2025/4694
Web version of 2024/74642

Government of **Western Australia**
School Curriculum and Standards Authority

ATAR course examination, 2024

Question/Answer booklet

MATHEMATICS METHODS

Section Two: Calculator-assumed

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

WA student number: In figures

In words

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	10	10	100	100	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2024: Part II Examinations*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (100 Marks)**

This section has **ten** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

See next page

Question 8 **(11 marks)**

John is given a prescription for blood pressure tablets by his doctor. Each tablet contains five milligrams of the active medication. The mass of active medication, A, remaining in John's body t hours after taking a single tablet is given by

$$A(t) = 5e^{-0.0173t}$$

where A is in milligrams.

(a) Calculate the mass of medication remaining in John's body 10 hours after taking a single tablet. (1 mark)

(b) After how many hours will the mass of medication remaining in John's body have halved? (2 marks)

(c) Determine at what rate the mass of medication remaining in John's body is decreasing 24 hours after taking a single tablet. (3 marks)

See next page

If a tablet is taken every T hours, the mass of medication, B, remaining in John's body immediately after taking a tablet will be given by

$$B(T) = \frac{5}{1 - e^{-0.0173T}}$$

where B is in milligrams.

(d) How frequently should John take a tablet so that the mass of medication remaining in his body immediately after taking each tablet is 8.85 mg? (2 marks)

(e) Use the increments formula to approximate the change in B if the time between taking tablets increased by 30 minutes from the time determined in part (d). (3 marks)

See next page

Question 9 **(8 marks)**

A mathematics teacher uses a coin flip activity to demonstrate confidence intervals to their class. They flip a fair coin 50 times in front of the class and observe 30 heads and 20 tails.

(a) Calculate a 90% confidence interval for the proportion of heads obtained when the coin is flipped. (2 marks)

As a homework exercise, the teacher asks all 20 students in the class to repeat the coin activity and calculate their own individual 90% confidence interval for the proportion of heads. Let X be a random variable that denotes the number of students whose confidence interval contains the true proportion of heads.

(b) State the distribution for X. (2 marks)

(c) Determine the expected value and variance of X. (2 marks)

(d) Calculate the probability that the confidence intervals of three students do not contain the true proportion. (2 marks)

Question 10 **(13 marks)**

A book called 'Why I Love Mathematics' is having its first print run. This is scheduled to last for one week, using four printing presses. The publisher claims that, historically, 10 books have printing errors for every two hundred that are printed.

A sample of 200 books is to be chosen to determine how many contained errors. The proposed sampling procedure is to select the first 200 books printed over a 6-hour window using the newest printing press.

(a) (i) Identify and explain **one** possible source of bias in the proposed sampling procedure. (2 marks)

(ii) Identify **two** changes to the sampling procedure that would reduce bias. (2 marks)

Assume that the sample was gathered using an improved procedure, and that the publisher's claim is correct.

(b) Use the approximate normality of the distribution of sample proportions to determine the probability that the sample proportion of books with errors is less than 0.04. (2 marks)

See next page

In a different random sample of 600 books, it is found that the proportion of books containing an error is 0.1, with a margin of error of 0.024.

(c) Determine a 95% confidence interval for the proportion of books that will have printing errors. (1 mark)

(d) On the basis of the confidence interval determined in part (c), is the proportion of books with printing errors different from what was claimed by the publisher? (2 marks)

(e) Suggest **two** changes that could be made in order to decrease the margin of error of the confidence interval. (2 marks)

(f) Determine the minimum sample size that would be necessary to guarantee that the margin of error of the resulting 95% confidence interval was at most 0.02. (2 marks)

See next page

Question 11 **(10 marks)**

A protest group looking to abolish the circulation of physical money (notes and coins) claims that 74% of Perth residents have not used physical money in the past 12 months. To investigate this claim, a data scientist surveys a random sample of Perth residents, with 70% claiming they had not used physical money in the past 12 months.

Let p denote the proportion of Perth residents who have not used physical money in the past 12 months.

