Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS METHODS

ATAR course

Year 11 syllabus for teaching from 2026

Acknowledgement of Country

Kaya. The School Curriculum and Standards Authority (the Authority) acknowledges that our offices are on Whadjuk Noongar boodjar and that we deliver our services on the country of many traditional custodians and language groups throughout Western Australia. The Authority acknowledges the traditional custodians throughout Western Australia and their continuing connection to land, waters and community. We offer our respect to Elders past and present.

Important information

As part of the Western Australian Certificate of Education (WACE) Refreshment, the School Curriculum and Standards Authority (the Authority) has revised the course rationale and aims, and updated the General Capabilities to create clearer connections with the syllabus content.

This syllabus is effective from 1 January 2026.

Users of this syllabus are responsible for checking its currency.

Syllabuses are formally reviewed by the Authority on a cyclical basis, typically every five years.

Contents

Overview of mathematics courses

There are six mathematics courses. Each course is organised into four units. Unit 1 and Unit 2 are taken in Year 11 and Unit 3 and Unit 4 in Year 12. The ATAR course examination for each of the three ATAR courses is based on Unit 3 and Unit 4 only.

The courses are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students.

Mathematics Preliminary is a course which focuses on the practical application of knowledge, skills and understandings to a range of environments that will be accessed by students with special education needs. Grades are not assigned for these units. Student achievement is recorded as 'completed' or 'not completed'. This course provides the opportunity for students to prepare for post-school options of employment and further training.

Mathematics Foundation is a course which focuses on building the capacity, confidence and disposition to use mathematics to meet the numeracy standard for the Western Australian Certificate of Education (WACE). It provides students with the knowledge, skills and understanding to solve problems across a range of contexts, including personal, community and workplace/employment. This course provides the opportunity for students to prepare for post-school options of employment and further training.

Mathematics Essential is a General course which focuses on using mathematics effectively, efficiently and critically to make informed decisions. It provides students with the mathematical knowledge, skills and understanding to solve problems in real contexts for a range of workplace, personal, further learning and community settings. This course provides the opportunity for students to prepare for post-school options of employment and further training.

Mathematics Applications is an ATAR course which focuses on the use of mathematics to solve problems in contexts that involve financial modelling, geometric and trigonometric analysis, graphical and network analysis, and growth and decay in sequences. It also provides opportunities for students to develop systematic strategies based on the statistical investigation process for answering questions that involve analysing univariate and bivariate data, including time series data.

Mathematics Methods is an ATAR course which focuses on the use of calculus and statistical analysis. The study of calculus provides a basis for understanding rates of change in the physical world, and includes the use of functions, their derivatives and integrals, in modelling physical processes. The study of statistics develops students' ability to describe and analyse phenomena that involve uncertainty and variation.

Mathematics Specialist is an ATAR course which provides opportunities, beyond those presented in the Mathematics Methods ATAR course, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. The Mathematics Specialist ATAR course contains topics in functions and calculus that build on and deepen the ideas presented in the Mathematics Methods ATAR course, as well as demonstrate their application in many areas. This course also extends understanding and knowledge of statistics and introduces the topics of vectors, complex numbers and matrices. The Mathematics Specialist ATAR course is the only ATAR mathematics course that should not be taken as a stand-alone course.

Rationale

The Mathematics Methods ATAR course is the study of calculus and statistical analysis in applied and theoretical contexts. It focuses on developing students' ability to use mathematical concepts and techniques to solve problems, analyse data and model real-world phenomena.

The course focuses on the representations and transformations of functions, with a strong emphasis on algebraic techniques and graphs, further uses of trigonometry, probability and combinatorics, and sequences and their applications in applied and theoretical contexts. The course introduces differential calculus through the exploration of rates of change.

By studying this course, students develop a strong foundation in mathematical reasoning, critical thinking and analytical problem-solving. They gain fluency in manipulating algebraic expressions, interpreting mathematical models, and using statistical methods and probability techniques to draw meaningful conclusions from data. The course fosters students' ability to communicate mathematical arguments clearly and logically.

The mathematical skills and concepts covered in the course have broad applications in daily life and various professional fields. Students apply their understanding of probability to make informed decisions based on data analysis and real-world uncertainties. Their knowledge of functions helps in solving practical problems related to measurements, growth patterns and periodic phenomena.

The Mathematics Methods ATAR course provides a strong foundation for students intending to pursue tertiary studies in engineering, computer science, physics, economics, health sciences and data analytics. Careers in areas such as actuarial science, medicine, finance and research heavily depend on mathematical reasoning and statistical analysis, making the study of Mathematics Methods highly beneficial.

Aims

The Mathematics Methods ATAR course aims to develop students':

- understanding of concepts and techniques drawn from algebra, functions, calculus, probability and statistics
- ability to apply mathematical knowledge to solve applied and theoretical problems
- proficiency in reasoning and interpretation within mathematical and statistical contexts
- capacity to communicate findings clearly and systematically, using precise and appropriate mathematical and statistical language
- confidence to select and use digital tools appropriately and efficiently to support problem-solving and data analysis.

Organisation

This course is organised into a Year 11 syllabus and a Year 12 syllabus. The cognitive complexity of the syllabus content increases from Year 11 to Year 12.

Structure of the syllabus

The Year 11 syllabus is divided into two units, each of one semester duration, which are typically delivered as a pair. The notional time for each unit is 55 class contact hours.

Organisation of content

Unit 1

Contains the three topics:

- Counting and probability
- Functions and graphs
- Trigonometric functions.

