2025/4684
Web version of 2024/81896

Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS APPLICATIONS

Calculator-free

ATAR course examination 2024

Marking key

Marking keys are an explicit statement about what the examining panel expect of candidates when they respond to particular examination items. They help ensure a consistent interpretation of the criteria that guide the awarding of marks.

Section One: Calculator-free **35% (54 Marks)**

Question 1 **(11 marks)**

A school is hosting a sports carnival to raise funds for charity. The first event is a 400 metre running race between a Year 8 student and a Year 12. The Year 12 student will start at the starting line and is expected to run at a speed of 8 metres per second. The Year 8 student will start 100 metres ahead of the starting line and is expected to run at a speed of 6 metres per second. Let n represent the number of seconds after the start of the race.

(a) (i) Write a recursive rule to model the total distance (in metres) the Year 12 student is from the starting line during the race. (2 marks)

Solution
$T_{n+1} = T_n + 8,\ T_0 = 0$
Specific behaviours
✓ states correct common difference
✓ states correct value for T_0

(ii) Deduce a rule for the n^{th} term to model the total distance (in metres) the Year 8 student is from the starting line during the race. (2 marks)

Solution
$T_n = 6n + 100$
Specific behaviours
✓ states correct coefficient of n
✓ correctly adds 100

(b) Determine how many metres the Year 8 student is ahead of the Year 12 student after 5 seconds. (2 marks)

Solution
Year 12: T_5 = 40 metres Year 8: T_5 = 130 metres Therefore the Year 8 student is 90 metres ahead
Specific behaviours
✓ determines correct T_5 values for both Year 8 and Year 12
✓ correctly determines the difference between the two values

After 30 seconds, the Year 8 student trips over. This results in a 5-second delay for the Year 8 student.

(c) Determine how much of a lead the Year 8 student will have after the 5-second delay. (3 marks)

Solution
Year 12: T_{35} = 280 metres Year 8: T_{30} = 280 metres Therefore the Year 8 student is equal with the Year 12 student
Specific behaviours
✓ determines T_{35} value for the Year 12 student ✓ determines T_{30} value for the Year 8 student ✓ correctly states it is equal after 35 seconds

(d) Determine who wins the race. Justify your answer. (2 marks)

Solution
Year 12: Time = 50 seconds Year 8: Time = 55 seconds Therefore the Year 12 student wins the race.
Specific behaviours
✓ determines correct time for both students ✓ states the Year 12 student wins the race

Question 2 **(9 marks)**

A computer repair shop recorded the number and types of complaints received from its customers regarding laptop computers with different purchase prices.

		Purchase price			
		Under $1000	**From $1000 to $1500**	**More than $1500**	**Total**
Complaints	**Battery failure**	50	50	16	**116**
	Overheating	25	**46**	**14**	85
	Update failures	**30**	70	12	112
	Viruses	45	**34**	38	117
	Total	150	200	80	430

(a) Complete the two-way table above. (3 marks)

Solution
see table above
Specific behaviours
✓ determines two correct values ✓ determines three correct values ✓ determines all correct values

(b) The manager of the computer repair shop believes that the purchase price of the laptop computers is being explained by the number and type of complaints.

(i) State why this belief is incorrect. (1 mark)

Solution
purchase price is set prior to the complaints; therefore, complaints cannot be the explanatory variable
Specific behaviours
✓ states purchase price is set
Accept other answers.

(ii) Identify the response variable for these data. (1 mark)

Solution
complaints
Specific behaviours
✓ identifies the correct response variable

(c) The incomplete two-way percentaged table is shown below.

		Purchase price			
		Under $1000	**From $1000 to $1500**	**More than $1500**	**Total**
Complaints	**Battery failure**	33.3	25	20	
	Overheating	16.7	**23**	17.5	
	Update failures	**20**	**35**	**15**	
	Viruses	**30**	**17**	**47.5**	
	Total	**100**	**100**	**100**	

(i) Complete the table above by using either row percentages **or** column percentages, as appropriate. (3 marks)