(a) Given that the width of the 95% confidence interval for p is 0.096

(i) determine the 95% confidence interval for p. (2 marks)

(ii) determine the number of people surveyed. (2 marks)

(b) What does the data scientist's confidence interval suggest about the protest group's claim? (2 marks)

The data scientist is interested in whether the proportion is similar to that in other Australian capital cities, and conducts surveys in Brisbane, Sydney and Hobart. The following information was recorded based on the survey results:

City	Sample size	Sample proportion
Brisbane	N	0.65
Sydney	$2N$	0.65
Hobart	N	0.75

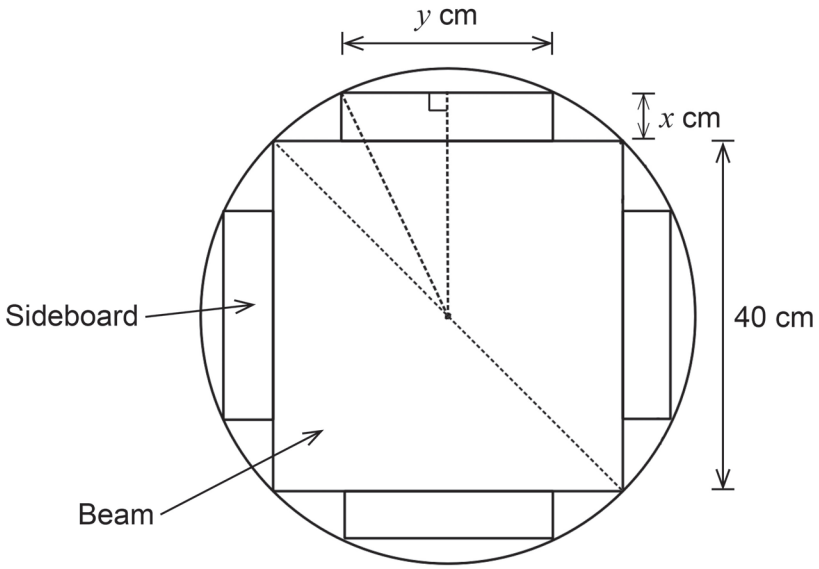
(c) For each of the following city pairs, identify which had the widest 95% confidence interval. Justify your answer.

(i) Brisbane and Sydney (2 marks)

(ii) Brisbane and Hobart (2 marks)

Question 12 **(9 marks)**

A log with a circular cross-section is being prepared for milling. The log is to be cut into five usable pieces, one large beam with a square cross-section and four sideboards, as shown in the diagram below.



(a) Determine the exact radius of the log. (2 marks)

(b) Using the variables defined in the diagram, show that the cross-sectional area, in cm^2, of a single sideboard is $A(x) = 2x\sqrt{400 - 40x - x^2}$. (3 marks)

See next page

(c) Use calculus techniques to determine the dimensions x and y that maximise the cross-sectional area of one sideboard. (4 marks)

See next page

Question 13 **(12 marks)**

Brianna is considering buying an electric vehicle from Zaprer Motors. The manufacturer claims that, on average, a driver will be able to travel 350 km before needing to recharge the vehicle, and that the probability of travelling more than 400 km before needing to recharge is 0.2525.

Let X denote the distance, in kilometres, that a Zaprer Motors vehicle will travel before needing to recharge. Assume that X is a normally distributed random variable.

(a) Determine the standard deviation of X. (2 marks)

Brianna will need to travel regularly to Albany, which is 420 km from her house.

(b) Calculate the probability that on any given day she will be able to drive to Albany without recharging the vehicle. (1 mark)

