question 13 **(8 marks)**

The equation $x^3 - x^2 - 5x = 3 - y^2$ implicitly defines the curve shown below. The line $y = \sqrt{24}$ intersects this curve as shown below.



It can be shown that the equation $x^3 - x^2 - 5x + 21 = 0$ will determine the intersection between the line $y = \sqrt{24}$ and the implicitly defined curve.

(a) Explain, with reference to the graph above, why we know that there is one real and two complex solutions (a conjugate pair) to this cubic equation. (2 marks)

(b) Determine the **two** exact complex solutions to the equation $x^3 - x^2 - 5x + 21 = 0$.
(2 marks)

See next page

(c) Calculate the area of the shaded region, correct to 0.001 square units. (4 marks)

See next page

Question 14 **(6 marks)**

The annual incomes (in thousands of dollars) of a random sample of n Australians is taken. The sample standard deviation is 10.98. A 99% confidence interval I_1 based on this sample is $90 \leq \mu \leq 94$.

(a) Calculate the value of the sample size n. (2 marks)

Another random sample of size n is taken and a 99% confidence interval I_2 is calculated.

(b) State **two** aspects in which the intervals I_1 and I_2 may be different. (2 marks)

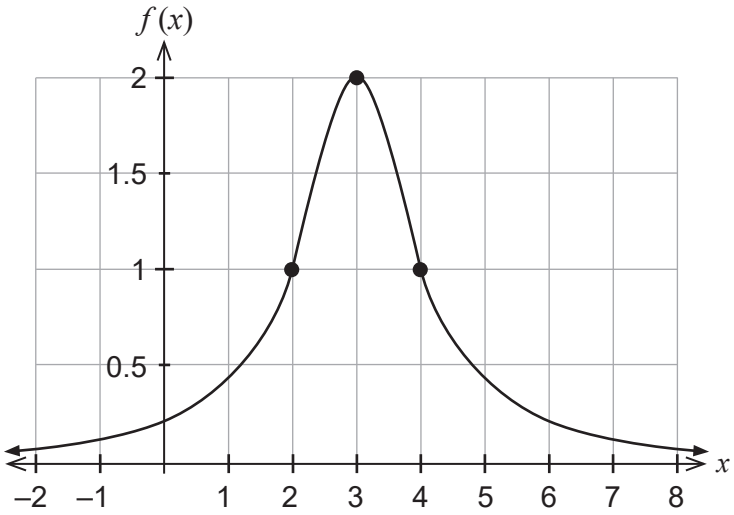
A third random sample of size 50 is taken and a 99% confidence interval I_3 is calculated. James suggests that since interval I_3 is the widest, it is more likely to contain the population mean Australian income μ.

(c) Is James correct? Justify your answer. (2 marks)

Question 15 **(4 marks)**

The graph of a rational function f is shown below. Function f has the form $f(x) = \dfrac{k}{q(x)}$ with the following properties:

- f has no x intercepts or vertical asymptotes
- $f(x) \to 0$ for $|x| \to \infty$
- f is symmetric about $x = 3$
- function q is quadratic
- k is a constant.

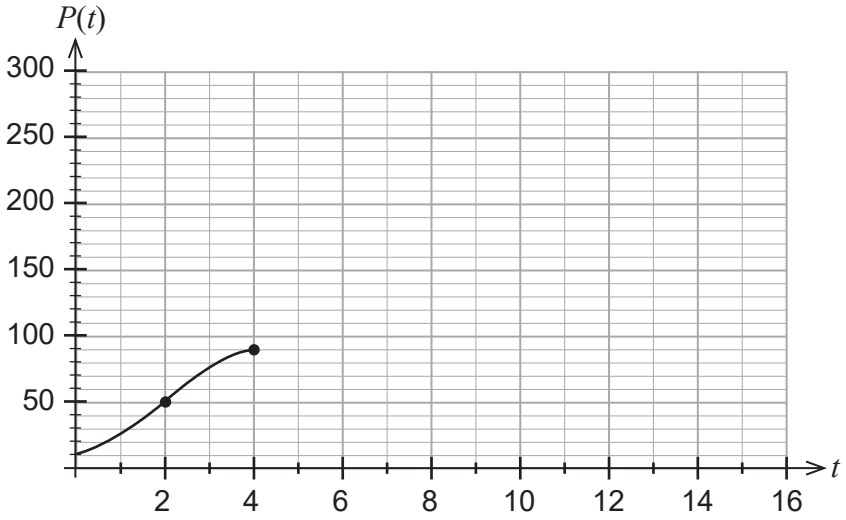


Determine the defining rule for f.

See next page

Question 16 **(4 marks)**

An ant colony population P at time t days grows at a rate given by the equation $\frac{dP}{dt} = 0.01P(100 - P)$, where $0 \le t \le 4$. The graph of this population is shown below.



(a) For $0 \le t \le 4$, using the growth rate equation explain the variation of the population. (2 marks)

At the end of the fourth day, the environment for the ant colony improves dramatically so that its limiting population is increased to 300.