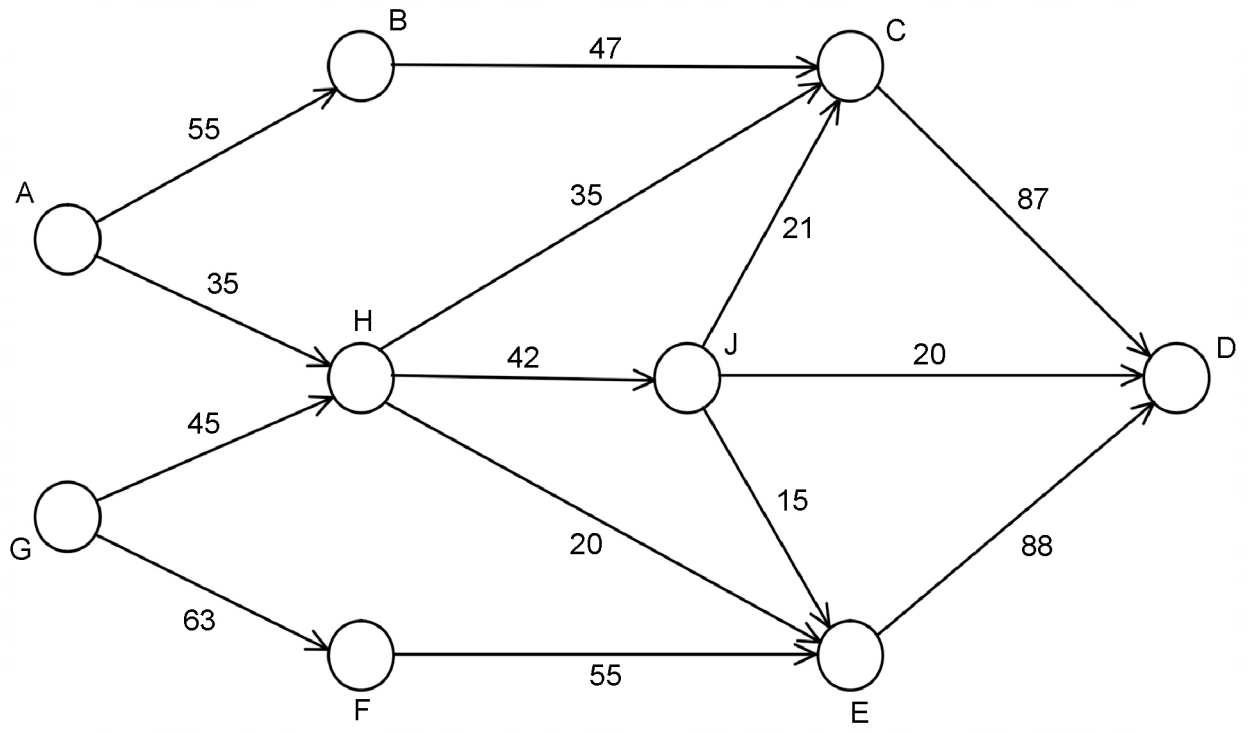
Solution
see table above
Specific behaviours
✓ determines column percentages are required ✓ determines three correct values ✓ determines all correct values

(ii) State an association that can be observed from the completed two-way percentaged table above. (1 mark)

Solution
as price increases, the percentage of battery failures decreases
Specific behaviours
✓ states correct association

Question 3 **(11 marks)**

An oil and gas company has two wells that produce natural gas. The gas is transported under pressure, through a network of pipes, to a refinery. Compressor stations are placed at intervals to ensure the gas remains pressurised for maximum flow through the pipes. The network below shows the capacity of the flow through each pipe in cubic metres per hour.



(a) Identify the source(s) and the sink(s). (2 marks)

