Government of **Western Australia**
School Curriculum and Standards Authority

ATAR course examination, 2020

Question/Answer booklet

MATHEMATICS APPLICATIONS

Section Two: Calculator-assumed

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

WA student number: In figures

In words _____

_____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	6	6	50	47	35
Section Two: Calculator-assumed	10	10	100	105	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2020: Part II Examinations*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (105 Marks)

This section has **10** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 7 (6 marks)

The world's tallest man was recorded as 60 cm long at birth. He grew 28 cm in his first year, 26 cm in his second year and so on, always 2 cm less than in the previous year until he stopped growing.

(a) Calculate his annual growth (in cm) in his fourth and fifth years. (1 mark)

(b) Deduce the rule for his annual growth in the nth year, until he stopped growing. (2 marks)

(c) In which year did he first not grow any taller? (1 mark)

(d) Calculate his maximum height. (2 marks)

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Question 8 **(9 marks)**

A farmer has a large lake on his farm and has started stocking it with fish of a variety that will flourish in the conditions in this lake. Monitoring has shown that the number of adult fish is increasing at a consistent rate of 9% per month and at the beginning of 2020 the lake holds 660 of the adult fish.

(a)   Write a recursive rule to give the number of adult fish in the lake at the end of each month from the beginning of 2020. (2 marks)

(b)   Deduce a rule for the nth term of this sequence. (2 marks)

The farmer plans to allow the general public to pay to fish in the lake. This will commence at the beginning of the next month after the adult fish population first reaches 4000.

(c)   Determine how many months after the beginning of 2020 fishing will commence. (2 marks)

(d)   The farmer wishes to maintain a steady state in the adult fish population once fishing commences. Calculate how many adult fish can be taken from the lake each month. (3 marks)

Question 9 **(11 marks)**

Giuseppe wishes to set up an annuity. He is told that an annuity with quarterly investment returns and quarterly payments is modelled by the recursive rule:

$A_{n+1} = A_n \times 1.019 - P$, $A_0 = Q$ with the values of P and Q consistent with the spreadsheet below.

Quarter	Opening balance	Investment gain	Payment	Closing balance
1	$648 000	$12 312	$15 000	X
2		Y	$15 000	
3				

(a) Determine the values of P, Q, X and Y and write them in the table below. (4 marks)

P	Q	X	Y

(b) What is the annual compound interest rate for this investment? (1 mark)

When the balance in the annuity first falls below $300 000, Giuseppe converts the payment to a perpetuity so that his children are left with some inherited benefits. The interest rate remains the same as that calculated in part (b).

(c) Determine the number of years the annuity operates before the perpetuity starts. (2 marks)

(d) What are the quarterly payments under this perpetuity? (2 marks)

(e) Giuseppe believes that his investment returns are at an effective interest rate of 7.93% p.a. Use a clear calculation to comment on the accuracy of this belief. (2 marks)
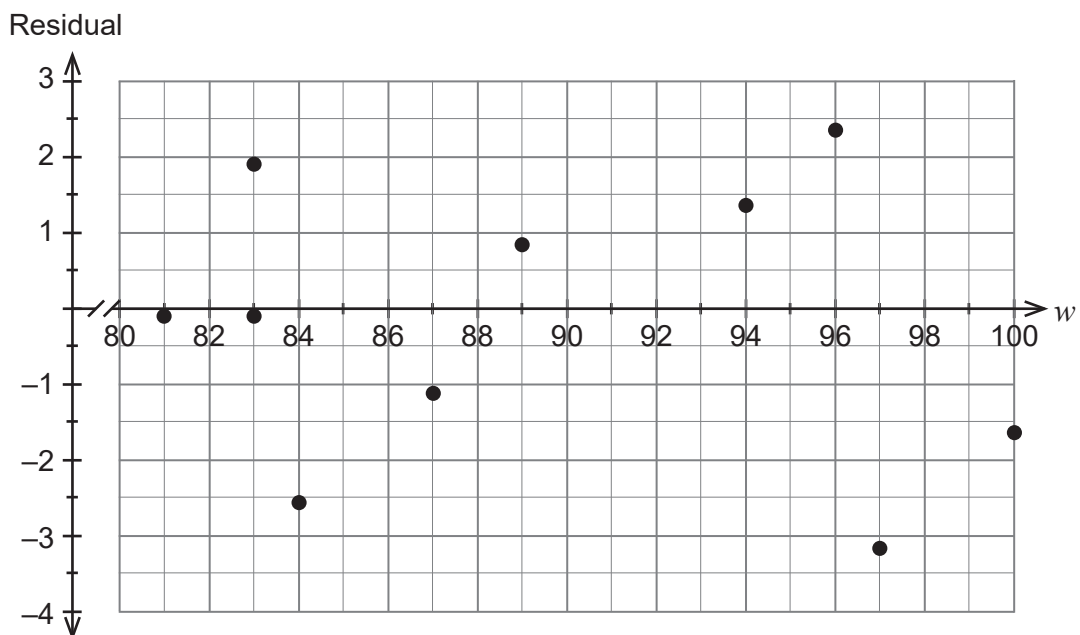
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Question 10 **(15 marks)**

