Government of **Western Australia**
School Curriculum and Standards Authority

ATAR course examination, 2018

Question/Answer booklet

MATHEMATICS SPECIALIST

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

Section Two: Calculator-assumed

Student number: In figures | | | | | | | | | |

In words _____

_____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	9	9	50	51	35
Section Two: Calculator-assumed	11	11	100	91	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2018*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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Section Two: Calculator-assumed **65% (91 Marks)**

This section has **11** questions. Answer all questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 10 **(4 marks)**

Consider the complex number $z = \sqrt{3} + i$.

Show that, for all positive integers n, $(z)^n - (\bar{z})^n = 2^{n+1} \sin\left(\frac{n\pi}{6}\right) i$.

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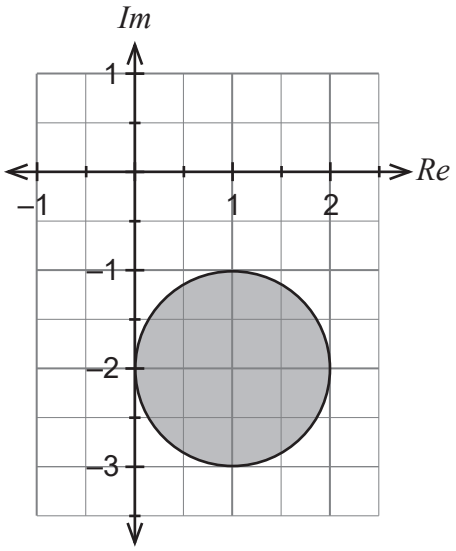
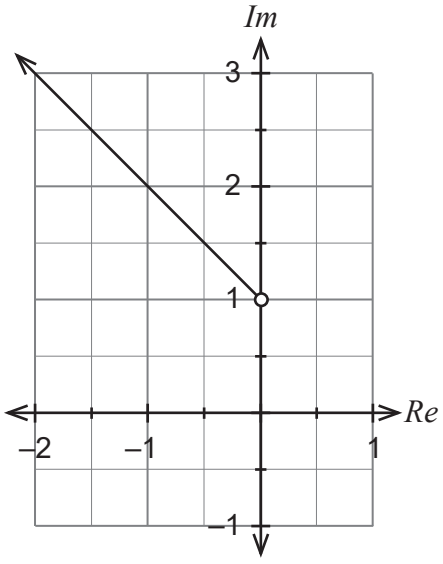
Question 11 **(8 marks)**

A sketch of the locus of a complex number z is shown below. Write equations or inequalities in terms of z (without using $x = \mathrm{Re}(z)$ or $y = \mathrm{Im}(z)$) for each of the following:

(a) (3 marks)



(b) (2 marks)



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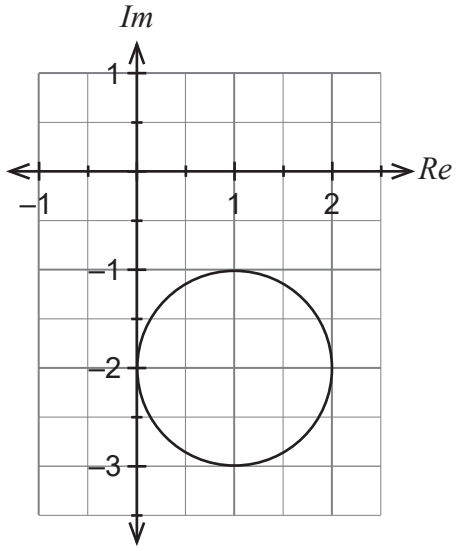
The sketch in Question 11 part (a) is repeated below, with only the circle indicated.



(c) For the locus from part (a), determine the maximum value for $arg\,(z)$ correct to 0.01, where $0 \leq arg\,(z) < 2\pi$. (3 marks)

Question 12 **(11 marks)**

The lifetime of an electronic device is distributed as an exponential random variable with mean $\mu = 20$ years and standard deviation $\sigma = 20$ years. A random sample of 50 of these devices is selected. Tam, a graduate electronics engineer, is interested in the sample mean lifetime $\overline{X}$ of these 50 devices.