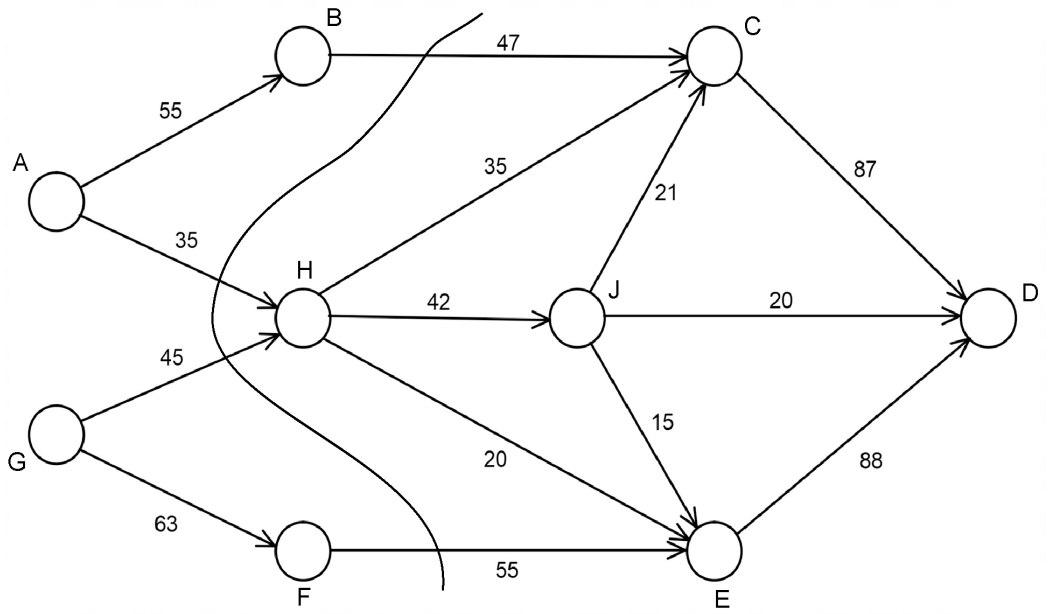
Solution
sources are A and G and the sink is D
Specific behaviours
✓ correctly identifies both sources ✓ correctly identifies the sink

(b) Determine the maximum flow of gas through the network. Show systematic workings. (4 marks)

Solution
One possibility: ABCD - 47 AHCD - 35 GHJCD - 5 GHJD - 20 GHED - 20 GFED - 55 Maximum flow = 182 m^3/hr
Specific behaviours
✓ shows systematic working ✓ shows four correct paths ✓ shows all correct paths ✓ states correct maximum flow
Accept other relevant answers.

(c) Draw the minimum cut that corresponds to the maximum flow on the network above.
(1 mark)

Solution

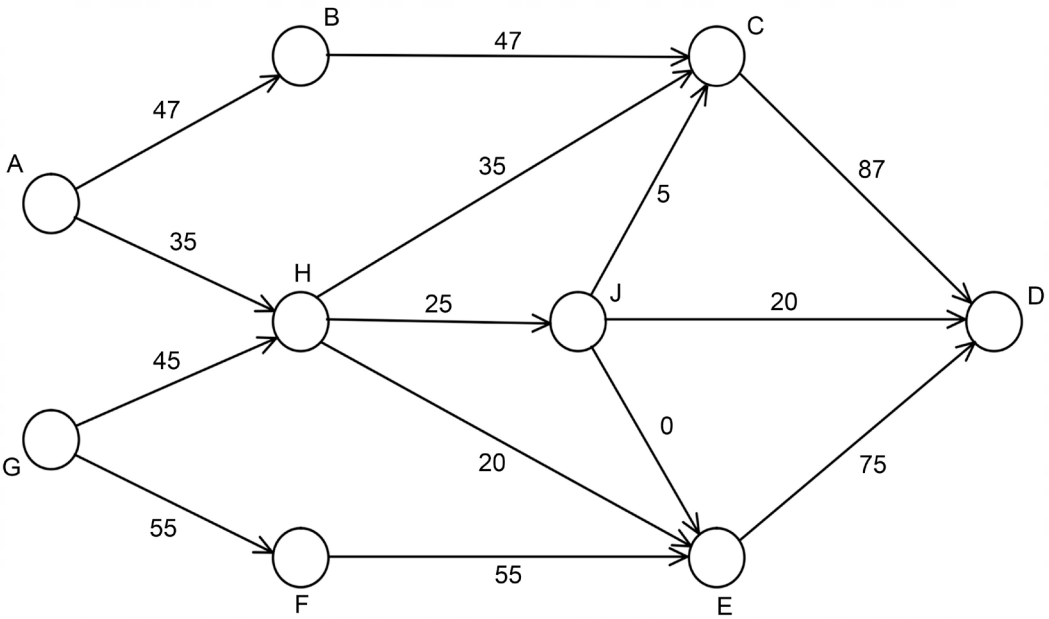


Specific behaviours

✓ draws correct cut

(d) Indicate on the network below the flow through each pipe corresponding to the maximum flow determined in part (b). (2 marks)

Solution



Specific behaviours

✓ indicates 10 correct flows
✓ indicates all correct flows

Question 3 (continued)