A football club records body measurements for all of their players. Shown below are the waistline measurements (cm) and percentage body fat for eleven players.

Player	1	2	3	4	5	6	7	8	9	10	11
Waistline measurement (w)	89	100	87	96	94	83	81	83	84	97	98
Percentage body fat (p)	14	17	11	19	17	12	9	10	8	14	19

Research has shown that estimates for percentage body fat can be determined by using waistline measurements.

(a) Calculate the correlation coefficient r_{wp} for these data. (1 mark)

(b) Determine the equation of the least-squares line for these data. (1 mark)

(c) In the context of this question, interpret the slope of the line found in part (b). (2 marks)

(d) The residual plot shown below is for the first 10 players' data. Calculate the residual for player number 11 and plot this point on the graph. (2 marks)

(e) Comment on the appropriateness of fitting a linear model to the data. Justify your answer.

(2 marks)

(f) What percentage of the variation in the percentage body fat measurements is **unexplained** by the variation in the waistline measurements? (2 marks)

(g) Wayne is player number 12 and has a waistline measurement of 105 cm.

(i) Determine his predicted percentage of body fat. (1 mark)

(ii) Comment on the validity of the prediction and give a justification for your answer.

(2 marks)

(h) Player number 13 has a residual of –2.6. What information does this provide about the percentage body fat for this player? (2 marks)

Question 11 **(9 marks)**

Shari requires a loan of $325 000 for the purchase of a new house. She wishes to make two equally-spaced repayments of $700 each month.

Shari is offered a choice of two loan options for the first three years, both of which have interest calculated daily.
- Option 1 An introductory compound interest rate of 2.55% per annum for the first year which changes to 2.99% per annum for the next two years.
- Option 2 A compound interest rate of 2.85% per annum fixed for the first three years.

(a) Describe briefly the benefit of making two repayments of $700 each month instead of one repayment of $1400 at the end of each month. (1 mark)

(b) For Option 1, calculate

  (i) the loan balance at the end of the first year. (3 marks)

  (ii) the loan balance at the end of the third year. (2 marks)

(c) Determine which option gives the best result for Shari after three years and by how much. (3 marks)

Question 12 **(10 marks)**

Jessica wants to borrow $15 000 from her parents to purchase a car. They will be charging her compound interest at the rate of 4% per annum, with interest added yearly.

(a) Jessica is currently studying so she will not want to be making any regular repayments.

(i) Complete the table below to show the amount she will owe her parents at the end of each year. (2 marks)

Number of years (n)	0	1	2	3
Amount owing ($)	15 000			

(ii) Write a recursive rule to calculate the amount owing at the end of each year. (2 marks)

Jessica's parents are encouraging her to get a part-time job so that she can make repayments along the way. Jessica estimates that she will be able to earn enough money to pay off $2400 each year.