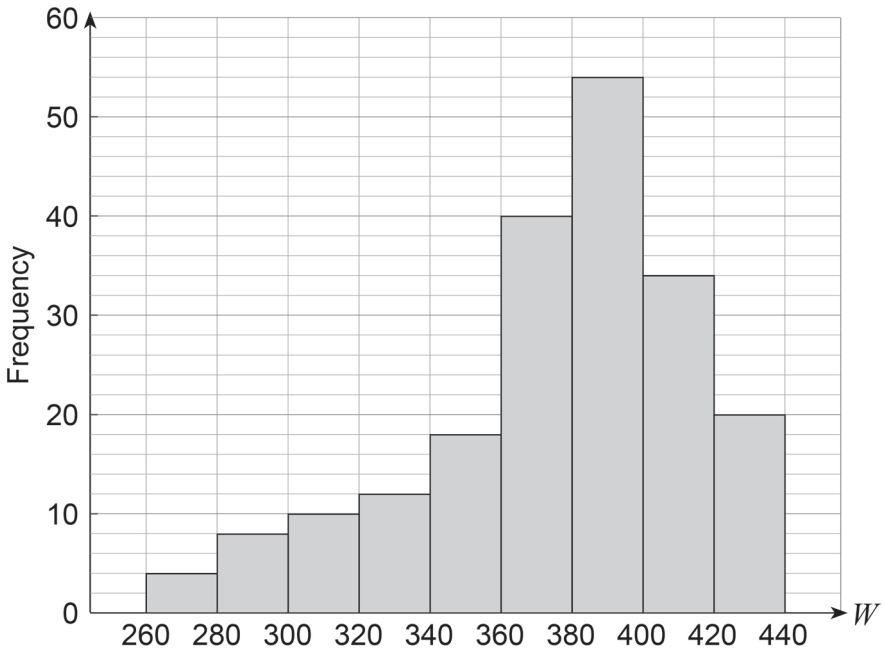
Matthew is interested in buying the same type of electric vehicle, but as he lives in England he would like to consider distances in miles (1 mile = 1.6 kilometres).

Let Y be a random variable that denotes the distance, in miles, that a Zaprer Motors vehicle will travel before needing to recharge.

(c) Determine the expected value and variance of Y. (3 marks)

See next page

Brianna decides to consider an electric vehicle from a rival company, Spruky Cars, that show her the histogram below, based on 200 trials of its electric vehicle.



Let W be a random variable that denotes the distance, in kilometres, that a Spruky Cars vehicle will travel before needing to recharge.

(d) On the basis of the histogram, is it appropriate to use a normal distribution to model the distance a Spruky Cars vehicle will travel between recharges? Justify your answer.

(2 marks)

(e) Assuming the distances are uniformly distributed within each interval, use the histogram to estimate the expected distance that a Spruky Cars vehicle will be able to travel before needing to recharge. (2 marks)

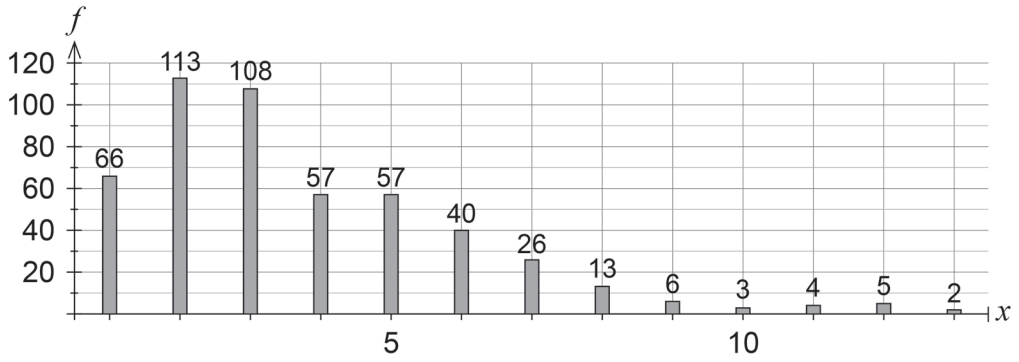
Question 13 (continued)

(f) In which company's vehicle (Zaprer or Spruky) would Brianna be more likely to drive to Albany without recharging? Justify your answer. (2 marks)

Question 14 **(14 marks)**

A mathematics teacher, Mr Ulam, devises a new game that uses four standard dice. A player rolls all the dice. Any dice with a 1 facing up are winning dice and are removed. The remaining dice are rolled again. The game is finished when a player has two or more winning dice.

Mr Ulam plays the game 500 times and records how many rolls it takes to win. Let X be a random variable denoting the number of rolls needed to win a game. The frequency results, f, are displayed in the graph below.