(b) Sketch, on the axes above, the expected variation of the population for $t > 4$ days, using the increased limiting population value. (2 marks)

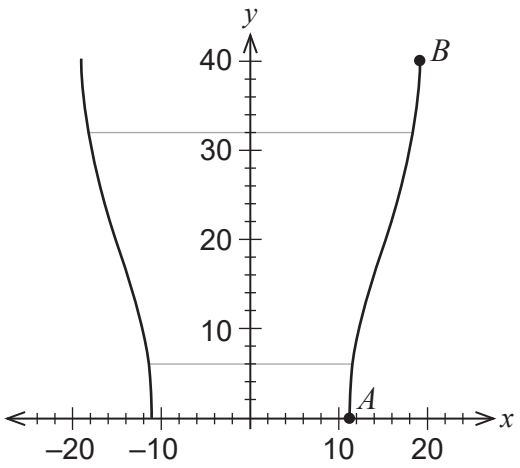
A spare grid is provided at the end of this Question/Answer booklet. If you need to use it, cross out this attempt and indicate that you have redrawn it on the spare grid.

See next page

Question 17 **(10 marks)**

A vase has water with a depth of 6 cm and needs to be filled to a depth of 32 cm. The cross-section of the vase is modelled by the curve AB where $x = 15 - 4\cos\left(\frac{\pi y}{40}\right),\ 0 \le y \le 40,$ and this curve is revolved about the y axis.
All dimensions are in centimetres.

Give all answers in this question to the nearest appropriate unit of measurement.



(a) Calculate the volume of water that needs to be added to increase the depth of water from 6 cm to 32 cm. (3 marks)

Josie, an interior designer, uses a hose to add water to the vase. This hose has a water-saving device that regulates the rate at which water flows into the vase. This rate is given by:

$$\frac{dV}{dt} = 300e^{-\frac{V}{12\,000}}$$

where $V(t)$ = the volume of water (cm^3) poured into the vase after t seconds of flow.

(b) If Josie has already poured 6000 cm^3, use the increments formula to calculate an approximation for the volume of water she will pour in the next 0.5 seconds. (2 marks)

See next page

To prevent an overflow of water, the device can be calibrated to switch off the flow after a set length of time using an in-built timer.

(c) Calculate the rate of flow into the vase at the instant when the depth becomes 32 cm. (2 marks)

(d) Using separation of variables, obtain the defining rule for $V(t)$. (3 marks)

Question 18 **(7 marks)**

(a) Show that for all positive integers n and complex numbers z where $0 \leq \theta \leq \frac{\pi}{2}$,

$(z^n - cis\,(\theta))(z^n + cis\,(-\theta)) = z^{2n} - (2i \sin\theta)\, z^n - 1$. (3 marks)

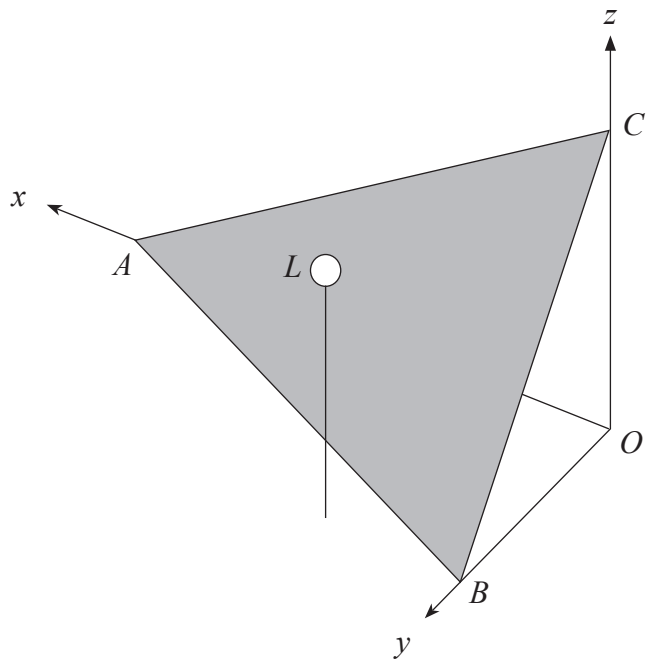
See next page

(b) Hence, using the result from part (a), obtain all the solutions to the equation $z^6 - (i)z^3 - 1 = 0$ in exact polar form. (4 marks)

See next page

Question 19 **(9 marks)**

A downward-sloping ramp is positioned according to the coordinate system shown.
$A\,(6, 0, 0)$, $B\,(0, 2, 0)$ and $C\,(0, 0, 3)$ are points on the ramp. A lamp L is positioned on top of a post at $\left(2, 2, \frac{5}{2}\right)$. All dimensions are measured in metres.



(a) Determine the Cartesian equation for the ramp. (2 marks)

At night, the lamp L emits a bright light and illuminates the ramp. The position that is closest to the lamp will be the most brightly illuminated.

