Government of **Western Australia**
School Curriculum and Standards Authority

ATAR course examination, 2024

Question/Answer booklet

MATHEMATICS SPECIALIST

Section Two: Calculator-assumed

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

WA student number: In figures

In words

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	47	35
Section Two: Calculator-assumed	10	10	100	85	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2024: Part II Examinations*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (85 Marks)**

This section has **10** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 9 **(6 marks)**

Consider the complex equation $z^n = 1$ where n is a positive integer.

(a) Show that $z = \dfrac{1+i}{\sqrt{2}}$ will be a solution of $z^{24} = 1$. (2 marks)

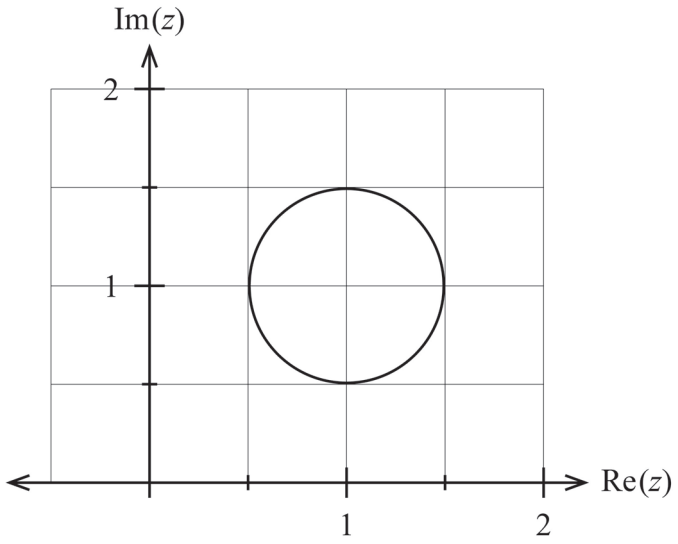
(b) Determine all the values of n so that $z = \dfrac{1+i}{\sqrt{2}}$ is a solution of $z^n = 1$. (2 marks)

Consider the smallest value of n from part (b).

(c) Explain how you could locate all the solutions to the equation $z^n = 1$ in the Argand plane for this smallest value of n. (2 marks)

Question 10 **(8 marks)**

(a) A circle is drawn in the Argand plane. Let z be any point on this circle.



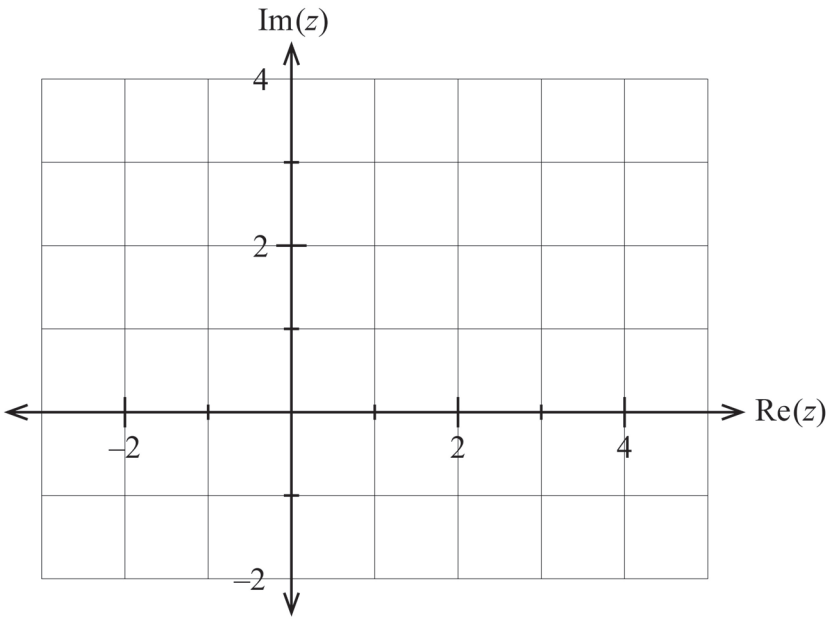
(i) State the equation for this circle. (2 marks)

(ii) Given that $a \leq |z - i| \leq b$, determine the exact values for a and b. (2 marks)

See next page

(b) Sketch, on the Argand diagram below, the locus given by the intersection of:

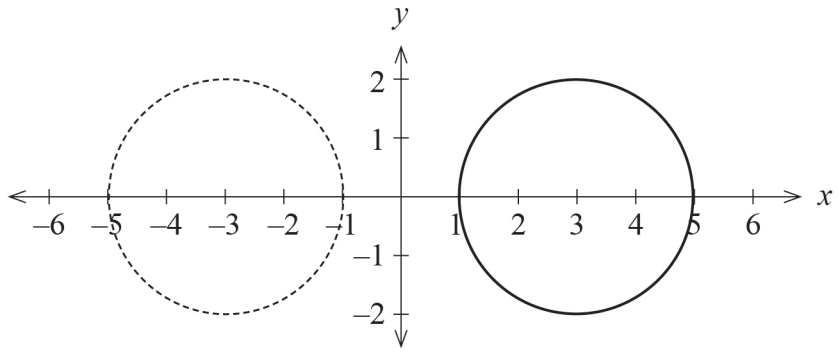
$z - \bar{z} \leq 6i$ and $\frac{\pi}{4} \leq Arg\,(z - 1 + i) \leq \frac{3\pi}{4}$. (4 marks)



A spare diagram is provided at the end of this Question/Answer booklet. If you need to use it, cross out this attempt and indicate that you have redrawn it on the spare diagram.

Question 11 **(6 marks)**

Scrummy Donuts are modelled by the circular region bounded by the circle $(x-3)^2+y^2=4$ being rotated about the y axis to produce a donut-shaped object called a torus. All dimensions are in centimetres.



(a) Show that the volume of this donut is given by $\int_a^b 12\pi\sqrt{4-y^2}\,dy$.

State the values for a and b. (4 marks)

Nutritional information about Scrummy Donuts*: Average density = 0.28 g/cm^3
Health recommendation: No more than 180 g to be eaten per day. (*eat responsibly)

(b) Calculate the maximum number of Scrummy Donuts that can be eaten each day, if the health recommendation is to be followed. (2 marks)

See next page

Question 12 **(5 marks)**

The vertical displacement, $y(t)$ centimetres, of the seat of a toy horse from the mean position after t seconds is measured as the toy horse moves up and down a pole during a ride on a merry-go-round.

The vertical motion of the toy horse obeys the equation $\frac{d^2y}{dt^2} = -\left(\frac{\pi}{2}\right)^2 y$.

The vertical motion of each toy horse spans 50 centimetres, and the seat of the toy horse is at the mean position at the start of the ride.

(a) Determine the vertical displacement function $y(t)$. (3 marks)

(b) Determine the maximum vertical speed of the toy horse, correct to the nearest centimetre per second. (2 marks)

See next page

Question 13 **(8 marks)**

A brumby is a free-roaming wild horse found in large numbers in parts of Australia. The culling of brumbies was banned in the year 2000. At this time the estimated population of brumbies in Kosciuszko National Park was 1600.

Scientists have modelled the population, $P(t)$, of brumbies in Kosciuszko National Park t years since the ban, by

$$P(t) = \frac{18\,000}{10.25e^{-0.15t} + 1}.$$

(a) Use the model to determine how long it will take the brumbies to increase to a number that is triple the number when the ban came into effect. (1 mark)

(b) From this model, determine the estimated long run number of brumbies in Kosciuszko National Park. (2 marks)

See next page

It can be shown that the growth rate of the population of brumbies can be expressed as

$$\frac{dP}{dt} = \frac{1}{r}P(k-P).$$

(c) Determine the values of the constants r and k. (3 marks)

(d) Determine the greatest growth rate for the population of brumbies. (2 marks)

Question 14 **(15 marks)**

Patients are anaesthetised before surgery. The time, in minutes, it takes for a patient to return to consciousness after surgery, called the 'recovery time', is an important measure of the procedure.

Let μ be the population mean recovery time. From historical data, the population standard deviation is $\sigma = 25$ minutes.

A random sample of 100 recovery times is taken of patients who have undergone a tonsillectomy. Let $\overline{X}$ denote the sample mean recovery time.

(a) State the approximate distribution for $\overline{X}$. (3 marks)

(b) Determine the probability that the sample mean is greater than the population mean by more than 3 minutes. (2 marks)

(c) If it is required that the sample mean is to have a 50% chance of being within m minutes of the population mean, determine the value of m correct to 0.1 minutes. (2 marks)

A particular sample of size 100 produces $\overline{x} = 15$ minutes.