(b) If interest is charged yearly and she repays the $2400 at the end of each year, write a recursive rule to calculate the amount owing at the end of each year. (1 mark)

(c) If interest is charged monthly and she makes equal monthly repayments,

(i) write a recursive rule to calculate the amount owing at the end of each month. (1 mark)

(ii) calculate how many months it will take to repay the loan. (1 mark)

(iii) calculate the total amount Jessica would pay over the duration of the loan. (3 marks)

Question 13 **(9 marks)**

The graph below represents a road transport network from a warehouse at A to seven retail outlets B, C, D, E, F, G and H. The number on each edge represents the distance, in kilometres, along each road.
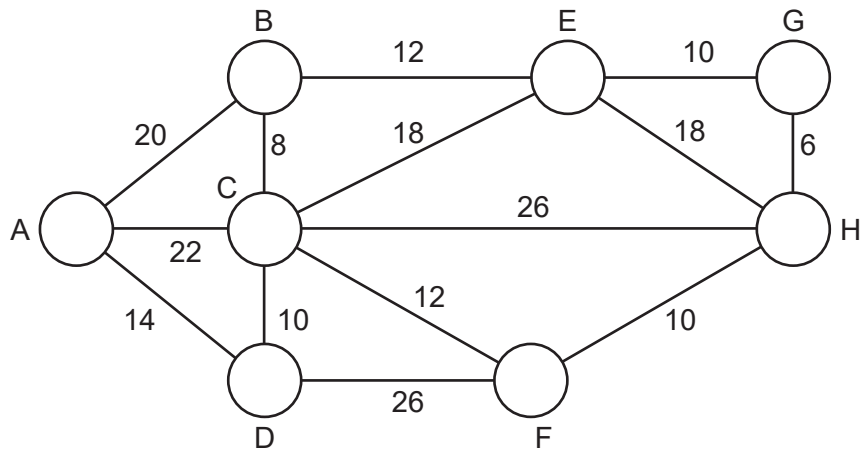

(a) Identify the shortest Hamiltonian path from the warehouse and state its length. (3 marks)

A special delivery must be made from the warehouse to retail outlet H.
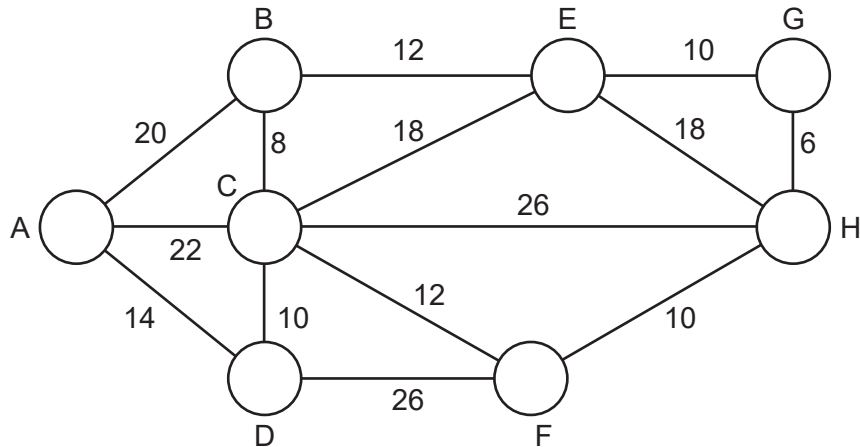
(b) Determine the shortest path and the distance travelled for this delivery. Working **must** appear on the network to show an appropriate method has been used. (3 marks)