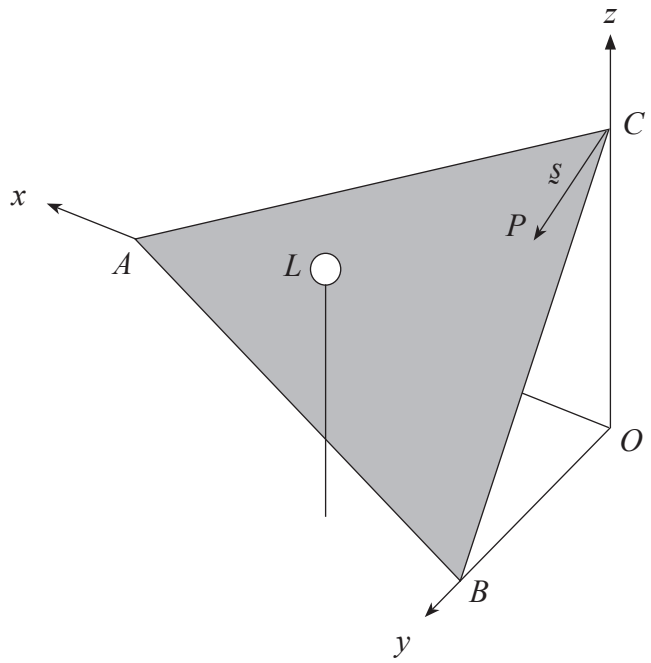
(b) Determine the coordinates for the point on the ramp that is the most brightly illuminated.

(4 marks)

See next page

Question 19 (continued)

If a ball is released from point C and is allowed to roll down the ramp, gravity will cause it to follow the path of steepest descent. Suppose the ball is allowed to roll exactly 1 metre from point C to point P, where $\underset{\sim}{s} = \overrightarrow{CP}$ is the direction of the steepest descent down the ramp.



(c) Determine vector $\underset{\sim}{s}$, giving components correct to 0.001. (3 marks)

End of questions

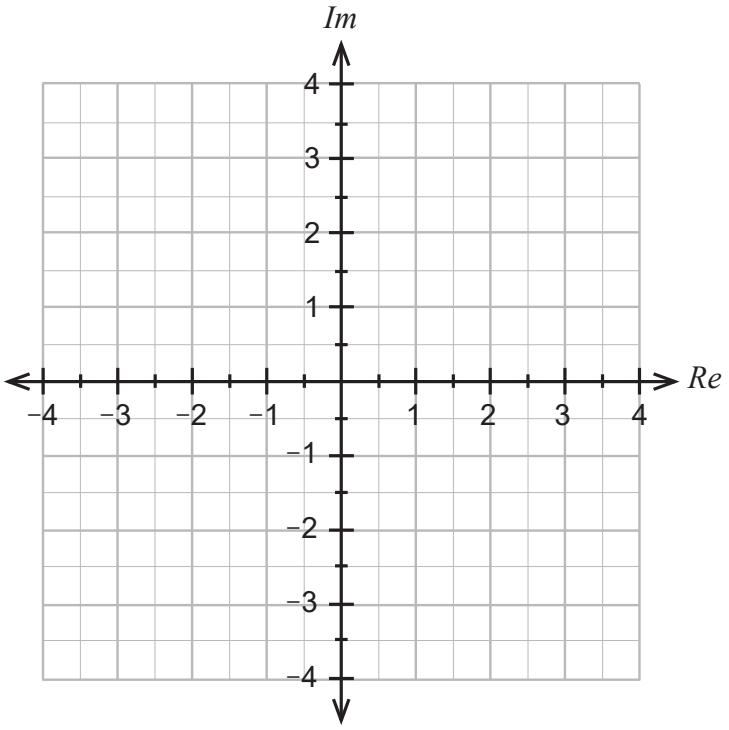
Supplementary page

Question number: _____

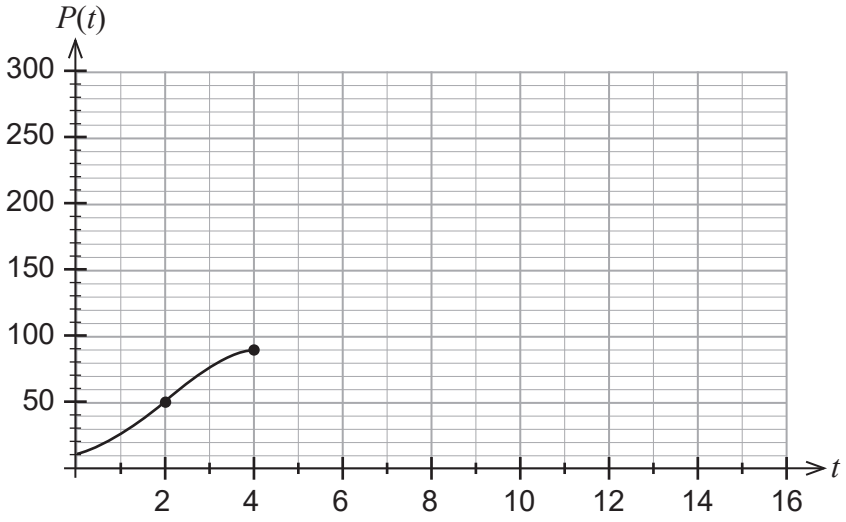
Supplementary page

Question number: _____

Spare grid for Question 9(a)



Spare grid for Question 16(b)



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