Government of **Western Australia**
School Curriculum and Standards Authority

ATAR course examination, 2024

Question/Answer booklet

MATHEMATICS SPECIALIST

Section One: Calculator-free

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

WA student number: In figures

In words

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

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Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	47	35
Section Two: Calculator-assumed	10	10	100	85	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2024: Part II Examinations*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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See next page

Section One: Calculator-free **35% (47 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

Question 1 **(3 marks)**

The complex number $z = r\,cis\,\theta = 3 + bi$, where $\tan\theta = \sqrt{2}$. Determine the exact values for r and b.

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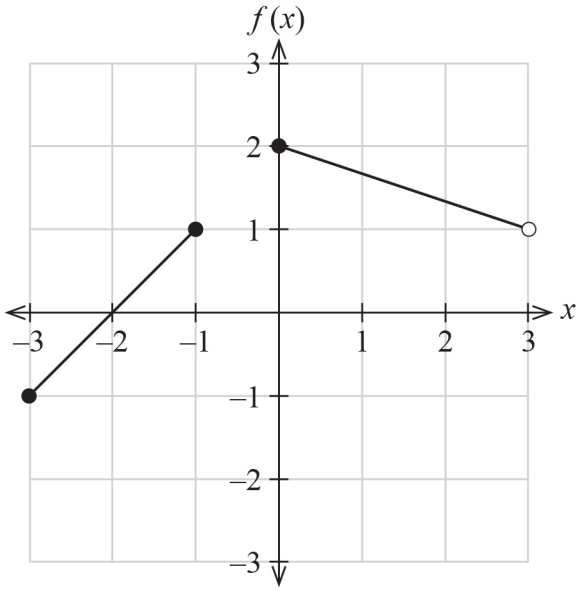
See next page

Question 2 **(5 marks)**

The graph of $y = f(x)$ is shown below.



(a) State the domain for $g(x) = \sqrt{f(x) - 1}$. Justify your answer. (3 marks)

(b) State the range for function $h(x) = \dfrac{1}{f(x)}$. (2 marks)

See next page

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Question 3 **(4 marks)**

Using the substitution $u = 1 - x$, evaluate exactly the definite integral $\int_0^1 15x\sqrt{1-x}\,dx$.

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See next page

Question 4 **(6 marks)**

(a) Given that $\frac{a}{x+1} + \frac{b}{(x+1)^2} = \frac{5x+3}{(x+1)^2}$, determine the values for a and b. (2 marks)

(b) Hence determine $\int \frac{5x+3}{(x+1)^2}\, dx$. (4 marks)

See next page

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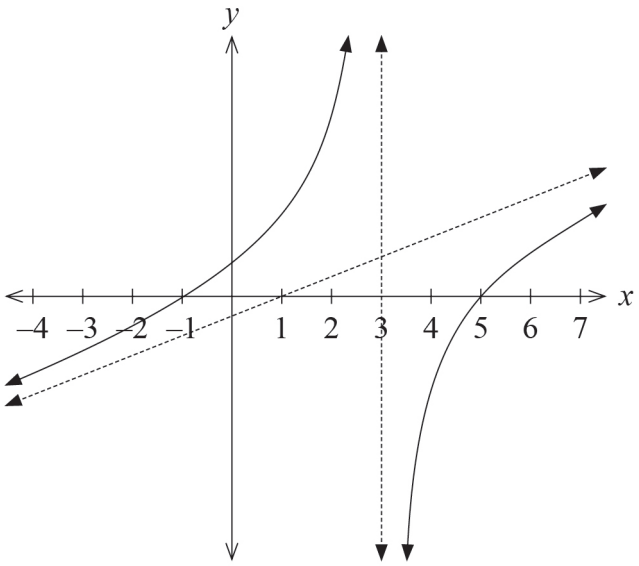
Question 5 **(5 marks)**

The graph of function $f(x) = \dfrac{(x+a)(x-b)}{2(x-c)}$ is shown below. The constants a, b and c are positive.



(a) Determine the values for a, b and c. Justify your answer for c. (3 marks)

The inclined asymptote for the graph of $y = f(x)$ is shown.

(b) Determine the equation for the inclined asymptote. (2 marks)

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Question 6 **(6 marks)**

(a) Solve the system of equations: (3 marks)

$$x + y + z = 4$$
$$x - y - z = 2$$
$$2x - 3y + z = 11.$$

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See next page

The third equation in part (a) on page 8 is changed to $2x - ky + z = 11$ where k is a real constant. The first two equations remain unchanged.

Ryan decided to solve this changed system of equations and obtained correctly the statement $(k+1)y = -4$.

(b) Determine the value of the constant k so that the changed system of equations does not have a unique solution. (1 mark)

(c) For the value of k determined from part (b), state the geometric interpretation of the solution of the three simultaneous equations. (2 marks)

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See next page

Question 7 **(5 marks)**

Consider the quartic polynomial $R(z) = z^4 - 6z^3 + 17z^2 - 22z + 14$ and $P(z) = z^2 - 2z + 2$ where $R(z) = P(z)(z^2 + az + b)$.