(d) Calculate a 99% confidence interval, I_1, for the population mean recovery time. (2 marks)

See next page

A new procedure for tonsillectomy using an assisting robot was trialled. A random sample of 400 recovery times was taken for patients who underwent the new procedure. The observed sample mean was 14.5 minutes.

The 99% confidence interval, I_2 , for the population mean recovery time for the new procedure, was found to be $11.28 < \mu\,(new) < 17.72$ minutes.

Two junior doctors made the following statements.

Anja: "The new procedure is superior as its sample mean of 14.5 minutes from 400 patients is lower than the sample mean of 15 minutes from 100 patients and by using a larger sample size we can be more confident."

Sanjeet: "Since the interval I_2 lies completely within interval I_1 , then it can be inferred that the population recovery time for the new procedure is the same as that for the old procedure."

(e) (i) State whether Anja's statement is true or false. Justify your answer. (2 marks)

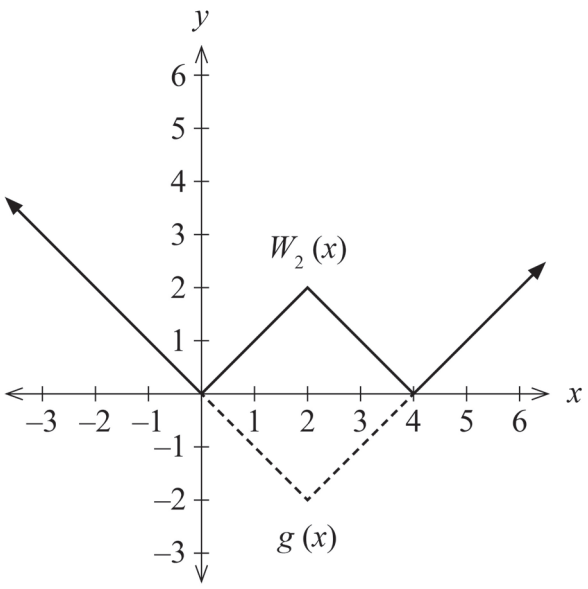
(ii) State whether Sanjeet's statement is true or false. Justify your answer. (2 marks)

(f) Calculate the minimum sample size required to estimate the mean recovery time for the new procedure with an interval width of at most 4 minutes using a 95% confidence level. (2 marks)

Question 15 **(7 marks)**

The graph of $W_k(x) = \left| \frac{k}{2} \left| x - k \right| - k \right|$ is called a W-graph where $k > 0$.

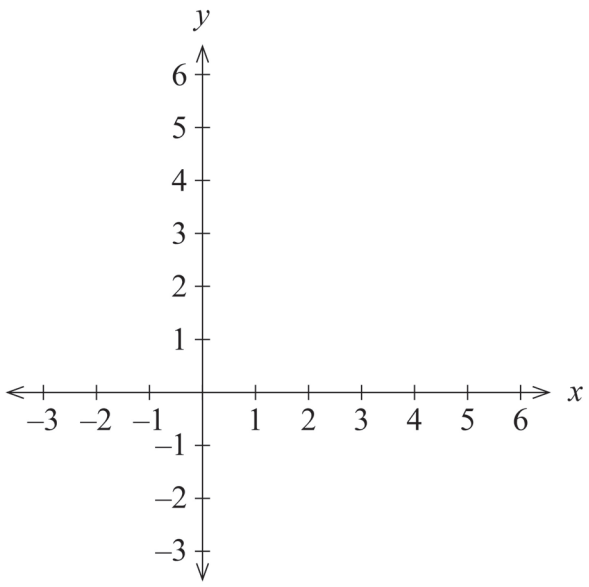
The graphs of $y = W_2(x)$ and $g(x) = \left| x - 2 \right| - 2$ are shown below for $k = 2$.



(a) On the axes below, sketch $y = W_3(x)$ i.e. the W-graph for $k = 3$.

i.e. $W_3(x) = \left| \frac{3}{2} \left| x - 3 \right| - 3 \right|$.