(e) As the refinery is not working at full capacity, the company plans to increase the capacity of the flow through **one** section of pipe (AB or FE) from 55 to 65 cubic metres per hour. Which section should they choose for the best improvement in total gas flow and how will this change the maximum flow? (2 marks)

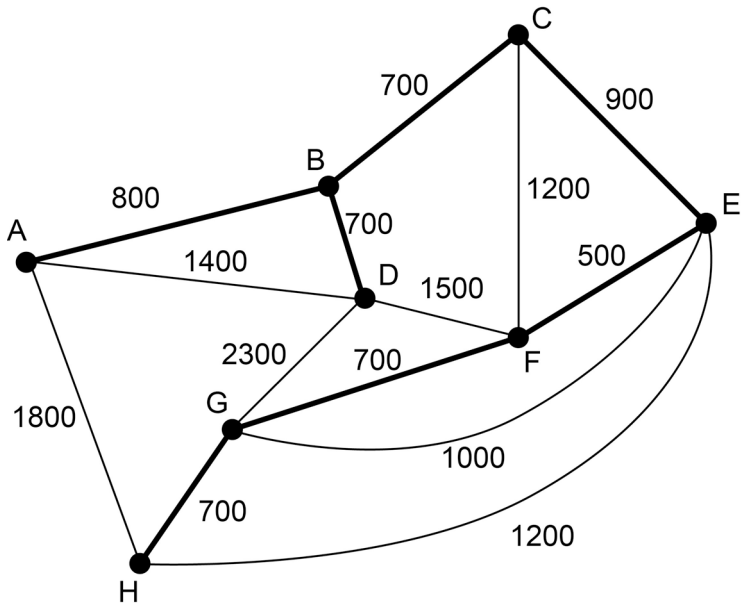
Solution
pipe section FE increase of 8 m^3/hr (new maximum flow is 190)
Specific behaviours
✓ identifies correct pipe ✓ determines correct increase

Question 4 **(13 marks)**

A council in the South West region of Western Australia has erected a number of water stations for hikers travelling through the forest on a walking trail. Over the years, the council has increased the number of water stations, with direct connecting paths to some other water stations, where the ground makes this feasible.

The table and graph below show the direct distances (in metres), between water stations of the paths that have been constructed.

	A	B	C	D	E	F	G	H
A		800		1400				1800
B	800		700	700				
C		700			900	1200		
D	1400	700				1500	2300	
E			900			500	1000	1200
F			1200	1500	500		700	
G				2300	1000	700		700
H	1800				1200		700	



(a) The council has decided to upgrade some of the paths between water stations. It aims to upgrade some of the paths to create the shortest distance connecting all water stations, using the upgraded path(s) for travel. Using Prim's algorithm, determine which paths should be upgraded to ensure the distance is minimised. (3 marks)

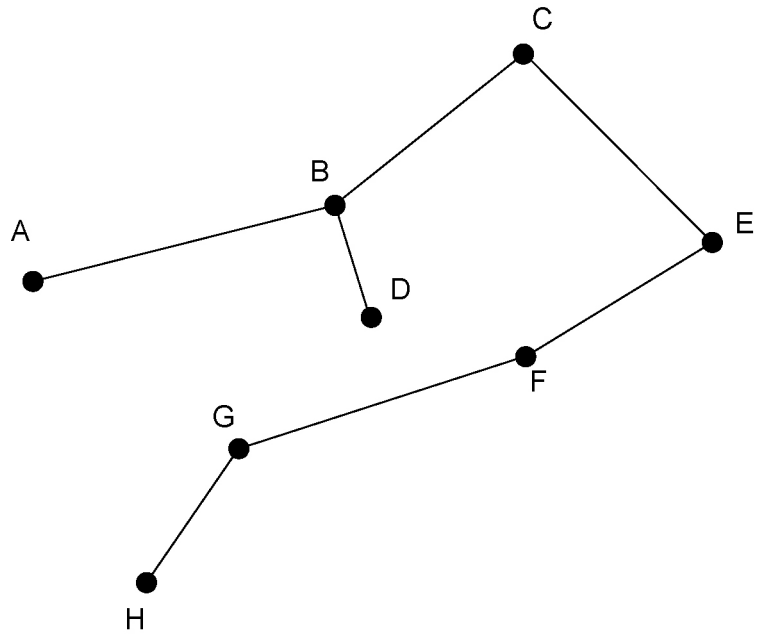
Solution
see table or diagram above for working upgrade AB, BC, BD, CE, EF, FG, GH
Specific behaviours
✓ shows systematic working (on diagram or table) ✓ determines five correct paths ✓ determines the seven correct paths