(a) Show that $(z - 1 - i)$ is a factor of $P(z)$. (2 marks)

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See next page

(b) Solve the equation $R(z) = 0$. (3 marks)

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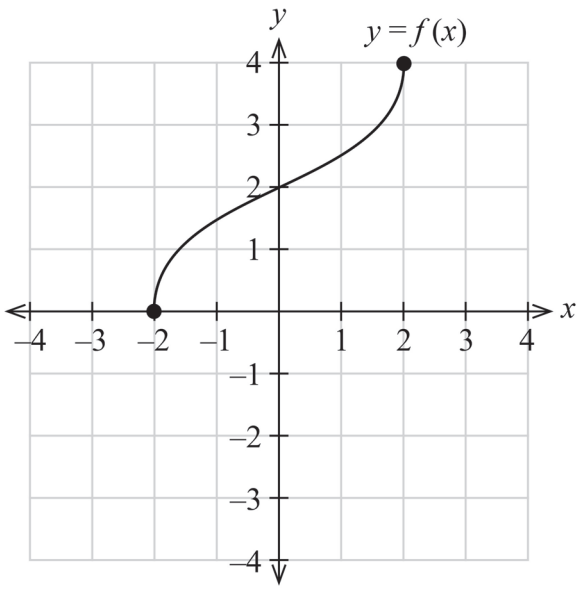
Question 8 **(13 marks)**

The statement $\cos\left(\frac{\pi}{3}\right)=\frac{1}{2}$ can be written as $\frac{\pi}{3}=\cos^{-1}\left(\frac{1}{2}\right)$, where $\cos^{-1}$ represents the inverse cosine function.

The graph of $y=f(x)=\frac{4}{\pi}\cos^{-1}\left(-\frac{x}{2}\right)$ for $-2\le x\le 2$ is shown below.



(a) Explain why $y=f^{-1}(x)$ exists. (1 mark)

(b) Determine the defining rule for $y=f^{-1}(x)$. (2 marks)

See next page

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(c) Sketch the graph of $y = f^{-1}(x)$ on the axes on page 12. (2 marks)

A spare grid is provided at the end of this Question/Answer booklet. If you need to use it, cross out this attempt and indicate that you have redrawn it on the spare grid.

(d) If $y = \frac{4}{\pi}\cos^{-1}\left(-\frac{x}{2}\right)$, using implicit differentiation, show that $\frac{dy}{dx} = \frac{4}{\pi\sqrt{4-x^2}}$. (5 marks)

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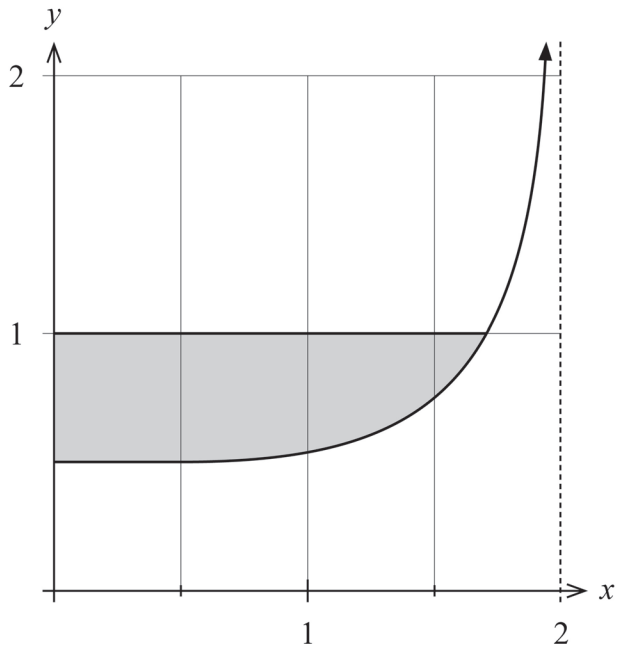
Question 8 (continued)

The graph of $y = \dfrac{1}{\sqrt{4 - x^2}}$ is shown below for $0 \leq x < 2$.



The shaded region is bounded by the curve $y = \dfrac{1}{\sqrt{4 - x^2}}$, the line $y = 1$ and the y axis.

(e) Determine the exact area of the shaded region. (3 marks)

End of section

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Supplementary page

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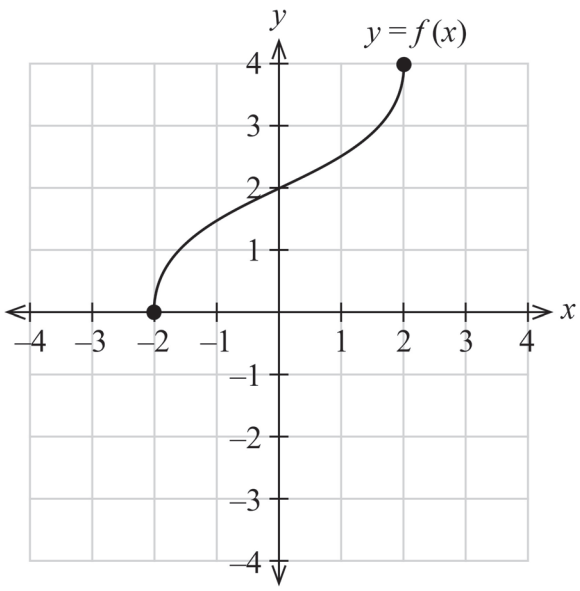
Spare grid for Question 8(c)



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