Unit 1 begins with the study of probability and statistics with a review of the fundamentals of probability, and the introduction of the concepts of conditional probability and independence. A review of the basic algebraic concepts and techniques required for a successful introduction to the study of functions and calculus is covered. Simple relationships between variable quantities are reviewed, and these are used to introduce the key concepts of a function and its graph. The study of the trigonometric functions begins with a consideration of the unit circle using degrees and the trigonometry of triangles and its application. Radian measure is introduced, and the graphs of the trigonometric functions are examined and their applications in a wide range of settings are explored.

Unit 2

Contains the three topics:

- Exponential functions
- Arithmetic and geometric sequences and series
- Introduction to differential calculus.

In Unit 2, exponential functions are introduced and their properties and graphs examined. Arithmetic and geometric sequences and their applications are introduced and their recursive definitions applied. Rates and average rates of change are introduced and this is followed by the key concept of the derivative as an 'instantaneous rate of change'. These concepts are reinforced numerically (by calculating difference quotients), geometrically (as slopes of chords and tangents), and algebraically. This first calculus topic concludes with derivatives of polynomial functions, using simple applications of the derivative to sketch curves, calculate slopes and equations of tangents, determine instantaneous velocities, and solve optimisation problems.

Each unit includes:

- a unit description – a short description of the focus of the unit
- learning outcomes – a set of statements describing the learning expected as a result of studying the unit
- unit content – the content to be taught and learned.

Role of technology

It is assumed that students will be taught this course with an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of the course. However, students also need to continue to develop skills that do not depend on technology. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills in this course.

Progression from the Years 7–10 curriculum

The Mathematics Methods ATAR course extends students' understanding, fluency, problem-solving and reasoning from all three strands of the Years 7–10 Mathematics curriculum. Students extend their use of the mathematical modelling process to investigate and understand real-world phenomena and solve problems in applied and theoretical contexts. To study the Mathematics Methods ATAR course, it is desirable that students have completed a selection of topics from the Year 10 Mathematics Optional content.

Representation of the General Capabilities

The General Capabilities encompass the knowledge, skills, behaviours and dispositions that will support students to live and work successfully now and into the future. They are not assessed unless identified within the specified unit content. Teachers should find opportunities to incorporate the following General Capabilities into the teaching and learning program for the Mathematics Methods ATAR course.

Critical and creative thinking

Students identify and clarify information from a range of sources, including visual information and digital sources. They identify the relevant aspects of a concept or problem, understanding that a single mathematical process can be used in seemingly different situations and that approaches may change depending on the context or nature of the problem. Students draw conclusions and make choices when completing tasks by connecting evidence from within and across the mathematical content to provide reasons and evaluate arguments for choices made.

Digital literacy

Students use appropriate digital tools to support the development of mathematical understanding and to apply mathematical knowledge to a range of problems. They use technologies aligned with areas of work they may be involved with such as statistical and graphical analysis, generation of algorithms, algebraic manipulation and complex calculations. Students use digital tools to make connections between mathematical theory, practice and application.

Literacy

Students develop literacy skills and strategies that enable them to express, interpret and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their ability to read, write, visualise and talk about complex situations involving a range of mathematical ideas. They can apply and further develop their literacy skills and strategies by shifting between verbal, graphical, numerical and symbolic forms of representing problems to formulate, understand and solve problems, and communicate results. Students learn to communicate their findings in different ways, using multiple systems of representation to illustrate the relationships they have observed or constructed.

Numeracy

Students develop their numeracy skills at a more sophisticated level, making decisions about the relevant mathematics to use, following through with calculations, selecting appropriate methods and being confident of their results. This course contains topics that will equip students for the increasing demands of the information age, developing the skills of critical evaluation of numerical information. They enhance their numerical operation skills via engagement with number sequences, trigonometric and probability calculations, algebraic relationships and the development of calculus concepts.

Addressing the other General Capabilities

Although the following General Capabilities have not been identified as a focus in the Mathematics Methods ATAR Year 11 syllabus, teachers may find opportunities to incorporate them into the teaching and learning program.

- Ethical understanding
- Intercultural understanding
- Personal and social capability

Such opportunities may occur through the application of different contexts, pedagogical practices and/or assessment strategies that relate to the syllabus as part of the teaching and learning program.

Summary representation of the General Capabilities in the Mathematics Methods ATAR course

The unit content and assessment types for this course provide students with the opportunity to develop the General Capabilities summarised in the table below.

Year	Course	Course type	General Capabilities						
			CCT	DL	EU	IU	L	N	PSC
Year 11	Mathematics Methods (AEMAM)	ATAR	✓	✓			✓	✓	
Year 12	Mathematics Methods (ATMAM)	ATAR	✓	✓			✓	✓	

Key
CCT: Critical and creative thinking, DL: Digital literacy, EU: Ethical understanding, IU: Intercultural understanding, L: Literacy, N: Numeracy, PSC: Personal and social capability

Representation of the Cross-curriculum Priorities

The Cross-curriculum Priorities address contemporary issues which students face in a globalised world. Teachers may find opportunities to incorporate them into the teaching and learning program for the Mathematics Methods ATAR course. The Cross-curriculum Priorities are not assessed unless they are identified within the specified unit content.

Aboriginal and Torres Strait Islander histories and cultures

Mathematics courses value the histories, cultures, traditions and languages of Aboriginal and Torres Strait Islander Peoples' past and ongoing contributions to contemporary Australian society and culture. Through the study of mathematics within relevant contexts, opportunities will allow for the development of students' understanding and appreciation of the diversity of Aboriginal and Torres Strait Islander Peoples' histories and cultures.

Asia and Australia's engagement with Asia

There are strong social, cultural and economic reasons for Australian students to engage with the countries of Asia and with the past and ongoing contributions made by the peoples of Asia in Australia. It is through the study of mathematics in an Asian context that students engage with Australia's place in the region. By analysing relevant data, students have opportunities to further develop an understanding of the diverse nature of Asia's environments and traditional and contemporary cultures.