(a) Using the experimental data above, estimate the probability of

(i) winning in exactly two rolls. (1 mark)

(ii) not winning in two or less rolls. (2 marks)

See next page

Question 14 (continued)

(b) State **two** reasons why the game cannot be modelled using a binomial distribution. (2 marks)

Mr Ulam uses a computer to simulate 10 000 games to better estimate the probability of each outcome. The results of the simulations are summarised in the table below.

x	1	2	3	4	5	6	7	8
$\mathrm{P}(X=x)$	0.134	0.215	0.208	0.153	0.106	0.067	0.047	0.030

x	9	10	11	12	13	14	15	16
$\mathrm{P}(X=x)$	0.016	0.012	0.005	0.003	0.002	0.001	0.001	0.000

To fundraise for a local charity, Mr Ulam charges \$1 for each game, and pays out winnings as follows:

- if the game is won in one or two rolls the player wins \$2
- if the game is won in three rolls the player wins \$1 (i.e. their money back)
- if the game is won in four or more rolls the player wins nothing.

Let Y be a random variable denoting the **profit** for a player who plays a \$1 game.

(c) Using the data above, complete the probability distribution table for Y. (3 marks)

y			
$\mathrm{P}(Y=y)$			

(d) Calculate the

(i) expected value of Y. (2 marks)

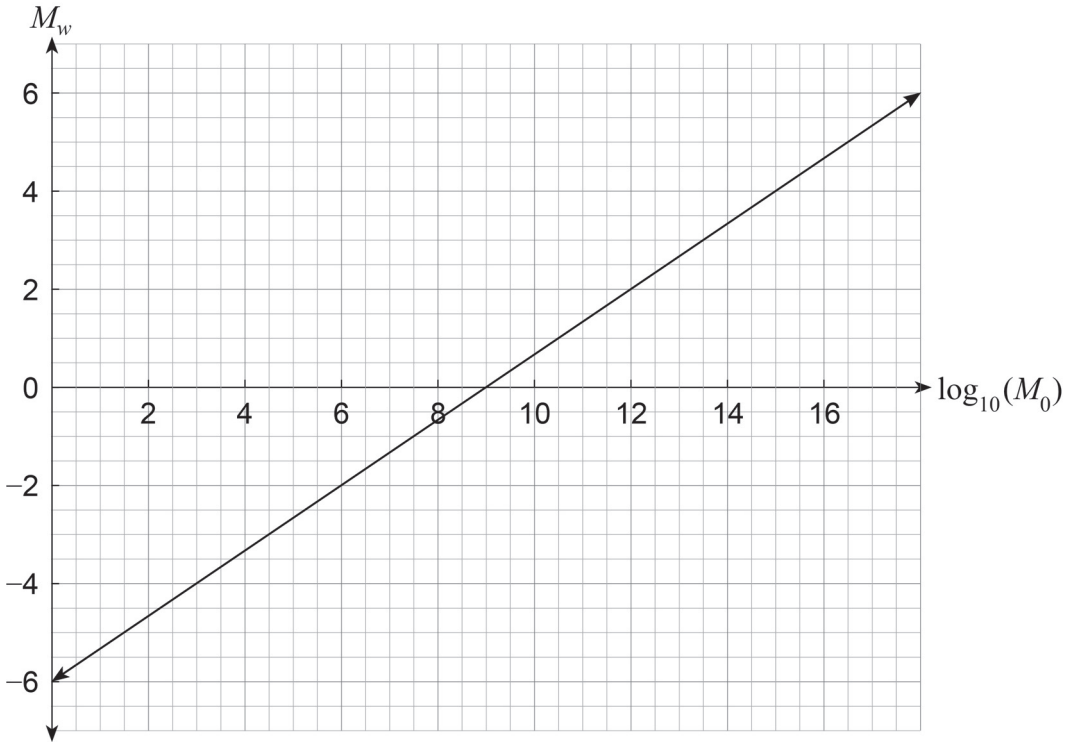
(ii) variance of Y. (2 marks)

(e) In the long run, do you expect that the game will be profitable for the charity? Justify your answer. (2 marks)

See next page

Question 15 **(9 marks)**

Earthquake magnitude can be measured using the moment magnitude scale (M_w), which is related to the seismic moment M_0 of an earthquake. The seismic moment has units of Nm. The graph of M_w versus $\log_{10}(M_0)$ is given below.