Question 4 (continued)

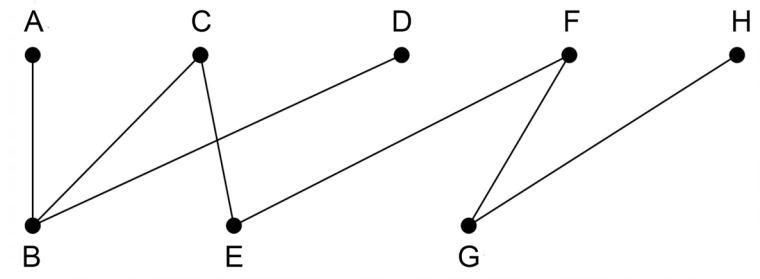
(b) Upgrading the paths is costed at \$30 per metre. If the budget is \$180 000, will this be sufficient to upgrade the appropriate paths? Justify your answer. (2 marks)

Solution
total length of paths is 5000 metres, 5000 × 30 = \$150 000 therefore under budget
Specific behaviours
✓ determines total length ✓ states that it is under budget

(c) (i) Draw the network identified by Prim's algorithm. (1 mark)

Solution

Specific behaviours
✓ draws correct network

(ii) Show this network is bipartite by drawing a diagram. (2 marks)

Solution

Specific behaviours
✓ shows two distinct groups of vertices ✓ shows correct connections

(d) Each day, a park ranger must check every path to ensure there are no obstacles interfering with a hiker's track. Can this be done using a semi-Eulerian or Eulerian trail? Justify your answer. (2 marks)

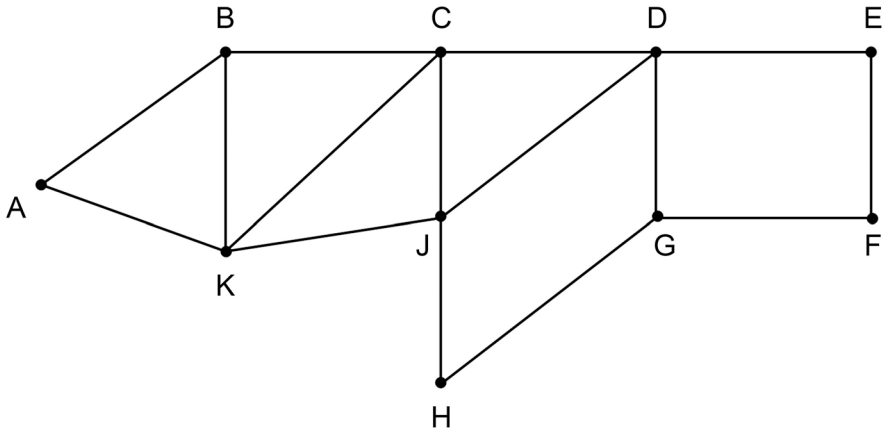
Solution
Neither. A, B, C, and H all have odd vertices
Specific behaviours
✓ correctly states neither is possible ✓ provides valid justification

(e) A park ranger is stopped at watering station G and receives a message to travel to watering station B to assist an injured hiker. Calculate the shortest distance the ranger must travel, and the pathway, to get from G to B. Working must be shown. (3 marks)

Solution
Path GDB, length 3000 metres. Path GFDB, length 2900 metres. Path GFECB, length 2800 metres. Path GFCB, length 2600 metres. Path GECB, length 2600 metres. Therefore, path GFCB or path GECB is the shortest distance of 2600 metres
Specific behaviours
✓ shows systematic working ✓ state a correct path ✓ states correct length

Question 5 **(10 marks)**

A volunteer tour guide has identified several historical places of interest around the Perth central business district. The network shown below has a place of interest at each vertex and the best possible connections between them.