Sustainability

Each of the mathematics courses provides the opportunity for the development of informed and reasoned points of view, discussion of issues, research and problem solving. Teachers are therefore encouraged to select contexts for discussion that are connected with sustainability. Through the analysis of data, students have the opportunity to research and discuss sustainability and learn the importance of respecting and valuing a wide range of world perspectives.

Unit 1

Unit description

The study of inferential statistics begins in this unit with a review of the fundamentals of probability and the introduction of the concepts of counting, conditional probability and independence. The unit covers a review of the basic algebraic concepts and techniques required for a successful introduction to the study of calculus. The basic trigonometric functions are then introduced. Simple relationships between variable quantities are reviewed, and these are used to introduce the key concepts of a function and its graph. Access to technology to support the computational and graphical aspects of these topics is assumed.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques in algebra, functions, graphs, trigonometric functions, counting and probability
- solve problems using algebra, functions, graphs, trigonometric functions, counting and probability
- apply reasoning skills in the context of algebra, functions, graphs, trigonometric functions, counting and probability
- interpret and evaluate mathematical information and ascertain the reasonableness of solutions to problems
- communicate their arguments and strategies when solving problems.

Unit content

This unit includes the knowledge, understandings and skills described below.

Topic 1.1: Counting and probability (18 hours)

Combinations

1.1.1 understand the notion of a combination as a set of r objects taken from a set of n distinct objects

1.1.2 use the notation $\binom{n}{r}$ and the formula $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ for the number of combinations of r objects taken from a set of n distinct objects

1.1.3 investigate Pascal's triangle and its properties to link $\binom{n}{r}$ to the binomial coefficients of the expansion of $(x+y)^n$ for small positive integers n

Language of events and sets

1.1.4 review the concepts and language of outcomes, sample spaces, and events, as sets of outcomes

1.1.5 use set language and notation for events, including:

a. $\bar{A}$ (or A') for the complement of an event A

b. $A \cap B$ and $A \cup B$ for the intersection and union of events A and B respectively

c. $A \cap B \cap C$ and $A \cup B \cup C$ for the intersection and union respectively of the three events A, B and C

d. recognise mutually exclusive events

1.1.6 use everyday occurrences to illustrate set descriptions and representations of events and set operations

Review of the fundamentals of probability

1.1.7 review probability as a measure of 'the likelihood of occurrence' of an event

1.1.8 review the probability scale: $0 \leq P(A) \leq 1$ for each event A, with $P(A) = 0$ if A is an impossibility and $P(A) = 1$ if A is a certainty

1.1.9 review the rules: $P(\overline{A}) = 1 - P(A)$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

1.1.10 use relative frequencies obtained from data as estimates of probabilities

Conditional probability and independence

1.1.11 understand the notion of a conditional probability and recognise and use language that indicates conditionality

1.1.12 use the notation $P(A|B)$ and the formula $P(A \cap B) = P(A|B)P(B)$

1.1.13 understand the notion of independence of an event A from an event B, where

$$P(A|B) = P(A)$$

1.1.14 establish and use the formula $P(A \cap B) = P(A)P(B)$ for independent events A and B, and recognise the symmetry of independence

1.1.15 use relative frequencies obtained from data as estimates of conditional probabilities and as indications of possible independence of events

Topic 1.2: Functions and graphs (22 hours)

Lines and linear relationships

1.2.1 recognise features of the graph of $y = mx + c$, including its linear nature, its intercepts and its slope or gradient

1.2.2 determine the equation of a straight line given sufficient information, including for parallel and perpendicular lines

Quadratic relationships

1.2.3 examine examples of quadratically related variables

1.2.4 recognise features of the graphs of $y = x^2$, $y = a(x-b)^2 + c$, and $y = a(x-b)(x-c)$, including their parabolic nature, turning points, axes of symmetry and intercepts

1.2.5 solve quadratic equations, including the use of quadratic formula and completing the square

1.2.6 determine the equation of a quadratic given sufficient information

1.2.7 determine turning points and zeros of quadratics and understand the role of the discriminant

1.2.8 recognise features of the graph of the general quadratic $y = ax^2 + bx + c$

Inverse proportion

1.2.9 examine the concept of inverse proportion

1.2.10 recognise features and determine equations of the graphs of $y = \frac{1}{x}$ and $y = \frac{a}{x-b}$, including their hyperbolic shapes and their asymptotes.

Powers and polynomials

1.2.11 recognise features of the graphs of $y = x^n$ for $n \in \boldsymbol{N}, n = -1$ and $n = ½$, including shape, and behaviour as $x \to \infty$ and $x \to -\infty$

1.2.12 identify the coefficients and the degree of a polynomial

1.2.13 expand factors to obtain quadratic and cubic polynomials

1.2.14 recognise features and determine equations of the graphs of $y = x^3$, $y = a(x-b)^3 + c$ and $y = k(x-a)(x-b)(x-c)$, including shape, intercepts and behaviour as $x \to \infty$ and $x \to -\infty$

1.2.15 factorise cubic polynomials in cases where all roots are given or easily obtained from the graph

1.2.16 solve cubic equations using technology, and algebraically in cases where all roots are given or easily obtained from the graph

Graphs of relations

1.2.17 recognise features and determine equations of the graphs of $x^2 + y^2 = r^2$ and $(x-a)^2 + (y-b)^2 = r^2$, including their circular shapes, their centres and their radii

1.2.18 recognise features of the graph of $y^2 = x$, including its parabolic shape and its axis of symmetry