(a) Use the graph to approximate the moment magnitude M_w of an earthquake with a seismic moment of 3.16×10^{13} Nm. You must show clearly how you have used the graph. (2 marks)

A spare grid is provided at the end of this Question/Answer booklet. If you need to use it, cross out this attempt and indicate that you have redrawn it on the spare grid.

(b) The relationship between M_w and M_0 can be expressed in the form

$$M_w = a\log_{10}(M_0) + b.$$

Determine the values of a and b. (2 marks)

See next page

(c) Hence, or otherwise, express the relationship between M_w and M_0 in the form

$$M_w = a \log_{10}\left(\frac{M_0}{c}\right).$$ (3 marks)

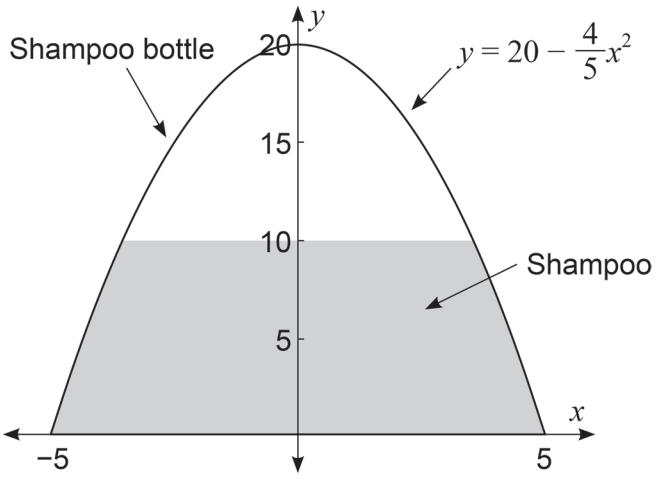
(d) Determine the seismic moment, M_0, of an earthquake with moment magnitude $M_w = 4$.

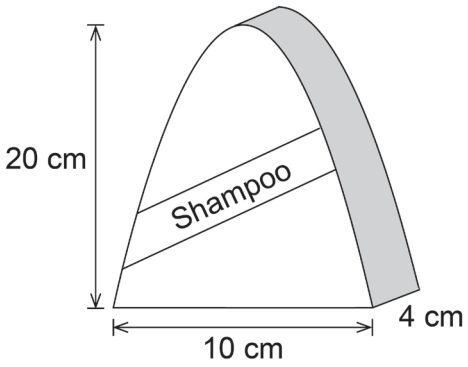
(2 marks)

See next page

Question 16 **(8 marks)**

A partially filled shampoo bottle has a uniform parabolic cross-section as shown in the diagram below.

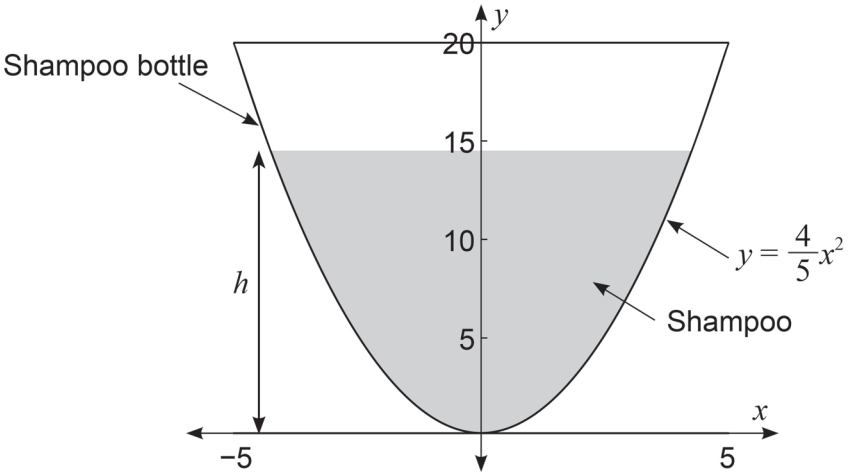


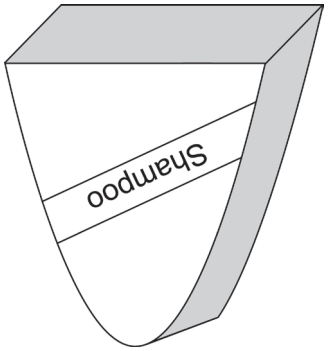


Both x and y are measured in centimetres. The width of the bottle perpendicular to the parabolic cross-section is 4 cm.