(a) (i) Verify Euler's formula holds true for the above graph. (2 marks)

Solution
v = 10, e = 15, f = 7 Euler's formula: v + f – e = 2 10 + 7 − 15 = 2 Therefore Euler's formula holds true for the above graph
Specific behaviours
✓ states correct values for v, e, f ✓ verifies Euler's formula

(ii) Given that part (a)(i) shows the above graph is planar, state what else this means about the graph. (1 mark)

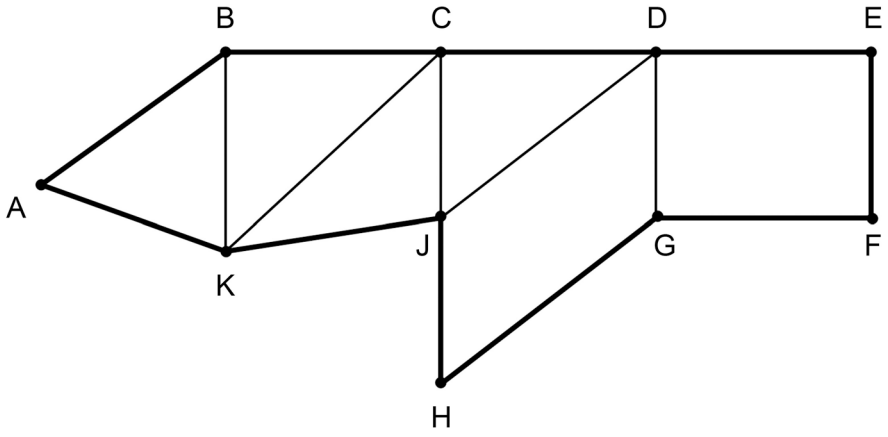
Solution
the graph is connected
Specific behaviours
✓ correctly states the graph is connected

The tour guide wants to offer two types of tours: one that just focuses on visiting each place of interest and the other that visits each place of interest and travels along every connection in order to show more of what Perth has to offer.

(b) Determine whether the tour is able to visit each place of interest and travel along every connection with an Eulerian or semi-Eulerian trail. Justify your response. (2 marks)

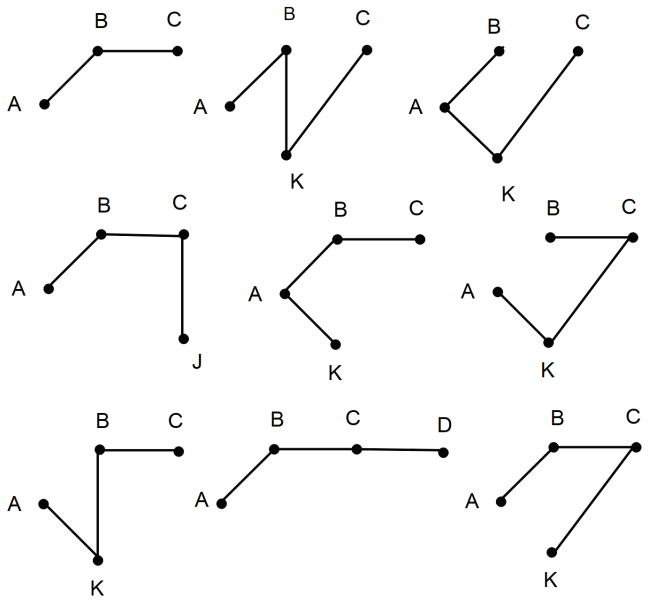
Solution
As the graph has 2 odd vertices and the rest are even, it can only be semi-Eulerian. Therefore the tour is able to visit each place of interest travelling along each connection
Specific behaviours
✓ identifies correct number of odd and even vertices ✓ determines the graph is semi-Eulerian and states the tour is able to visit each place of interest travelling along each connection

(c) Show that the graph below is Hamiltonian by highlighting a possible tour that visits every place of interest. (2 marks)



Solution
it contains a closed path that includes every vertex once only
Specific behaviours
✓ highlights a possible tour that visits every place of interest once only ✓ identifies the graph contains a closed path

(d) Determine the number of subgraphs with at most three edges, that contain all of the vertices A, B and C. Each subgraph must be a continuous path. Show a neat sketch of each subgraph. (3 marks)

Solution
 Nine subgraphs
Specific behaviours
✓ draws three correct subgraphs ✓ draws six correct subgraphs ✓ draws nine correct subgraphs

Published by the School Curriculum and Standards Authority of Western Australia
303 Sevenoaks Street
CANNINGTON WA 6107