Functions

1.2.19 understand the concept of a function as a mapping between sets and as a rule or a formula that defines one variable quantity in terms of another

1.2.20 use function notation; determine domain and range; recognise independent and dependent variables

1.2.21 understand the concept of the graph of a function

1.2.22 examine translations and the graphs of $y = f(x) + a$ and $y = f(x - b)$

1.2.23 examine dilations and the graphs of $y = cf(x)$ and $y = f(dx)$

1.2.24 recognise the distinction between functions and relations and apply the vertical line test

Topic 1.3: Trigonometric functions (15 hours)

Cosine and sine rules

1.3.1 review sine, cosine and tangent as ratios of side lengths in right-angled triangles

1.3.2 understand the unit circle definition of $\cos\theta$, $\sin\theta$ and $\tan\theta$ and periodicity using degrees

1.3.3 examine the relationship between the angle of inclination of a line and the gradient of that line

1.3.4 establish and use the cosine and sine rules, including consideration of the ambiguous case and the formula $Area = \frac{1}{2}bc\ sin\ A$ for the area of a triangle

Circular measure and radian measure

1.3.5 define and use radian measure and understand its relationship with degree measure

1.3.6 use radian measure to calculate lengths of arcs and areas of sectors and segments in a circle

Trigonometric functions

1.3.7 understand the unit circle definition of $\sin\theta$, $\cos\theta$ and $\tan\theta$ and periodicity using radians

1.3.8 recognise the exact values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ at integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$

1.3.9 recognise the graphs of $y = \sin x, y = \cos x$, and $y = \tan x$ on extended domains

1.3.10 examine amplitude changes and the graphs of $y = a\sin x$ and $y = a\cos x$

1.3.11 examine period changes and the graphs of $y = \sin bx$, $y = \cos bx$ and $y = \tan\ bx$

1.3.12 examine phase changes and the graphs of $y = \sin(x - c)$, $y = \cos(x - c)$ and $y = \tan\ (x - c)$

1.3.13 examine the relationships $\sin\left(x + \frac{\pi}{2}\right) = \cos x$ and $\cos\left(x - \frac{\pi}{2}\right) = \sin x$

1.3.14 prove and apply the angle sum and difference identities

1.3.15 identify contexts suitable for modelling by trigonometric functions and use them to solve practical problems

1.3.16 solve equations involving trigonometric functions using technology, and algebraically in simple cases

Unit 2

Unit description

The algebra section of this unit focuses on exponentials. Their graphs are examined and their applications in a wide range of settings are explored. Arithmetic and geometric sequences are introduced and their applications are studied. Rates and average rates of change are introduced, and this is followed by the key concept of the derivative as an 'instantaneous rate of change'. These concepts are reinforced numerically, by calculating difference quotients both geometrically as slopes of chords and tangents, and algebraically. Calculus is developed to study the derivatives of polynomial functions, with simple application of the derivative to curve sketching, the calculation of slopes and equations of tangents, the determination of instantaneous velocities and the solution of optimisation problems. The unit concludes with a brief consideration of anti-differentiation.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques used in algebra, sequences and series, functions, graphs and calculus
- solve problems in algebra, sequences and series, functions, graphs and calculus
- apply reasoning skills in algebra, sequences and series, functions, graphs and calculus
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems
- communicate arguments and strategies when solving problems.

Unit content

This unit builds on the content covered in Unit 1.

This unit includes the knowledge, understandings and skills described below.

Topic 2.1: Exponential functions (10 hours)

Indices and the index laws

2.1.1 review indices (including fractional and negative indices) and the index laws

2.1.2 use radicals and convert to and from fractional indices

2.1.3 understand and use scientific notation and significant figures

Exponential functions

2.1.4 establish and use the algebraic properties of exponential functions

2.1.5 recognise the qualitative features of the graph of $y = a^x$ $(a > 0)$, including asymptotes, and of its translations ($y = a^x + b$ and $y = a^{x-c}$)

2.1.6 identify contexts suitable for modelling by exponential functions and use them to solve practical problems

2.1.7 solve equations involving exponential functions using technology, and algebraically in simple cases

Topic 2.2: Arithmetic and geometric sequences and series (15 hours)

Arithmetic sequences

2.2.1 recognise and use the recursive definition of an arithmetic sequence: $t_{n+1} = t_n + d$

2.2.2 develop and use the formula $t_n = t_1 + (n-1)d$ for the general term of an arithmetic sequence and recognise its linear nature

2.2.3 use arithmetic sequences in contexts involving discrete linear growth or decay, such as simple interest

2.2.4 establish and use the formula for the sum of the first n terms of an arithmetic sequence

Geometric sequences

2.2.5 recognise and use the recursive definition of a geometric sequence: $t_{n+1} = t_n r$

2.2.6 develop and use the formula $t_n = t_1 r^{n-1}$ for the general term of a geometric sequence and recognise its exponential nature

2.2.7 understand the limiting behaviour as $n \to \infty$ of the terms t_n in a geometric sequence and its dependence on the value of the common ratio r

2.2.8 establish and use the formula $S_n = t_1 \frac{r^n - 1}{r - 1}$ for the sum of the first n terms of a geometric sequence

2.2.9 use geometric sequences in contexts involving geometric growth or decay, such as compound interest

Topic 2.3: Introduction to differential calculus (30 hours)

Rates of change

2.3.1 interpret the difference quotient $\frac{f(x+h)-f(x)}{h}$ as the average rate of change of a function f

2.3.2 use the Leibniz notation δx and δy for changes or increments in the variables x and y

2.3.3 use the notation $\frac{\delta y}{\delta x}$ for the difference quotient $\frac{f(x+h)-f(x)}{h}$ where $y = f(x)$

