2025/4682
Web version of 2024/81897

Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS APPLICATIONS

Calculator-assumed

ATAR course examination 2024

Marking key

Marking keys are an explicit statement about what the examining panel expect of candidates when they respond to particular examination items. They help ensure a consistent interpretation of the criteria that guide the awarding of marks.

Section Two: Calculator-assumed **65% (99 Marks)**

Question 6 **(10 marks)**

Jenny has organised a housing loan. She has modelled the balance owing (in dollars) at the end of each month by the recursive rule $T_{n+1} = 1.0055T_n - 3200, \quad T_0 = 430\,000$.

(a) State

(i) the amount borrowed. (1 mark)

Solution
$430 000
Specific behaviours
✓ correctly states amount borrowed

(ii) the monthly repayment. (1 mark)

Solution
$3200
Specific behaviours
✓ correctly states monthly repayment

(b) Determine the annual interest rate. (1 mark)

Solution
0.0055 × 12 = 6.6%
Specific behaviours
✓ correctly determines annual interest rate

(c) Assuming the interest rate remains unchanged,

(i) how long will it take to pay off the loan? (1 mark)

Solution
245 months
Specific behaviours
✓ correctly states length of time

(ii) determine the final repayment. (1 mark)

Solution
$3005.90
Specific behaviours
✓ correctly determines the final repayment

At the beginning of the eighth year, Jenny makes an extra lump sum payment of $50 000 and increases her repayments by $100 per month.

(d) Calculate how much interest will be saved compared to the original loan arrangement. Assume that the interest rate remains unchanged. (5 marks)

Solution
Balance after 7 years (84 months) = $341 150.33 New recursive rule: $T_{n+1} = 1.0055T_n - 3300, \quad T_0 = 291150.33 \ \left(341\,150.33 - 50\,000\right)$ New loan = 121 × 3300 + 240.33(final payment: 3300 – 3059.67) + 84 × 3200 + 50 000 = 718 340.33 Original loan = 244 × 3200 + 3005.90 = 783 805.90 Therefore, a saving of 783 805.90 – 718 340.33 = $65 465.57
Specific behaviours
✓ calculates balance after 7 years ✓ determines new recursive rule ✓ calculates total repayment after the changes ✓ calculates total repayment under original conditions ✓ calculates savings

Question 7 **(12 marks)**

A new entertainment streaming service is concerned because it is receiving cancellations and not gaining any new subscriptions. The number of subscriptions (in thousands) at the end of each week have been recorded in the table below.

Week (n)	**1**	**2**	**3**	**4**	**5**
Subscriptions (in thousands)		97	94.09	91.267	

(a) Show that the number of subscriptions is decreasing by 3% each week. (2 marks)

Solution
$\frac{94.09}{97} = 0.97$ therefore a 3% decrease.
Specific behaviours
✓ shows correct ratio ✓ states correct percentage decrease

(b) Complete the table above. (1 mark)

Solution					
Week (n)	**1**	**2**	**3**	**4**	**5**
Subscriptions (in thousands)	**100**	97	94.09	91.267	**88.529**
Specific behaviours					
✓ correctly completes table					

(c) Deduce a rule for the n^{th} term for the number of subscriptions at the end of each week. (2 marks)

Solution
$T_n = 100(0.97)^{n-1}$
Specific behaviours
✓ uses correct form for a geometric sequence ✓ states correct a and r values

(d) During which week does the number of subscriptions first fall below 70 000? (2 marks)

Solution
$70 = 100(0.97)^{n-1}$ $n = 12.71$ Therefore, the number of new subscriptions falls below 70 000 during week 13
Specific behaviours
✓ solves correctly for n ✓ states correct week

In an attempt to increase the total number of subscriptions to the streaming service, two possible loyalty programs are being developed, with a plan for one of these to be launched when the number of subscriptions falls to 50 000.

(e) It is hoped that the first loyalty program will change the percentage decrease of subscriptions per week from 3% to 1.4% and also increase the number of new subscriptions by 100 000 each week.