(a) Calculate the volume V of shampoo in the bottle, if it is partially filled to a height of 10 cm. (4 marks)

The bottle is turned upside down and the entire volume of the shampoo allowed to settle, as shown in the diagram below.





(b) Determine the shampoo level h. (4 marks)

Question 17 **(6 marks)**

Speed cubers are people who try to solve the Rubik's cube as quickly as possible by using pre-learned algorithms (sequences of turns). Knowing more algorithms has the potential to result in faster solution times. A speed cuber aims to learn the full ZBLL algorithm set, consisting of 493 algorithms.

Assume that the total number of ZBLL algorithms learnt, A, can be modelled by the equation

$$A(t) = b\log_4(t+1) + c$$

where t is the time in weeks since learning commenced, and b and c are constant parameters.

(a) Determine the parameters b and c, given that the speed cuber already knows 21 of the ZBLL algorithms (at $t = 0$) and learnt an additional 32 algorithms by the end of the first week. (3 marks)

(b) Determine how many of the ZBLL algorithms the speed cuber will have learnt after 26 weeks. (1 mark)

(c) Based on the assumed model, will the speed cuber learn the entire ZBLL algorithm set within their lifetime? Justify your answer. (2 marks)

End of questions

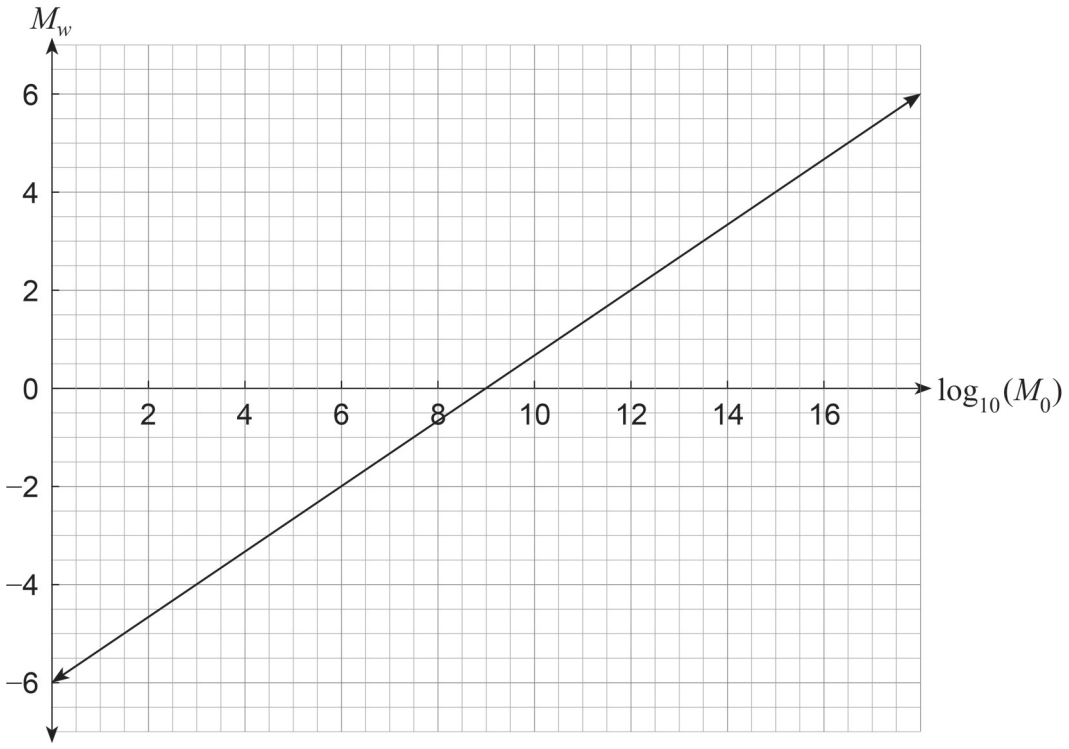
Supplementary page

Question number: _____

Supplementary page

Question number: _____

Spare grid for Question 15(a)



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