2.3.4 interpret the ratios $\frac{f(x+h)-f(x)}{h}$ and $\frac{\delta y}{\delta x}$ as the slope or gradient of a chord or secant of the graph of $y = f(x)$

The concept of the derivative

2.3.5 examine the behaviour of the difference quotient $\frac{f(x+h)-f(x)}{h}$ as $h \to 0$ as an informal introduction to the concept of a limit

2.3.6 define the derivative $f'(x)$ as $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

2.3.7 use the Leibniz notation for the derivative: $\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$ and the correspondence $\frac{dy}{dx} = f'(x)$ where $y = f(x)$

2.3.8 interpret the derivative as the instantaneous rate of change

2.3.9 interpret the derivative as the slope or gradient of a tangent line of the graph of $y = f(x)$

Computation of derivatives

2.3.10 estimate numerically the value of a derivative for simple power functions

2.3.11 examine examples of variable rates of change of non-linear functions

2.3.12 establish the formula $\frac{d}{dx}(x^n) = nx^{n-1}$ for non-negative integers n expanding $(x+h)^n$ or by factorising $(x+h)^n - x^n$

Properties of derivatives

2.3.13 understand the concept of the derivative as a function

2.3.14 identify and use linearity properties of the derivative

2.3.15 calculate derivatives of polynomial functions

Applications of derivatives

2.3.16 determine instantaneous rates of change

2.3.17 determine the slope of a tangent and the equation of the tangent

2.3.18 construct and interpret position-time graphs with velocity as the slope of the tangent

2.3.19 recognise velocity as the first derivative of displacement with respect to time

2.3.20 sketch curves associated with simple polynomials, determine stationary points, and local and global maxima and minima, and examine behaviour as $x \to \infty$ and $x \to -\infty$

2.3.21 solve optimisation problems arising in a variety of contexts involving polynomials on finite interval domains

Anti-derivatives

2.3.22 calculate anti-derivatives of polynomial functions

School-based assessment

The *Western Australian Certificate of Education (WACE) Manual* contains essential information on principles, policies and procedures for school-based assessment that needs to be read in conjunction with this syllabus.

Teachers design school-based assessment tasks to meet the needs of students. The table below provides details of the assessment types for the Mathematics Methods ATAR Year 11 syllabus and the weighting for each assessment type.

Assessment table – Year 11

Type of assessment	Weighting
Response Students apply mathematical knowledge and understanding of concepts, techniques and relationships to solve a mix of routine and non-routine questions, demonstrating their interpretation of concepts and results in applied and theoretical contexts. Response tasks can include: tests, assignments and multimedia representations.	40%
Investigation Students use the mathematical thinking process to plan, research, conduct and communicate the findings of an investigation. They can investigate problems to identify the underlying mathematics, or select, adapt and apply models and procedures to solve problems. This assessment type provides for the assessment of the mathematical thinking process using course-related knowledge and modelling skills. Evidence can include: observation and interview, written work or multimedia presentations.	20%
Examination Students apply mathematical understanding and skills to analyse, interpret and respond to questions and situations. Examinations provide for the assessment of conceptual understandings, knowledge of mathematical facts and terminology, problem-solving skills, and the use of algorithms. Examination questions can range from those of a routine nature, assessing lower level concepts, through to those that require responses at the highest level of conceptual thinking. Typically conducted at the end of each semester and/or unit. In preparation for Unit 3 and Unit 4, the examination should reflect the examination design brief included in the ATAR Year 12 syllabus for this course. Where a combined assessment outline is implemented, the Semester 2 examination should assess content from both Unit 1 and Unit 2. However, the combined weighting of Semester 1 and Semester 2 should reflect the respective weightings of the course content as a whole.	40%

Teachers are required to use the assessment table to develop an assessment outline for the pair of units (or for a single unit where only one is being studied).

The assessment outline must:

- include a set of assessment tasks
- include a general description of each task
- indicate the unit content to be assessed
- indicate a weighting for each task and each assessment type
- include the approximate timing of each task (for example, the week the task is conducted, or the issue and submission dates for an extended task).

In the assessment outline for the pair of units, each assessment type must be included at least once over the year/pair of units. In the assessment outline where a single unit is being studied, each assessment type must be included at least once.

The set of assessment tasks must provide a representative sampling of the content for Unit 1 and Unit 2.

Assessment tasks not administered under test/controlled conditions require appropriate validation/authentication processes. This may include observation, annotated notes, checklists, interview, presentations or in-class tasks assessing related content and processes.

Grading

Schools report student achievement in terms of the following grades:

Grade	Interpretation
A	Excellent achievement
B	High achievement
C	Satisfactory achievement
D	Limited achievement
E	Very low achievement

The teacher prepares a ranked list and assigns the student a grade for the pair of units (or for a unit where only one unit is being studied). The grade is based on the student's overall performance as judged by reference to a set of pre-determined standards. These standards are defined by grade descriptions and annotated work samples. The grade descriptions for the Mathematics Methods ATAR Year 11 syllabus are provided in Appendix 1. They can also be accessed, together with annotated work samples, through the Guide to Grades link on the course page of the Authority website at www.scsa.wa.edu.au.

To be assigned a grade, a student must have had the opportunity to complete the education program, including the assessment program (unless the school accepts that there are exceptional and justifiable circumstances).

Refer to the *WACE Manual* for further information about the use of a ranked list in the process of assigning grades.