The total number of subscriptions can be represented by the recursive rule

$T_{n+1} = aT_n + b, \quad T_0 = 50.$

Determine the value of a and b. (2 marks)

Solution
a = 0.986 b = 100
Specific behaviours
✓ determines correct value for a ✓ determines correct value for b

(f) The total number of subscriptions from the second loyalty program can be represented by the recursive rule, $T_{n+1} = 0.99T_n + 95, \quad T_0 = 50.$

(i) Interpret the number 0.99 in the context of this question. (1 mark)

Solution
1% rate of cancellations
Specific behaviours
✓ states correct interpretation

(ii) The streaming service will need to maintain 8.5 million total subscriptions if it is to be considered sustainable. Determine whether this is achievable with the second loyalty program. (2 marks)

Solution
$x = 0.99x + 95 \Rightarrow x = 9500$ The steady-state value is 9.5 million subscriptions Therefore, 8.5 million subscriptions is achievable
Specific behaviours
✓ determines the correct steady-state value ✓ states 8.5 million subscriptions are achievable

Question 8 **(13 marks)**

The owners of a small family business that sells farm-fresh honey decide to advertise their produce online. A website has been created and they monitor the number of visits to the website each day for three weeks. The table below shows time series analysis of the collected data using a seven-day period.

Week	Day	Time (*t*)	Weekly mean	Visits as a percentage of the weekly mean	Deseasonalised data
1	Monday	1	43.7	86.9	42.4
	Tuesday	2		91.5	45.3
	Wednesday	3		77.8	45.4
	Thursday	4		**A**	41.2
	Friday	5		116.7	42.6
	Saturday	6		125.8	42.0
	Sunday	7		130.4	47.0
2	Monday	8	45.9	89.4	45.8
	Tuesday	9		85.0	44.1
	Wednesday	10		72.0	44.1
	Thursday	11		76.3	46.5
	Friday	12		124.3	47.6
	Saturday	13		135.2	47.4
	Sunday	14		117.8	44.5
3	Monday	15	50.9	92.4	52.5
	Tuesday	16		88.5	**B**
	Wednesday	17		74.7	50.8
	Thursday	18		78.7	53.1
	Friday	19		118.0	50.1
	Saturday	20		131.7	51.2
	Sunday	21		116.0	48.6

(a) Complete the following table. (2 marks)

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Daily index	89.6%	88.3%	74.8%	75.3%	119.7%	**130.9%**	**121.4%**

Solution
see table above
Specific behaviours
✓ calculates one correct value ✓ calculates second correct value

(b) Determine the value of **A** in the table above. (2 marks)

Solution
$\frac{A+76.3+78.7}{3}=75.3 \quad \therefore A=70.9$
Specific behaviours
✓ uses correct calculation ✓ correctly determines A

(c) Given the actual website visits on Tuesday of week 3 was 45, determine the value of **B** in the table above. (2 marks)

Solution
B = $\frac{45}{0.883} = 50.96 \approx 51.0$
Specific behaviours
✓ sets up correct fraction ✓ correctly determines B

(d) Interpret the daily index for Friday in the context of this question. (1 mark)

Solution
Fridays are 19.7% above the daily mean for the week
Specific behaviours
✓ interprets index correctly

(e) Given there were 62 website visits on Friday of week 4, estimate the total of number of website visits for week 4. (2 marks)

Solution
$\frac{62}{1.197} = 51.796\ldots \quad \therefore 51.796 \times 7 = 362.57\ldots \approx 363$ Therefore there were approximately 363 visits in week 4
Specific behaviours
✓ calculates deseasonalised number ✓ calculates approximate weekly total

(f) Calculate the actual number of visits to the website on Monday of week 1. (1 mark)

Solution
42.4 × 0.896 = 38
Specific behaviours
✓ calculates correct value

The least-squares line for the deseasonalised data and time is $y = 0.47t + 41.70$.