(a) State the distribution of the sample mean lifetime $\overline{X}$. Justify your answer. (3 marks)

(b) Determine the probability that the sample mean lifetime is between 15 and 25 years. (2 marks)

Jai, the chief engineer, informs Tam that the lifetimes may not be exponentially distributed but could be a more complicated distribution, yet still having mean $\mu = 20$ years and standard deviation $\sigma = 20$ years.

(c) If Jai is correct, will your answer to part (b) change? Explain. (2 marks)

See next page

A different random sample of size n of these devices was selected. Repeated sampling with this sample size shows that there is a 3% chance of obtaining a sample mean greater than 25 years.

(d) Determine the value of n. (4 marks)

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Question 13 **(5 marks)**

A particle travels in a straight line so that its velocity v cm/sec and displacement x cm are related by the equation:

$$v = \frac{2}{x}$$

(a) Determine the acceleration a in terms of its displacement x. (2 marks)

It is known that the initial displacement of the particle is $x = 2$ cm.

(b) Determine the displacement x as a function of time t. (3 marks)

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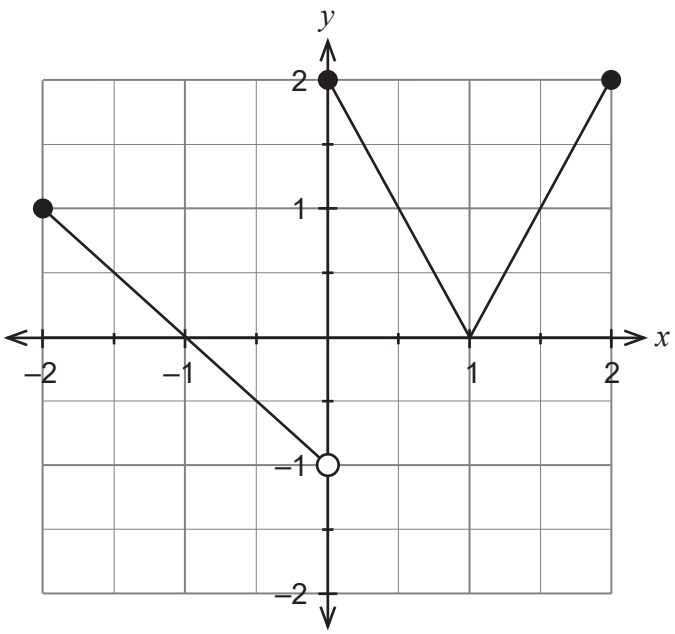
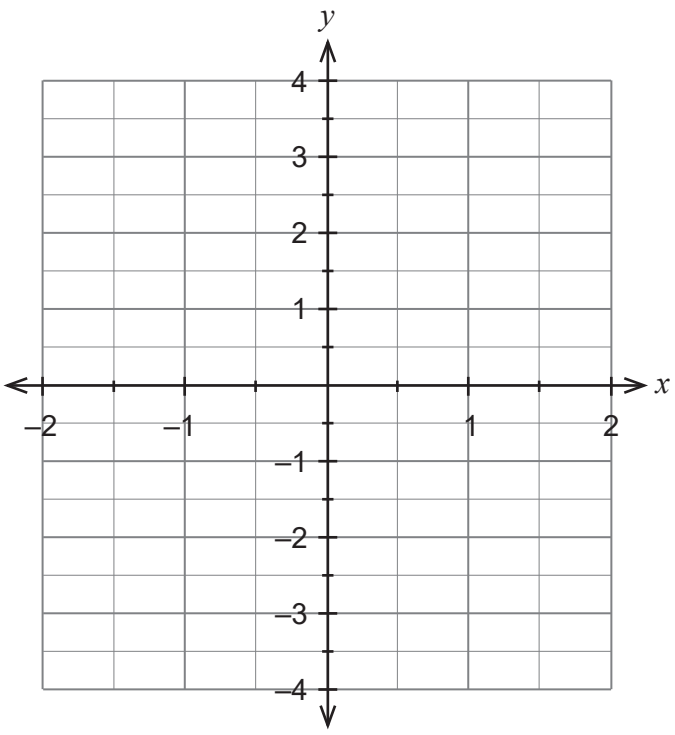
Question 14 **(9 marks)**

The graph of $y = f(x)$ is shown below.