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Road CH presently goes around what is now a dry salt lake. It is proposed that a direct road be constructed that will reduce the distance between retail outlets C and H.
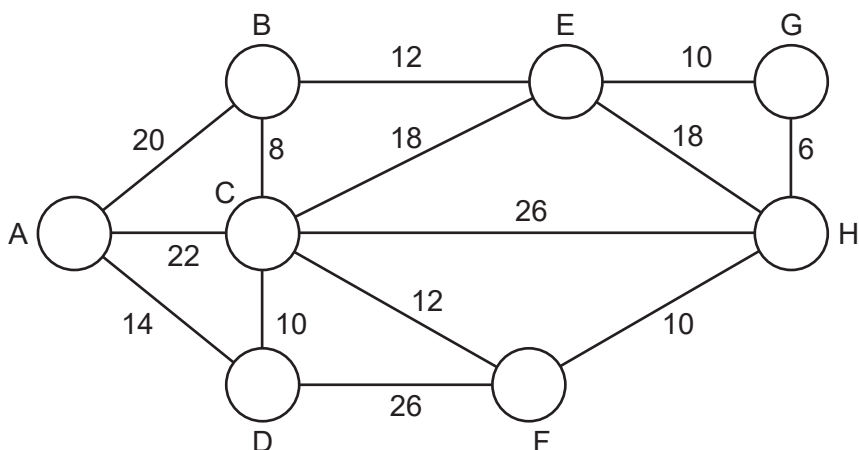
(c)   By how much can the direct road between C and H be reduced, so that the shortest path from the warehouse to H includes the direct road CH?   (3 marks)

Question 14   **(14 marks)**

The table below shows the number of sprinkler systems installed by a local reticulation business over the past four years.

Year	Season	n	Number of systems	Seasonal mean	Number of systems as a percentage of the seasonal mean	Seasonally adjusted figures
2017	Summer	1	**A**	14	71.4	10.4
	Autumn	2	18		**B**	15.7
	Winter	3	11		78.6	14.7
	Spring	4	17		121.4	14.7
2018	Summer	5	15	**C**	105.3	15.7
	Autumn	6	16		112.3	14.0
	Winter	7	11		77.2	14.7
	Spring	8	15		105.3	13.0
2019	Summer	9	13	11.75	110.6	13.6
	Autumn	10	12		102.1	10.5
	Winter	11	8		68.1	10.7
	Spring	12	14		119.1	12.1
2020	Summer	13	16	—	—	—
	Autumn	14	15		—	—

(a)   Calculate the value of **A**, **B** and **C**.   (3 marks)

(b)   Complete the table showing the seasonal index for each season.   (2 marks)

Season	Summer	Autumn	Winter	Spring
Seasonal index	95.8	114.3		

(c) Show how the seasonally adjusted figure of 13.6 for Summer 2019 was calculated.

(2 marks)

(d) During which season could more employees be given annual holidays with least disruption to sprinkler installations? Use mathematical evidence to support your answer.

(2 marks)

(e) Determine the least-squares line using the seasonally adjusted figures. (1 mark)

(f) Using your line from part (e), estimate the number of sprinkler systems that will be installed in Summer 2021. (2 marks)

(g) Comment on the long-term prospects of the business. (2 marks)

Question 15 **(8 marks)**

The graph below shows a network of sewage pipes. The numbers on the edges indicate the number of litres per minute that can flow along each pipe.
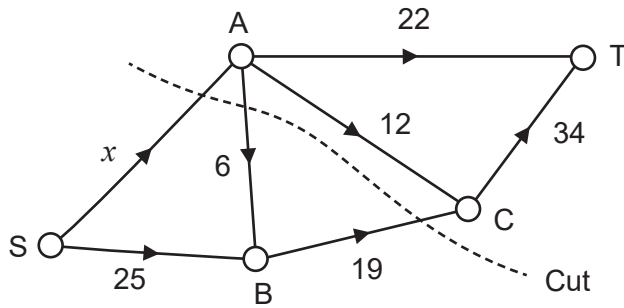
Diagram 1

(a) Show that the value of x is 38, given that the value of the cut is 57. (2 marks)

(b) Calculate the value of the maximum flow through the network. (2 marks)

(c) Indicate on **Diagram 2** below a possible flow along each pipe corresponding to the maximum flow calculated in part (b). (2 marks)
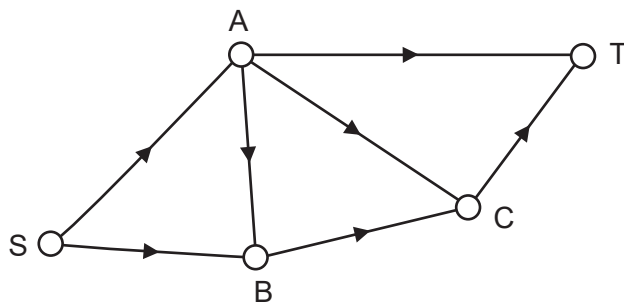
Diagram 2