(g) Predict the number of website visits on Tuesday of week 5. (3 marks)

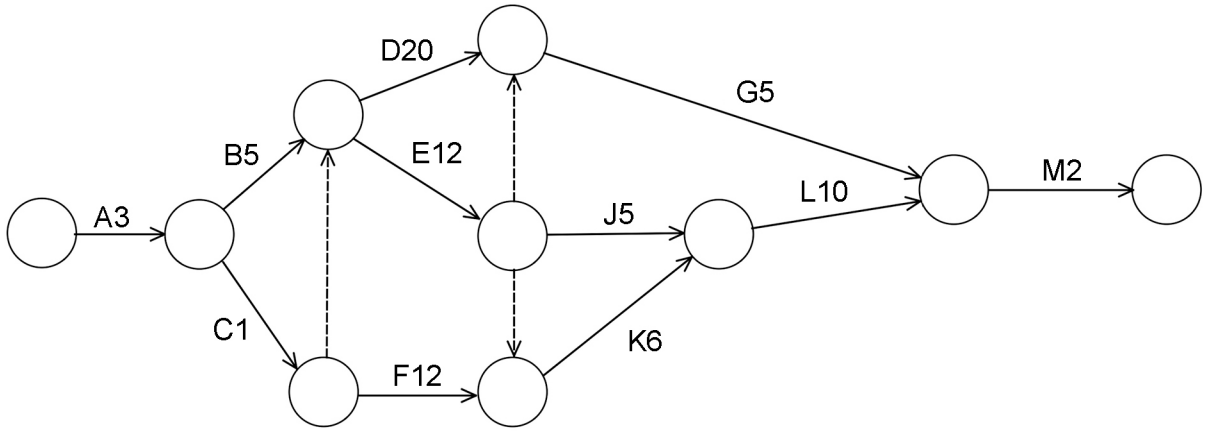
Solution
$t = 30 \quad y = 0.47(30) + 41.70 = 55.8$ $\therefore$ predicted value $= 55.8 \times 0.883 = 49.27 \approx 49$
Specific behaviours
✓ identifies correct value of t ✓ calculates correct trend value ✓ calculates correct predicted value

Question 9 **(10 marks)**

A builder is planning the renovation of a bathroom. The table below shows each task, the immediate predecessors and the duration of the task, for the project.

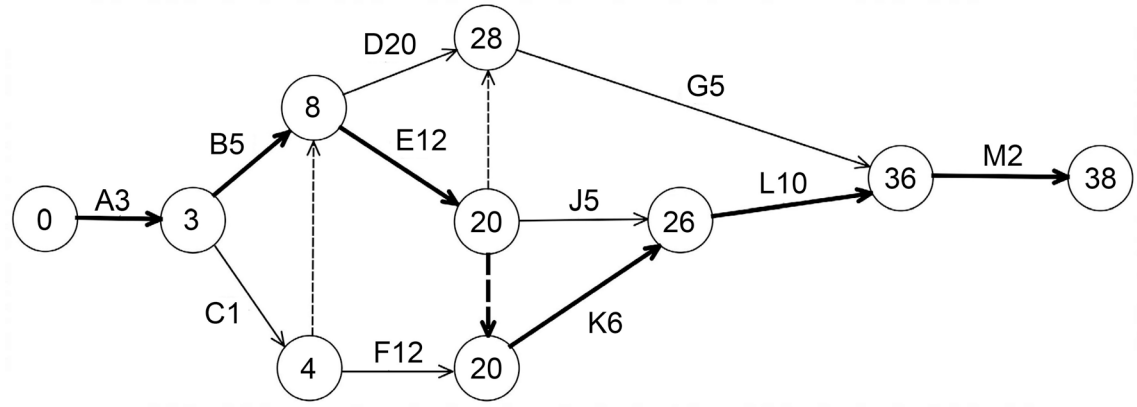
Task	Immediate predecessor(s)	Duration (days)
A	-	3
B	A	5
C	A	1
D	B, C	20
E	B, C	12
F	C	12
G	D, E	5
J	E	5
K	E, F	6
L	J, K	10
M	G, L	2

(a) Complete the project network diagram for the bathroom renovation. (3 marks)



Solution
See network above
Specific behaviours
✓ shows two correct dummy links ✓ completes the network ✓ shows all direction arrows, task names and durations

(b) Determine the critical path and the minimum completion time for the project. (3 marks)

Solution
 Critical path: ABEKLM Minimum completion time is 38 days
Specific behaviours
✓ shows systematic working on network ✓ determines critical path ✓ determines minimum completion time

(c) Determine the float time for each of the following tasks.