(a) On the axes below, sketch the graph of $y = \frac{1}{f(x)}$. (4 marks)



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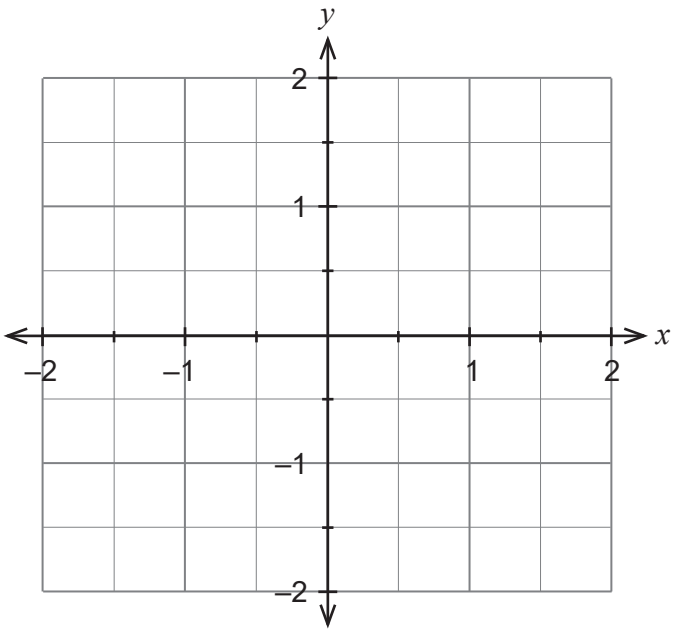
(b) On the axes below, sketch the graph of $y = f(-|x|)$. (3 marks)

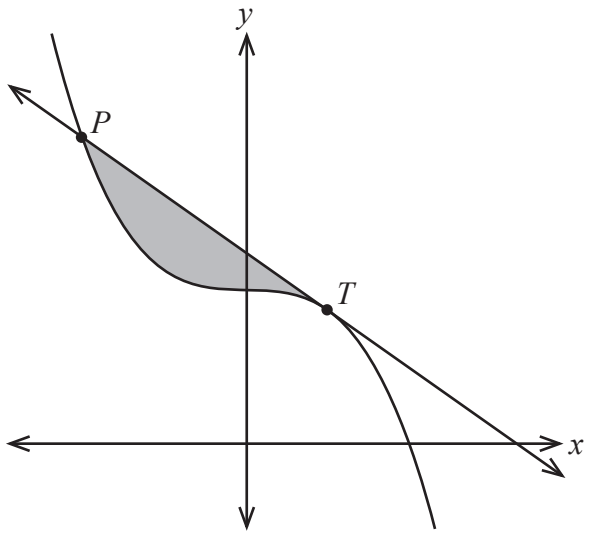


(c) Solve the equation $|f(x) - 1| = 1$. (2 marks)

See next page

Question 15 **(8 marks)**

Part of the graph of $x^3 + 8y = 64$ is shown below. A tangent is drawn to the curve at point $T(2,7)$, intersecting the curve again at point P.



(a) Determine the equation of the tangent to the curve at point T. (2 marks)

(b) Determine the area of the shaded region. (3 marks)

See next page

The shaded region is then rotated about the x axis.