(2 marks)



A spare grid is provided at the end of this Question/Answer booklet. If you need to use it, cross out this attempt and indicate that you have redrawn it on the spare grid.

See next page

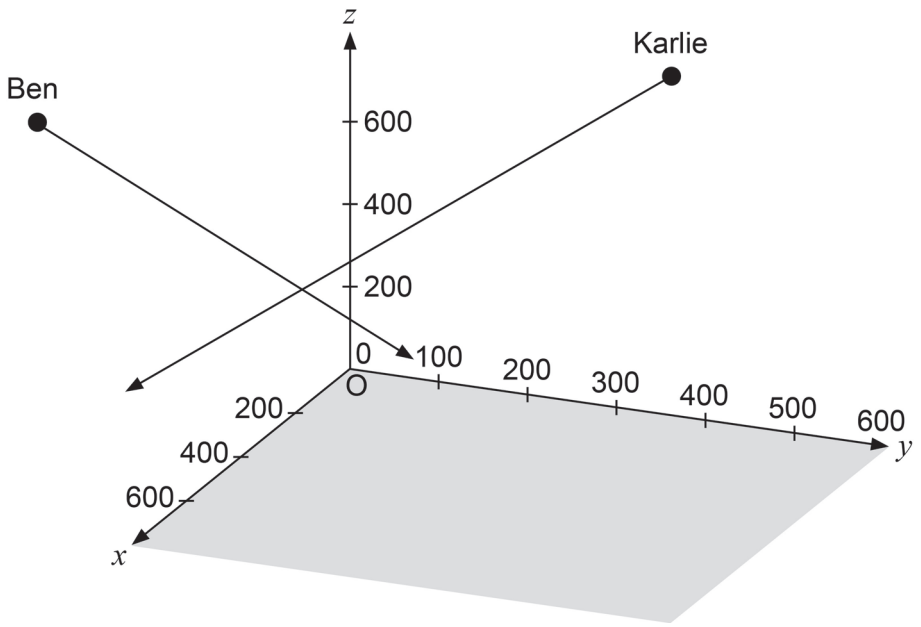
(b) Determine how many solutions the equation $W_k(x) = k$ will have. Justify your answer. (2 marks)

(c) By considering the general W-graph, develop an expression for $\int_{k-2}^{k+2} W_k(x)\,dx$ in terms of the constant k. (3 marks)

Question 16 **(9 marks)**

Two thrill-seekers, Karlie and Ben, are each attached to straight wires that allow them to slide down within a wide canyon.

A co-ordinate system is defined showing the positive co-ordinate axes with O being the origin. At exactly 10.30 am, Karlie is at a position of $-200\underset{\sim}{i} + 300\underset{\sim}{j} + 700\underset{\sim}{k}$ metres and is sliding down her wire with velocity $2\underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}$ metres per second. Meanwhile, Ben is at position $500\underset{\sim}{i} - 200\underset{\sim}{j} + 800\underset{\sim}{k}$ and is sliding down his wire at a velocity of $-0.5\underset{\sim}{i} + \underset{\sim}{j} - 1.5\underset{\sim}{k}$ metres per second.



(a) Determine Ben's speed, correct to the nearest 0.01 metres per second. (2 marks)

(b) Calculate the angle of Ben's descent to the horizontal, correct to the nearest degree.
(3 marks)

It was found that the closest that Karlie and Ben approached one another was approximately 57.74 metres after 266.67 seconds of motion (from 10.30 am).

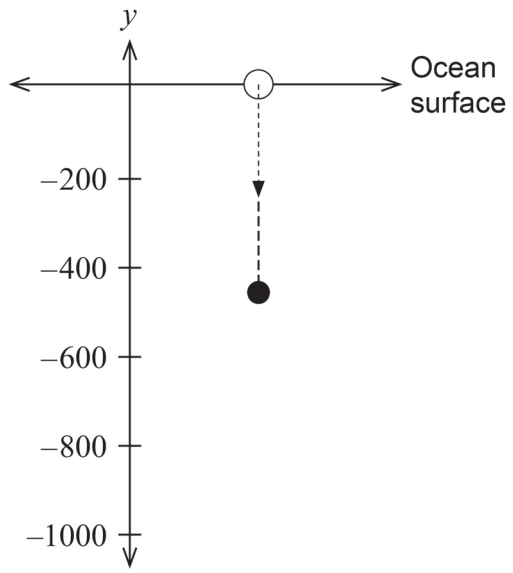
Suppose that Ben is able to adjust the speed of his descent and the time at which he commences sliding down his wire.