(i) C (1 mark)

Solution
4 days
Specific behaviours
✓ determines correct number of days

(ii) E (1 mark)

Solution
0 days
Specific behaviours
✓ determines correct number of days

It is possible that one or more tasks can be delayed without affecting the minimum completion time of the project.

(d) Identify which task(s) will give the maximum possible time delay without affecting the minimum completion time and state the time delay. (2 marks)

Solution
4 days, tasks C or F
Specific behaviours
✓ states correct time ✓ identifies correct tasks

Question 10 (13 marks)

Matt is saving up to purchase a new boat. He deposits \$14 500 into a savings account which is compounded monthly. The account pays an annual interest rate of 4.8% and he also deposits \$300 into the account at the end of each month.

(a) (i) Calculate the monthly interest rate. (1 mark)

Solution
4.8 ÷ 12 = 0.4%
Specific behaviours
✓ calculates correct rate

(ii) Determine a recursive rule to model the balance of the savings account at the end of each month. (2 marks)

Solution
$T_{n+1} = 1.004T_n + 300, \quad T_0 = 14\,500$
Specific behaviours
✓ states correct rule ✓ states correct initial value

(b) After how many months will the balance of Matt's account first exceed \$20 000? (2 marks)

Solution
$T_{14} = 19\,644.42$ $T_{15} = 20\,023.00$ Therefore after 15 months OR using finance app I = 4.8, PV = –14 500, PMT = –300, FV = 20 000, P/Y = C/Y = 12 N = 14.939… Therefore after 15 months
Specific behaviours
✓ correctly calculates 14th and 15th terms ✓ correctly concludes it is 15 months or ✓ correctly states the parameters ✓ correctly concludes that it is 15 months

After four years, Matt makes a one-off deposit of $2500 into the savings account. His goal is to have a total of $50 000 by the end of the fifth year.

(c) Determine the equal monthly deposits during the fifth year he will need to make to reach this amount. (5 marks)

Solution
$T_{48} = 33\,402.99$ 33 402.99 + 2500 = 35 902.99 N = 12, I = 4.8, PV = –35 902.99, FV = 50 000, P/Y = 12, C/Y = 12 PMT = –1005.52 Therefore, deposits of $1005.52 per month
Specific behaviours
✓ correctly calculates balance after 4 years ✓ adds 2500 to balance after 4 years ✓ states correct PV ✓ correctly states the remaining parameters ✓ determines correct monthly deposit

(d) Matt purchases his new boat, which costs him $47 500. He decides to take the remaining money and re-invest it in one of the following high-interest savings accounts.

Option 1: 5.52% per annum, compounded six-monthly.

Option 2: 5.5% per annum, compounded quarterly.

Determine which option Matt should choose, by calculating the effective annual rates of interest. (3 marks)

Solution
Option 1: $i_e = \left(1 + \frac{0.0552}{2}\right)^2 - 1 = 0.05596 \times 100 = 5.60$ Option 2: $i_e = \left(1 + \frac{0.055}{4}\right)^4 - 1 = 0.0561 \times 100 = 5.61$ Therefore option 2 is the better choice as it has a higher effective interest rate.
Specific behaviours
✓ correctly calculates effective interest rate for option 1 ✓ correctly calculates effective interest rate for option 2 ✓ correctly states option 2 is the better choice

Question 11 (19 marks)

University students from the faculty of Human Movement were tasked with determining the relationship between grip strength and age in adults who participate in regular exercise. They visited a health club and collected data from female volunteers. There is an expectation that grip strength will eventually decrease with age.

(a) After data was collected and analysed from 13 female volunteers, the students stated: "51% of the variation in grip strength cannot be explained by the variation in age". On the basis of this statement, determine the correlation coefficient between age and grip strength. (3 marks)

Solution
$r^2 = 1 - 0.51 = 0.49 \therefore r = \pm 0.7$ Reject r = 0.7 since there is an expectation that grip strength will eventually decrease Therefore r = –0.7
Specific behaviours
✓ correctly determines r^2 ✓ correctly determines r ✓ correctly accepts negative value

The students decided to omit one particular data point from all calculations.

(b) What is the most likely reason for removing this data point? (1 mark)

Solution
it is an outlier
Specific behaviours
✓ states correct reason

The table below shows the remaining 12 data points.