(c) Calculate the volume of the resulting solid, correct to 0.01 cubic units. (3 marks)

See next page

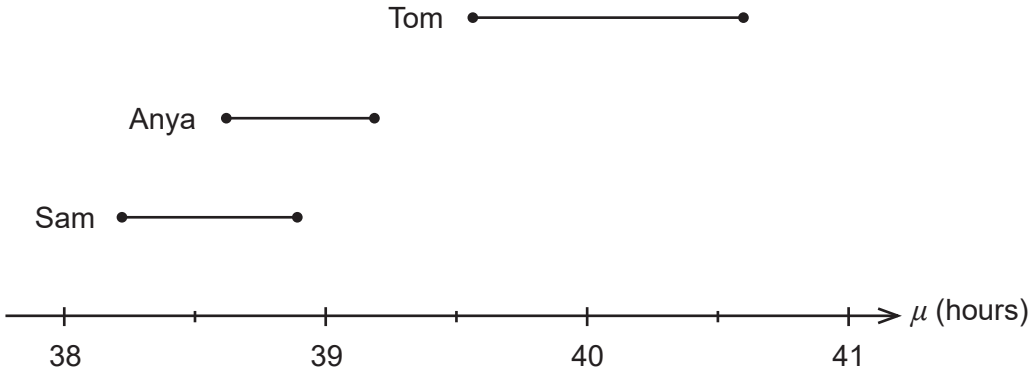
Question 16 **(9 marks)**

Tom wants to estimate the population mean number of hours, μ, worked by Australians per week. He takes a random sample of 400 workers and determines a 99% confidence interval for μ. The upper limit of this interval is 40.62 hours and the width of this interval is 1.08 hours.

(a) Determine the sample mean for this sample of 400 workers. (2 marks)

(b) Calculate, correct to 0.01 hours, the sample standard deviation for the sample of 400 workers. (3 marks)

Two of Tom's colleagues, Anya and Sam, each take different random samples of size 400 and similarly determine 99% confidence intervals for the population mean μ. These confidence intervals, together with Tom's, are shown below.



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(c) Anya makes the following statements based on these confidence intervals. Indicate **why** each of her statements is true or false.

(i) 'Tom's sample has a larger standard deviation compared with that of Sam's and mine.' (1 mark)

(ii) 'Tom's method of sampling must be biased since his confidence interval does not overlap with mine or Sam's.' (1 mark)

(d) Which of these three confidence intervals contains the value for μ? Justify your answer. (2 marks)

Question 17 **(7 marks)**

Plane Π is represented by the equation: $\underset{\sim}{r} = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$.

(a) Determine $\begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ and describe what this represents. (1 mark)

(b) Show that the equation of plane Π can be written as $x - 2y + 2z = 11$. (2 marks)

Consider sphere S with its centre at point $A\ (3, 4, -1)$.

(c) Determine the Cartesian equation for S if plane Π is tangential to this sphere. (4 marks)

See next page

Question 18 **(13 marks)**

A rumour that the Federal Government plans to cut university funding begins to spread around a campus. There is a combined total of 1600 students and staff at this university.

One hundred people know of this rumour via a social media post at 8 am one morning.

Let $N(t)$ = the number of people at the university who have heard the rumour at t hours after 8 am. It is found that the rate at which the rumour spreads is given by the equation:

$$\frac{dN}{dt} = k\,N\,(1600 - N).$$

At 8 am the rumour was spreading at a rate of 60 people per hour.

(a) Show that $k = 0.0004$. (2 marks)

(b) At 11 am there were approximately 500 people who had heard the rumour. Using the increments formula, determine the approximate number of people who learn of this rumour between 11 am and 11.15 am. (3 marks)

See next page

(c) Given that $\frac{1}{N(1600-N)} = \frac{1}{1600}\left(\frac{1}{N} + \frac{1}{1600-N}\right)$, use the separation of variables technique to show that $N(t)$ is given by $\frac{N}{1600-N} = \frac{e^{0.64t}}{15}$. (4 marks)

(d) At what time, correct to the nearest minute, does the rumour spread at the fastest rate? (4 marks)