Appendix 1 – Grade descriptions Year 11

A	**Interprets the task, identifies and organises relevant information, and chooses strategies** Identifies and organises relevant information from previous parts of a problem and brings them together to solve additional problems. Selects an appropriate strategy and demonstrates mathematical conventions to solve non-routine, unfamiliar, unstructured and/or multi-step problems. Clarifies an investigative task, identifies the key information and relevant assumptions, and chooses the appropriate mathematics.
	Uses mathematical knowledge and understanding to obtain solutions Completes concise and accurate solutions to mathematical problems set in a variety of applied and theoretical contexts. Navigates between numerical, graphical and symbolic representations appropriately to solve problems in unfamiliar contexts. Uses digital tools and their appropriate functionalities effectively to solve problems in unfamiliar contexts. Applies changed conditions, correctly determines and explains the effect on the solution. Applies comprehensive knowledge and understanding of relevant concepts and relationships to extensively investigate a problem.
	Communicates mathematical reasoning, interprets results and draws conclusions Sets out the steps of the solution in a succinct and logical sequence, including suitable justification and explanation of methods and processes used. Correctly uses mathematical notation, terminology, units and appropriate rounding consistently. Interprets mathematical results and draws conclusions in the context of the problem. Comprehensively interprets and clearly communicates mathematical findings in the context of an investigation. Identifies the strengths and limitations of an investigation and includes consideration of these in refining the results to draw sensible conclusions.

B	**Interprets the task, identifies and organises relevant information, and chooses strategies** Identifies and organises relevant information for problems involving a few steps or processes. Selects an appropriate strategy and demonstrates mathematical conventions to solve non-routine, familiar, and/or partly structured problems. Clarifies an investigative task, identifies the key information and some assumptions, and mostly chooses the appropriate mathematics.
	Uses mathematical knowledge and understanding to obtain solutions Produces mostly accurate and complete solutions to mathematical problems set in applied and theoretical contexts. Navigates between numerical, graphical and symbolic representations appropriately to solve problems in familiar contexts. Uses digital tools and their appropriate functionalities effectively to solve problems in familiar contexts. Applies changed conditions and attempts to determine the effect on the solution. Applies some depth of knowledge and understanding of relevant concepts and relationships to investigate a problem.
	Communicates mathematical reasoning, interprets results and draws conclusions Sets out the steps of the solution in a logical sequence, including some justification and explanation of methods and processes used. Mostly correct use of mathematical notation, terminology, units and appropriate rounding. Interprets most mathematical results and draws conclusions in the context of the problem. Interprets and communicates mathematical findings in the context of an investigation. Identifies the strengths and limitations of an investigation.

C	**Interprets the task, identifies and organises relevant information, and chooses strategies** Identifies and extracts key information needed to solve a familiar problem. Selects an appropriate strategy and demonstrates mathematical conventions to solve routine, familiar and/or structured problems. Clarifies an investigative task, identifies the key information, and mostly chooses appropriate mathematics.
	Uses mathematical knowledge and understanding to obtain solutions Produces some accurate and mostly complete solutions to mathematical problems set in applied or theoretical contexts. Recognises and uses numerical, graphical and symbolic representations appropriately to solve routine problems. Uses digital tools and their appropriate functionalities to solve routine problems. Applies changed conditions to determine a solution in routine problems. Applies competent knowledge and understanding of concepts and relationships to investigate a problem.
	Communicates mathematical reasoning, interprets results and draws conclusions Sets out the steps of the solution and supports methods and processes with simple or routine statements. Some correct use of mathematical notation, terminology, units and appropriate rounding. Interprets some mathematical results and draws some conclusions in the context of the problem. Communicates mathematical findings in the context of an investigation. Attempts to identify the limitations of an investigation.

D	**Interprets the task, identifies and organises relevant information, and chooses strategies** Uses given information to solve simple routine problems. Selects inappropriate strategies and/or demonstrates limited mathematical conventions in an attempt to solve familiar problems. Clarifies an investigative task, identifies some key information and chooses some appropriate mathematics.
	Uses mathematical knowledge and understanding to obtain solutions Produces partly accurate and incomplete solutions to mathematical problems set in applied or theoretical contexts. Uses digital tools in an attempt to solve routine problems. Attempts to apply changed conditions to determine a solution in routine problems. Attempts to apply knowledge and understanding of concepts and relationships to investigate a problem.
	Communicates mathematical reasoning, interprets results and draws conclusions Attempts to set out the steps of the solution. Limited use of mathematical notation, terminology, units and appropriate rounding. Limited interpretation of mathematical results and conclusions not clearly formulated Minimal communication of findings from the results of an investigation.

E	Does not meet the requirements of a D grade and/or has completed insufficient assessment tasks to be assigned a higher grade.

Appendix 2 – Glossary

This glossary is provided to enable a common understanding of the key terms in this syllabus.

Unit 1

Functions and graphs

Asymptote
A line is an asymptote to a curve if the distance between the line and the curve approaches zero as they 'tend to infinity'. For example, the line with equation $x = \frac{\pi}{2}$ is a vertical asymptote to the graph of $y = \tan x$, and the line with equation $y = 0$ is a horizontal asymptote to the graph of $y = \frac{1}{x}$.

Binomial distribution
The expansion $(x + y)^n = x^n + \binom{n}{1} x^{n-1} y + \cdots + \binom{n}{r} x^{n-r} y^r + \cdots + y^n$ is known as the binomial theorem. The numbers $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n \times (n-1) \times \cdots \times (n-r+1)}{r \times (r-1) \times \cdots \times 2 \times 1}$ are called binomial coefficients.

Completing the square
The quadratic expression $ax^2 + bx + c$ can be rewritten as $a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$. Re-writing it in this way is called completing the square.

Discriminant
The discriminant of the quadratic expression $ax^2 + bx + c$ is the quantity $b^2 - 4ac$.