Age (a) (years)	34	61	20	41	49	26	63	38	56	55	46	35
Grip strength (s) (kg)	49.4	33.6	46.6	43.1	43.1	47.1	30.2	47.7	39.5	41.3	40.7	49.6

(c) Calculate r_{as} the correlation coefficient between age and grip strength. (1 mark)

Solution
r = –0.85
Specific behaviours
✓ calculates correct value

(d) Determine the equation of the least-squares line between age and grip strength. (1 mark)

Solution
$s = -0.38a + 59.15$
Specific behaviours
✓ determines correct equation

(e) Interpret the gradient of the least-squares line in the context of this question. (2 marks)

Solution
for every year increase in age, grip strength decreases by 0.38 kg
Specific behaviours
✓ refers to age and grip strength ✓ correctly states grip strength decrease by 0.38 kg

(f) Explain why the vertical intercept of the least-squares line is meaningless in the context of this question. (1 mark)

Solution
a newborn baby would not have a grip strength of 59.15 kg
Specific behaviours
✓ gives an appropriate answer
Accept other relevant answers.

(g) Use your least-squares line to predict the grip strength of a female aged 64 who participates in regular exercise. (1 mark)

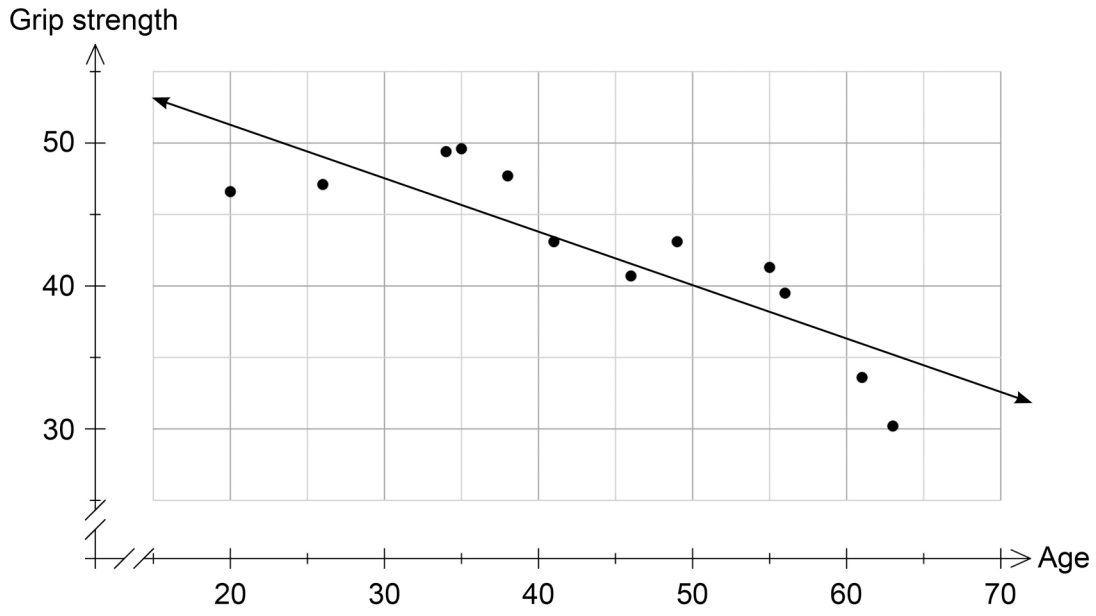
Solution
$s = -0.38(64) + 59.15 = 34.83$ kg
Specific behaviours
✓ calculates correct answer

(h) Discuss the validity of the prediction in part (g) above. (2 marks)