(c) Calculate the minimum distance (correct to the nearest 0.01 metres) that Ben could be separated from Karlie, if he was able to adjust the speed and timing of his motion. Show all evidence of your working. (4 marks)

See next page

Question 17 **(9 marks)**

A pressure sensitive device measures depth as it sinks toward the seabed. The device is released from rest at the ocean surface, and as it sinks downward, the water exerts a resistance force to oppose its motion.



Let $t =$ the time (in seconds) elapsed from release.

$y(t) =$ the displacement of the device relative to the surface (metres).

$v(t) =$ the velocity of the device (metres per second).

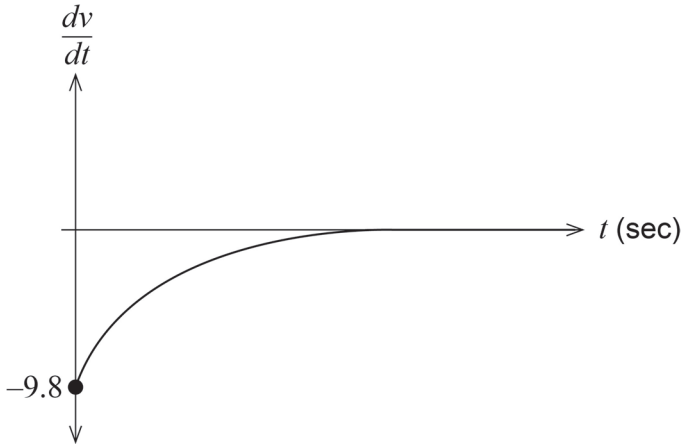
$a(t) =$ the acceleration of the device (metres/second^2).

The diagram shows that after 95 seconds, the device is 463.05 metres below the surface i.e. $y(95) = -463.05$.

The acceleration of the device, at any point in time, is given by $\frac{dv}{dt} = -9.8 - 2v$.

(a) Calculate the acceleration of the device, when the device is falling at a rate of 3 metres per second. (2 marks)

The graph of the acceleration $\frac{dv}{dt}$ is shown below.



See next page

(b) Explain **two** features of the graph of the acceleration $a(t)$ on page 16, referring to the differential equation $\frac{dv}{dt} = -9.8 - 2v$ or to the resistance force. (2 marks)

(c) Show, using the separation of variables technique, that $v(t) = 4.9(e^{-2t} - 1)$. (3 marks)

Question 17 (continued)

At a particular location, the device is released from rest at the surface of the ocean and falls until it strikes the seabed.

(d) If the device takes exactly 2 minutes 30 seconds to hit the seabed, calculate the depth of the seabed at this location, correct to the nearest metre. (2 marks)

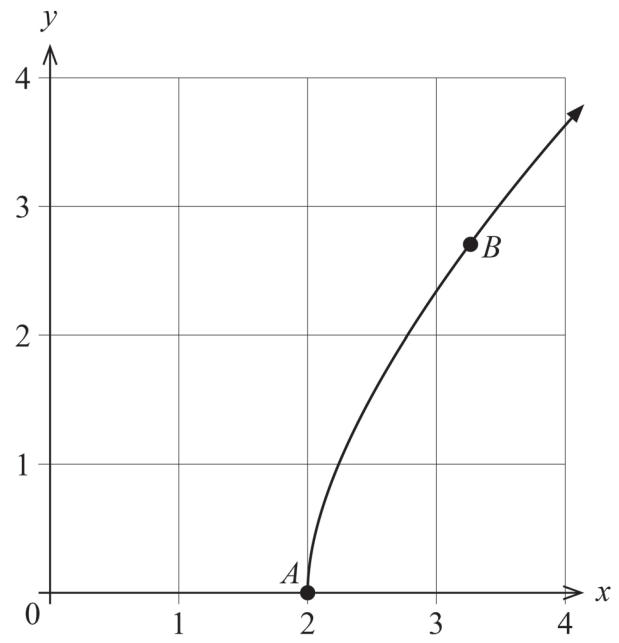
See next page

This page has been left blank intentionally

See next page

Question 18 **(12 marks)**

A drone's position vector is given by $\underset{\sim}{r}(t) = \begin{pmatrix} e^t + e^{-t} \\ e^t - e^{-t} \end{pmatrix}$ metres where t is measured in seconds for $0 \leq t \leq 5$. A plot of the path of the drone is shown below.