Function
A function f is a rule that associates with each element x in a set S, a unique element $f(x)$ in a set T. We write $x \longmapsto f(x)$ to indicate the mapping of x to $f(x)$. The set S is called the domain of f and the set T is called the codomain. The subset of T consisting of all the elements $f(x): x \in S$ is called the range of f. If we write $y = f(x)$ we say that x is the independent variable and y is the dependent variable.

Graph of a function
The graph of a function f is the set of all points (x, y) in Cartesian plane where x is in the domain of f and $y = f(x)$.

Quadratic formula
If $ax^2 + bx + c = 0$ with $a \neq 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. This formula for the roots is called the quadratic formula.

Vertical line test
A relation between two real variables x and y is a function and $y = f(x)$ for some function f, if and only if each vertical line, i.e. each line parallel to the y-axis, intersects the graph of the relation in, at most, one point. This test to determine whether a relation is, in fact, a function is known as the vertical line test.

Trigonometric functions

Angle sum and difference identities

The angle sum and difference identities for sine and cosine are given by

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

Area of a sector

The area of a sector of a circle is given by $A = \frac{1}{2}r^2\theta$, where A is the sector area, r is the radius and θ is the angle subtended at the centre, measured in radians.

Area of a segment

The area of a segment of a circle is given by $A = \frac{1}{2}r^2(\theta - \sin\theta)$, where A is the segment area, r is the radius and θ is the angle subtended at the centre, measured in radians.

Circular measure

Circular measure is the measurement of angle size in radians.

Length of an arc

The length of an arc in a circle is given by $\ell = r\theta$, where ℓ is the arc length, r is the radius and θ is the angle subtended at the centre, measured in radians. This is simply a rearrangement of the formula defining the radian measure of an angle.

Length of a chord

The length of a chord in a circle is given by $\ell = 2r\sin\frac{1}{2}\theta$, where ℓ is the chord length, r is the radius and θ is the angle subtended at the centre, measured in radians.

Period of a function

The period of a function $f(x)$ is the smallest positive number p with the property that $f(x + p) = f(x)$ for all x. The functions $\sin x$ and $\cos x$ both have period 2π and $\tan x$ has period π.

Radian measure

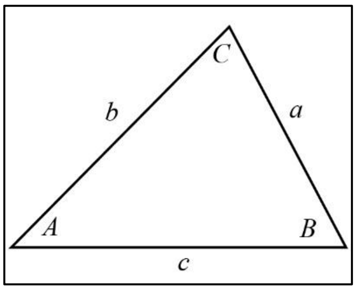
The radian measure θ of an angle in a sector of a circle is defined by $\theta = \frac{\ell}{r}$, where r is the radius and ℓ is the arc length. Thus, an angle whose degree measure is 180 has radian measure π.

Sine rule and cosine rule

The lengths of the sides of a triangle are related to the sine of its angles by the equations

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

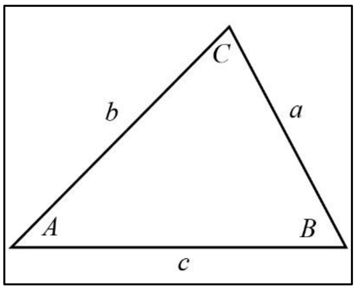
This is known as the sine rule.



The lengths of the sides of a triangle are related to the cosine of one of its angles by the equation

$$c^2 = a^2 + b^2 - 2ab\cos C.$$

This is known as the cosine rule.



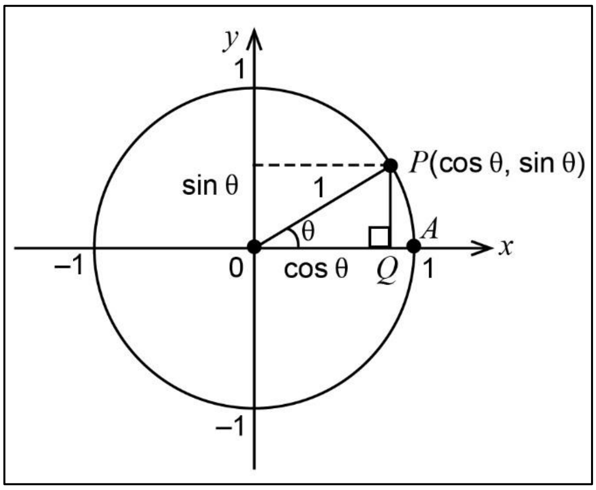
Sine, cosine and tangent functions

Since each angle θ measured anticlockwise from the positive x-axis determines a point P on the unit circle, we will define

the cosine of θ to be the x-coordinate of the point P

the sine of θ to be the y-coordinate of the point P

the tangent of θ is the gradient of the line segment OP.



Counting and probability

Conditional probability

The probability that an event A occurs can change if it becomes known that another event B occurs. The new probability is known as a conditional probability and is written as $P(A|B)$. If B has occurred, the sample space is reduced by discarding all outcomes that are not in the event B. The new sample space, called the reduced sample space, is B. The conditional probability of event A is given by $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

Independent events

Two events are independent if knowing that one occurs tells us nothing about the other. The concept can be defined formally using probabilities in various ways: events A and B are independent if $P(A \cap B) = P(A)P(B)$, if $P(A|B) = P(A)$ or if $P(B) = P(B|A)$. For events A and B with non-zero probabilities, any one of these equations implies any other.

Mutually exclusive

Two events are mutually exclusive if there is no outcome in which both events occur.

Pascal's triangle

Pascal's triangle is a triangular arrangement of binomial coefficients. The n^{th} row consists of the binomial coefficients $\binom{n}{r}$, for $0 \le r \le n$, each interior entry is the sum of the two entries above it, and sum of the entries in the n^{th} row is 2^n

1							
1	1						
1	2	1					
1	3	3	1				
1	4	6	4	1			
1	5	10	10	5	1		
1	6	15	20	15	6	1	
1	7	21	35	35	21	7	1

For example, 10 = 4 + 6.