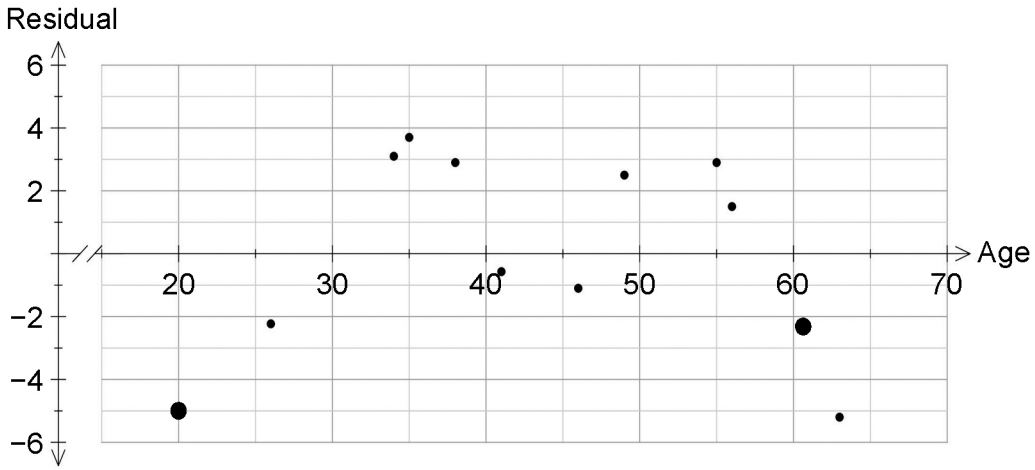
Solution
extrapolation, therefore not valid
Specific behaviours
✓ correctly states extrapolation ✓ correctly states it is not valid

Question 11 (continued)

(i) Draw the least-squares line for these data on the scatterplot below. (1 mark)

Solution

Specific behaviours
✓ correctly draws least-squares line

The incomplete residual plot for these data is drawn below.



(j) Determine the **two** missing residuals and plot them on the graph above. (3 marks)

Solution
(20, 46.6) residual = –5.0 (61, 33.6) residual = –2.5 See graph above for plot
Specific behaviours
✓ determines one correct residual ✓ determines second correct residual ✓ correctly plots residuals

(k) On the basis of your analysis of the data, make **three** reasonable statements about the connection between age and grip strength for females. (3 marks)

Solution
Answers could include: • grip strength declines from the mid-thirties • linear model not appropriate as there is a pattern in the residual plot • negative relationship • strong relationship.
Specific behaviours
✓ identifies one valid statement ✓ identifies two valid statements ✓ identifies three valid statements
Accept other relevant answers.

Question 12 **(9 marks)**

The sales manager of a large retail store plans to put the best salesperson as a sales specialist in each of the four departments within the store. Over a period of time and by rotating them through each of the four areas of the store, the sales ($'000) made by the five people being considered for these positions have been summarised in the table below.

	Computing	**Kitchen and laundry**	**Audio-visual**	**Furnishing**
Thomas	41	33	53	49
Megan	41	33	43	35
Lim	39	27	49	37
Le-Tong	27	33	37	43
Heidi	37	41	39	45

(a) Use the Hungarian algorithm to determine the allocation of a salesperson to each of the four departments within the store to maximise sales. (4 marks)

Solution

1

$$\begin{bmatrix} 41 & 33 & 53 & 49 & 0 \\ 41 & 33 & 43 & 35 & 0 \\ 39 & 27 & 49 & 37 & 0 \\ 27 & 33 & 37 & 43 & 0 \\ 37 & 41 & 39 & 45 & 0 \end{bmatrix}$$

2

$$\begin{bmatrix} 12 & 20 & 0 & 4 & 0 \\ 12 & 20 & 10 & 18 & 0 \\ 14 & 26 & 4 & 16 & 0 \\ 26 & 20 & 16 & 10 & 0 \\ 16 & 12 & 14 & 8 & 0 \end{bmatrix}$$

3

$$\begin{bmatrix} 0 & 8 & 0 & 0 & 0 \\ 0 & 8 & 10 & 14 & 0 \\ 2 & 14 & 4 & 12 & 0 \\ 14 & 8 & 16 & 6 & 0 \\ 4 & 0 & 14 & 4 & 0 \end{bmatrix}$$

4

$$\begin{bmatrix} 4 & 12 & 0 & \textcircled{0} & 4 \\ \textcircled{0} & 8 & 6 & 10 & 0 \\ 2 & 14 & \textcircled{0} & 8 & 0 \\ 14 & 8 & 12 & 2 & 0 \\ 4 & \textcircled{0} & 10 & 0 & 0 \end{bmatrix}$$

Specific behaviours

- ✓ correctly adds extra column of zeros
- ✓ subtracts each score from 53
- ✓ subtracts lowest score in each column from all scores in the column
- ✓ adjusts scores as only 4 lines needed to cover all zeros