See next page

Extra pipes, BD, DC and DT, are added to form a new system shown below. The capacities of the new pipes are indicated on the diagram. The original pipes have the same capacity as shown in **Diagram 1**.
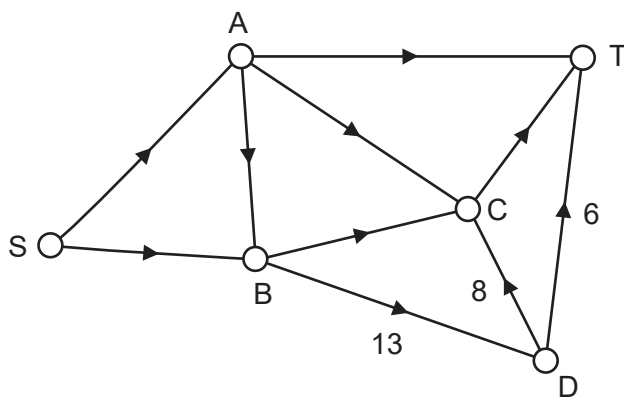
(d)   How will the addition of these pipes affect the new maximum flow through the system?

(2 marks)

Question 16 **(14 marks)**

Valeska and her sister Katrin are planning a small building project. The table below shows the required activities, together with the times taken (in days) and the immediate predecessors for each activity.

Activity	A	B	C	D	E	F	G	H
Time (days)	2	4	5	3	6	3	8	2
Immediate predecessors	–	–	A, B	B	C	C	D	D, F

(a) Complete the project network below, showing all activities and durations. (3 marks)
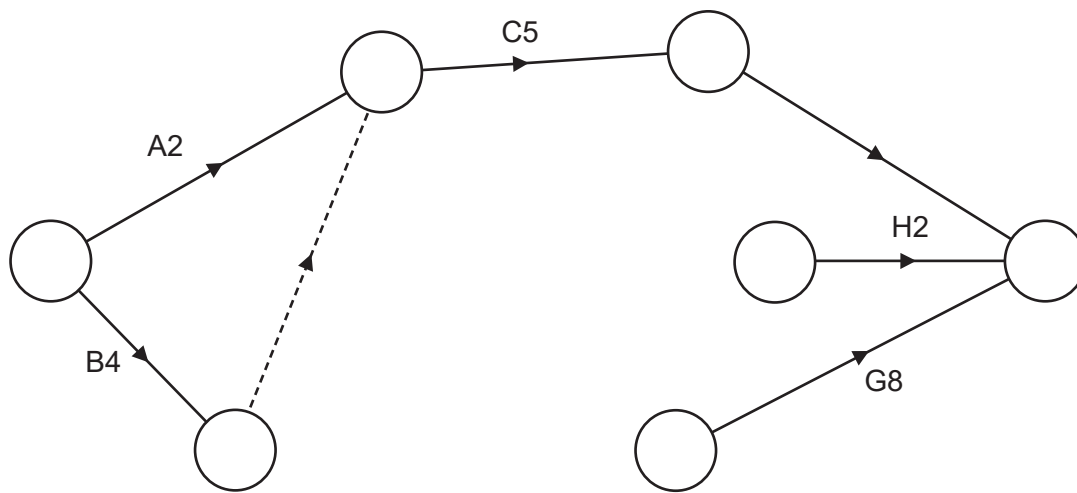
(b) Determine all critical activities and the minimum completion time for the project.
(3 marks)

(c) Calculate the float times for each of the non-critical activities. (3 marks)

(d)   If Activity H is delayed by three days, what effect will this have on the minimum completion time and the critical activities?   (2 marks)

(e)   Extra resources become available that can be used to shorten the duration of **one** of Activities B, E or F (on the original network) by one day. Which of these activities should be shortened and why?   (3 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____