Relative frequency

If an event E occurs r times when a chance experiment is repeated n times, the relative frequency of E is $\frac{r}{n}$.

Unit 2

Exponential functions

Algebraic properties of exponential functions

The algebraic properties of exponential functions are the index laws: $a^x a^y = a^{x+y}$, $a^{-x} = \frac{1}{a^x}$, $(a^x)^y = a^{xy}$, $a^0 = 1$, for any real numbers x, y, and a, with $a > 0$.

Index laws

The index laws are the rules: $a^x a^y = a^{x+y}$, $a^{-x} = \frac{1}{a^x}$, $(a^x)^y = a^{xy}$, $a^0 = 1$, and $(ab)^x = a^x b^x$, for any real numbers x, y, a and b, with $a > 0$ and $b > 0$.

Arithmetic and geometric sequences and series

Arithmetic sequence

An arithmetic sequence is a sequence of numbers such that the difference of any two successive members of the sequence is a constant. For instance, the sequence

2, 5, 8, 11, 14, 17, …

is an arithmetic sequence with common difference 3.

If the initial term of an arithmetic sequence is a and the common difference of successive members is d, then the n^{th} term t_n of the sequence, is given by:

$t_n = a + (n-1)d$ for $n \geq 1$.

A recursive definition is

$t_1 = a, t_{n+1} = t_n + d$, where d is the common difference and $n \geq 1$.

Geometric sequence

A geometric sequence is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed number called the common ratio. For example, the sequence

3, 6, 12, 24, ...

is a geometric sequence with common ratio 2. Similarly the sequence

40, 20, 10, 5, 2.5, …

is a geometric sequence with common ratio $\frac{1}{2}$.

If the initial term of a geometric sequence is a and the common ratio of successive members is r, then the n^{th} term t_n of the sequence, is given by:

$t_n = ar^{n-1}$ for $n \geq 1$.

A recursive definition is

$t_1 = a, t_{n+1} = rt_n$ for $n \geq 1$ and where r is the constant ratio.

Partial sums of a geometric sequence (geometric series)

The partial sum S_n of the first n terms of a geometric sequence with first term a and common ratio r,

$a, ar, ar^2, \ldots \ldots ar^{n-1}$ … is $S_n = \frac{a(r^n-1)}{r-1}, r \neq 1$.

The partial sums form a sequence with $S_{n+1} = S_n + t_n$ and $S_1 = t_1$

Partial sum of an arithmetic sequence (arithmetic series)
The partial sum S_n of the first n terms of an arithmetic sequence with first term a and common difference d.

$a, a + d, a + 2d, \dots\dots, a + (n - 1)d, \dots\dots$

is $S_n = \frac{n}{2}(a + t_n) = \frac{n}{2}(2a + (n - 1)d)$ where $t_n$is the n^{th} term of the sequence.

The partial sums form a sequence with $S_{n+1} = S_n + t_n$ and $S_1 = t_1$

Partial sums of a sequence (series)
The sequence of partial sums of a sequence $t_{1,\dots,}t_{s\dots}$is defined by $S_n = t_1 + \dots + t_n$

Introduction to differential calculus

Anti-differentiation
An anti-derivative, primitive or indefinite integral of a function $f(x)$ is a function $F(x)$ whose derivative is $f(x)$, i.e. $F'(x) = f(x)$.

The process of solving for anti-derivatives is called anti-differentiation.

Anti-derivatives are not unique. If $F(x)$ is an anti-derivative of $f(x)$, then so too is the function $F(x) + c$ where c is any number. We write $\int f(x)\,dx = F(x) + c$ to denote the set of all anti-derivatives of $f(x)$. The number c is called the constant of integration. For example, since $\frac{d}{dx}(x^3) = 3x^2$, we can write $\int 3x^2\,dx = x^3 + c$.

Gradient (Slope)
The gradient of the straight line passing through points (x_1, y_1) and (x_2, y_2) is the ratio $\frac{y_2 - y_1}{x_2 - x_1}$. Slope is a synonym for gradient.

Linearity property of the derivative
The linearity property of the derivative is summarised by the equations:
$\frac{d}{dx}(ky) = k\frac{dy}{dx}$ for any constant k and $\frac{d}{dx}(y_1 + y_2) = \frac{dy_1}{dx} + \frac{dy_2}{dx}$.

Local and global maximum and minimum
A stationary point on the graph $y = f(x)$ of a differentiable function is a point where $f'(x) = 0$.

We say that $f(x_0)$ is a local maximum of the function $f(x)$ if $f(x) \le f(x_0)$ for all values of x near x_0. We say that $f(x_0)$ is a global maximum of the function $f(x)$ if $f(x) \le f(x_0)$ for all values of x in the domain of f.

We say that $f(x_0)$ is a local minimum of the function $f(x)$ if $f(x) \ge f(x_0)$ for all values of x near x_0. We say that $f(x_0)$ is a global minimum of the function $f(x)$ if $f(x) \ge f(x_0)$ for all values of x in the domain of f.

Secant
A secant of the graph of a function is the straight line passing through two points on the graph. The line segment between the two points is called a chord.

Simple polynomial
A simple polynomial is one which is easily factorised and whose stationary points may be easily determined using traditional calculus techniques.

Tangent line

The tangent line (or simply the tangent) to a curve at a given point P can be described intuitively as the straight line that 'just touches' the curve at that point. At P where the curve meets the tangent, the curve has 'the same direction' as the tangent line. In this sense, it is the best straight-line approximation to the curve at the point P.