The drone starts its motion at point A and is at point B when $t = \ln 3$.

(a) Determine the position vector for point B exactly. (2 marks)

(b) Determine the velocity vector $\underset{\sim}{v}(t)$. (2 marks)

See next page

(c) Calculate the distance travelled from point A to point B, correct to 0.001 metres. (3 marks)

(d) After 2 seconds of motion, calculate correct to 0.1 degrees, the direction in which the drone is travelling. (2 marks)

See next page

Question 18 (continued)

(e) Determine the Cartesian equation for the path of the drone. (3 marks)

End of questions

Supplementary page

Question number: _____

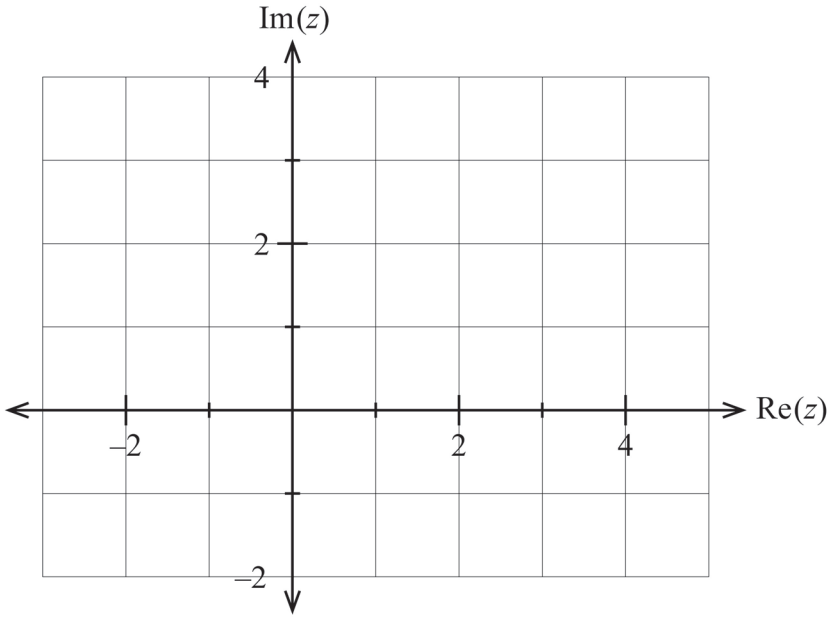
Supplementary page

Question number: _____

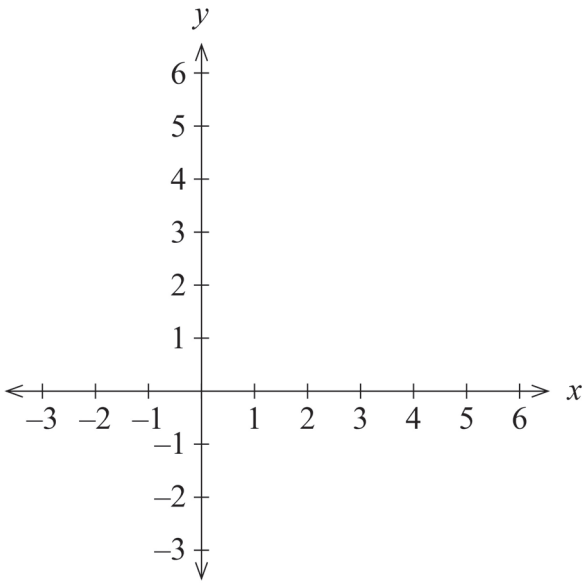
Supplementary page

Question number: _____

Spare diagram for Question 10(b)



Spare grid for Question 15(a)



ACKNOWLEDGEMENTS

Question 13 Hermaion. (2018). *Horse Head Grayscale Photo*. Retrieved April, 2024, from https://www.pexels.com/photo/horse-head-grayscale-photo-1058357/

Published by the School Curriculum and Standards Authority of Western Australia
303 Sevenoaks Street
CANNINGTON WA 6107