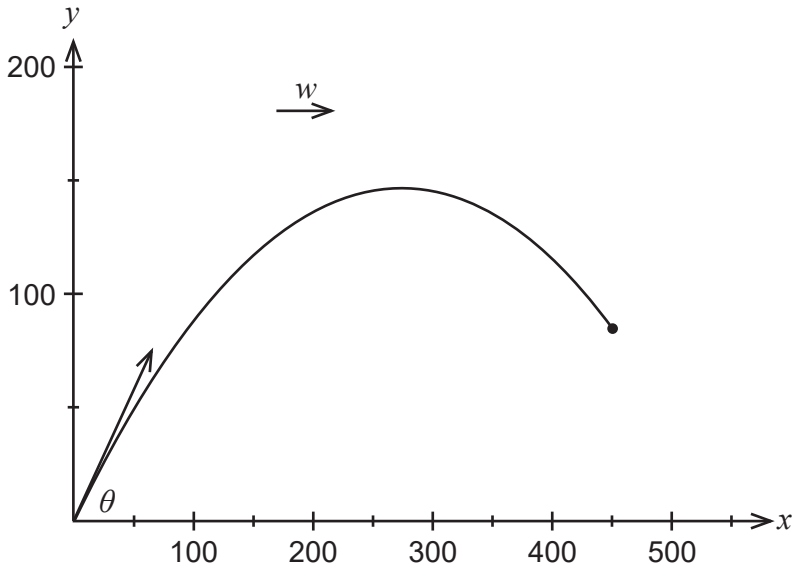
See next page

Question 19 **(9 marks)**

A small rocket is fired from the ground at an angle of $\theta°$ to the horizontal with a speed of 70 metres per second. The rocket has the assistance of a steady wind that is blowing horizontally at w metres per second.

A coordinate system is set up to track the path of the rocket as shown below.

Let t = the number of seconds elapsed after the rocket is fired
$\underset{\sim}{r}(t)$ = the position vector (metres)
$\underset{\sim}{v}(t)$ = the velocity vector (ms^{-1})
$\underset{\sim}{a}(t)$ = the acceleration vector (due to gravity) $= \begin{pmatrix} 0 \\ -9.8 \end{pmatrix}$ (ms^{-2})



(a) Given $\underset{\sim}{a}(t) = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix}$, show that $\underset{\sim}{r}(t) = \begin{pmatrix} (70\cos\theta + w)t \\ (70\sin\theta)t - 4.9t^2 \end{pmatrix}$. (3 marks)

See next page

(b) Obtain the Cartesian equation for the path of the rocket, in terms of θ and w. (2 marks)

The range of the rocket is defined as the horizontal distance travelled from its launch to the point at which it strikes the ground.

(c) Assuming that the wind speed $w = 2$ metres per second, determine the optimum angle θ so that the range of the rocket is maximised, correct to the nearest 0.1 degree. (4 marks)

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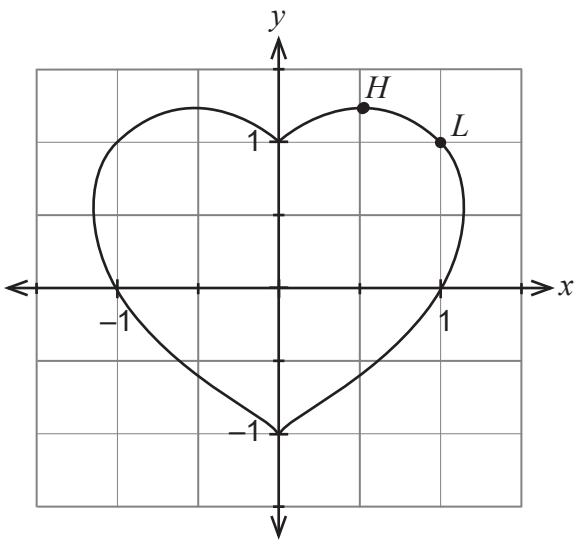
Question 20 **(8 marks)**

The graph of $(x^2 + y^2 - 1)^3 = x^2y^3$ is shown below.



(a) By implicitly differentiating the given equation, obtain an equation relating x, y and $\frac{dy}{dx}$ (3 marks)

(Note: Do **not** attempt to obtain $\frac{dy}{dx}$ as the subject of this equation.)

See next page

(b) Determine the exact slope of the tangent to the curve at the point $L\ (1,1)$. (2 marks)

At point H on the graph the curve is horizontal.

(c) Determine the coordinates of point H, correct to 0.001. (3 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____

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