(b) Show the allocation of departments to salespeople in the table below. (1 mark)

Solution

Salesperson	**Thomas**	**Megan**	**Lim**	**Le-Tong**	**Heidi**
Department	Furnishing	Computing	Audio Visual	-	Kitchen & laundry

Specific behaviours

- ✓ shows correct allocation

(c) Calculate the maximum sales figure determined in part (a). (1 mark)

Solution

$180 000

Specific behaviours

- ✓ calculates maximum sales figure

(d) After the allocations were made, Lim was dissatisfied and resigned from her job. Determine how this would affect the allocation and maximum sales figure. (3 marks)

Solution
New allocations after Lim resigns: • audio visual – Thomas • furnishings – Le-Tong • computing – Megan • kitchen and laundry – Heidi New maximum sales figure: $178 000
Specific behaviours
✓ correctly allocates Thomas to audio visual ✓ correctly allocates Le-Tong to furnishings ✓ states new sales figure

Question 13 (13 marks)

Ellie has retired from her job and plans to live off her acquired wealth. She has a superannuation balance of \$1 205 788 and a housing loan of \$205 749. Ellie sets up an annuity after paying off her housing loan. The annuity will have monthly investment returns and payments, and is modelled by the recursive rule

$$T_{n+1} = 1.005T_n - 6000, \quad T_0 = Z$$

(a) State how much Ellie receives as a monthly payment. (1 mark)

Solution
\$6000
Specific behaviours
✓ states correct monthly payment

(b) Determine the value of Z in the recursive rule. (1 mark)

Solution
Z = 1 205 788 – 205 749 = 1 000 039
Specific behaviours
✓ determines the correct value of Z

(c) Calculate what annual interest rate the annuity pays. (1 mark)

Solution
0.005 × 12 = 6%
Specific behaviours
✓ calculates the correct annual interest rate

The annuity has a start date of 1 March 2024. At the end of the month when the balance of the annuity first drops below \$700 000, Ellie plans to convert the annuity to a perpetuity, paid at the end of each month, so that she will leave behind some family inheritance.

(d) (i) Determine the year and month when the annuity will end. (3 marks)

Solution
Using the recursive rule: $T_{n+1} = 1.005T_n - 6000, \quad T_0 = 1\,000\,039$ n = 184 Therefore the annuity ends at the end of June 2039 or beginning of July 2039 Note: the value of n can also be calculated using the finance app with I = 6, PV = 1 000 039, PMT = –6000, FV = – 700 000, P/Y = C/Y = 12, gives N = 183.76
Specific behaviours
✓ correctly calculates the value of n ✓ states correct year ✓ states correct month

(ii) Determine the final balance at the end of the annuity. (1 mark)

Solution
$699 388.68
Specific behaviours
✓ determines correct value

Ellie receives advice that a perpetuity could now be arranged, receiving 5.95% per annum compounded daily.

(e) Determine the monthly perpetuity payment. (3 marks)

Solution
Uses finance app with I = 5.95, PV = –699 388.68, N = any positive value, FV = 699 388.68, P/Y = 12, C/Y = 365, gives PMT = 3476.13 i.e. $3476.13
Specific behaviours
✓ states correct values for parameters PV and FV where PV = -FV ✓ states correct values for parameters P/Y and C/Y ✓ determines correct monthly payment

(f) Ellie is curious as to whether the annuity or the perpetuity pays the better annual rate of interest. Determine which pays the higher interest rate. (3 marks)

Solution
Annuity: Uses effective annual rate of interest formula $i_{effective} = \left(1 + \frac{0.06}{12}\right)^{12} - 1 = 6.17\%$ Perpetuity: Uses effective annual rate of interest formula $i_{effective} = \left(1 + \frac{0.0595}{365}\right)^{365} - 1 = 6.13\%$ Annuity pays the higher rate of interest
Specific behaviours
✓ correctly calculates effective annuity rate ✓ correctly calculates effective perpetuity rate ✓ compares effective rates to conclude annuity is the higher rate

Published by the School Curriculum and Standards Authority of Western Australia
303 Sevenoaks Street
CANNINGTON WA 6107