Government of **Western Australia**
**School Curriculum and Standards Authority**

# Western Australian Certificate of Education
# ATAR course examination, 2016

## Question/Answer booklet

# MATHEMATICS APPLICATIONS

## Section Two: Calculator-assumed

Place one of your candidate identification labels in this box.
Ensure the label is straight and within the lines of this box.

Student number: In figures

In words

## Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

## Materials required/recommended for this section

***To be provided by the supervisor***
This Question/Answer booklet
Formula sheet (retained from Section One)

***To be provided by the candidate***
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.



Ref: 16-047
MAA-S2

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|---|---|---|---|---|---|
| Section One: Calculator-free | 6 | 6 | 50 | 50 | 35 |
| Section Two: Calculator-assumed | 10 | 10 | 100 | 100 | 65 |
| | | | | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specified to a particular question.

4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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**Section Two: Calculator-assumed** **65% (100 Marks)**

This section has **10** questions. Answer **all** questions. Write your answers in the spaces provided.

Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.

Working time: 100 minutes.

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**Question 7** **(9 marks)**

Julie buys a car with a purchase price of $13 000. However, she has been told to expect the car to depreciate in value. The value of the car after $n$ years can be determined by using the recursive rule.

$$T_{n+1} = 0.85T_n,\ \ T_0 = 13\ 000$$

(a) Complete the table below to show the value of the car at the end of each year, to the nearest dollar. (2 marks)

| $n$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Value of car after $n$ years ($) | 13 000 | | | |

(b) Use the information above to determine the rate of depreciation of Julie's car per year. (1 mark)

(c) Determine a rule for the $n^{\text{th}}$ term of the sequence of values found in part (a). (2 marks)

(d) Determine the value of Julie's car after eight years, correct to the nearest dollar. (2 marks)

(e) Julie decides that she will sell her car at the end of the year in which its value drops to half of the purchase price. After how many years should she sell her car? (2 marks)

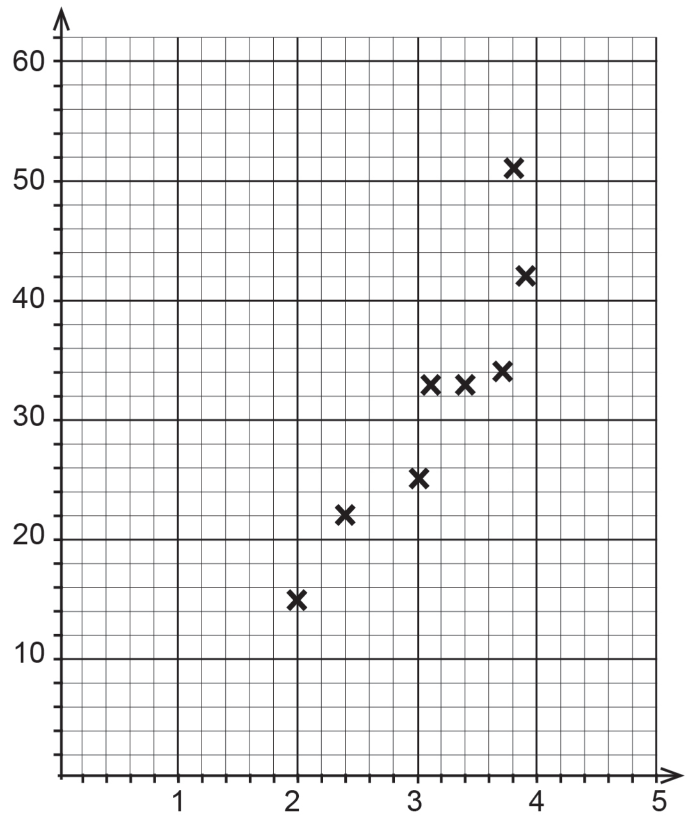
**Question 8** **(17 marks)**

An experiment was conducted to determine whether there was any relationship between the maximum tidal current, in centimetres per second, and the tidal range, in metres, at a particular marine location. (The tidal range is the difference between the height of high tide and the height of low tide.) Readings were taken over a period of 12 days and the results are shown in the following table.

| **Tidal range** | 2.0 | 2.4 | 3.0 | 3.1 | 3.4 | 3.7 | 3.8 | 3.9 | 4.0 | 4.5 | 4.6 | 4.9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Maximum tidal current** | 15.2 | 22.0 | 25.2 | 33.0 | 33.1 | 34.2 | 51.0 | 42.3 | 45.0 | 50.7 | 61.0 | 59.2 |

(a) State the explanatory variable. (1 mark)

(b) Complete the scatterplot below by plotting the last four data points and labelling the horizontal axis and the vertical axis clearly. (2 marks)




(c) Calculate the correlation coefficient for the data, and comment briefly on your answer with reference to the appearance of the scatterplot in part (b). (2 marks)

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(d) (i) Determine the equation for the least-squares line that models these data. State the slope and vertical-intercept correct to one decimal place. (2 marks)

(ii) Draw this line on the scatterplot in part (b) by showing two calculated points on the graph. (2 marks)

(iii) Interpret the slope of the least-squares line. (2 marks)

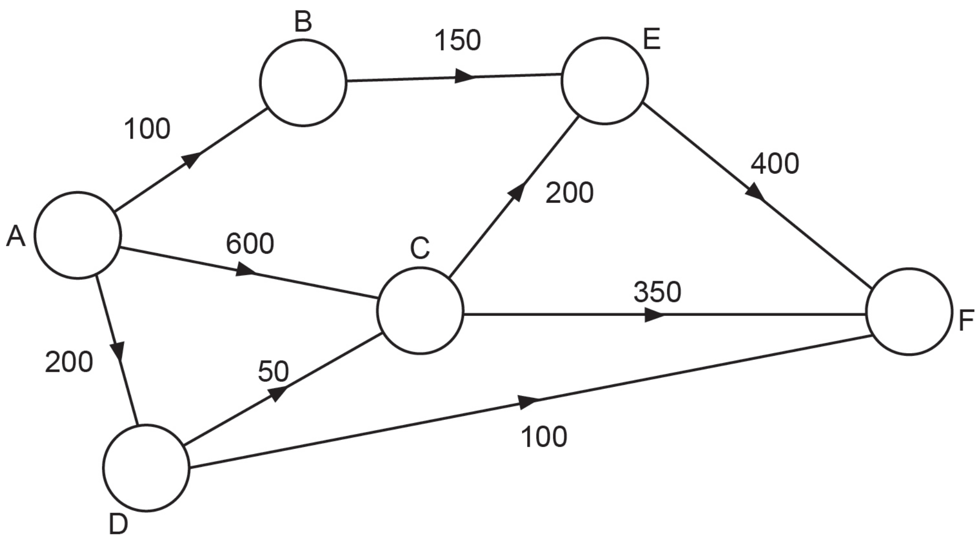
(e) Calculate the coefficient of determination and interpret it. (2 marks)

(f) (i) Estimate the maximum tidal current on a day when the tidal range is 4.2 m and comment on the reliability of this estimate. (3 marks)

(ii) It is suggested that the equation found in part (d)(i) could be used to predict the maximum tidal current on a day when the tidal range is 15 m. Comment briefly on the validity of this suggestion. (1 mark)

**Question 9** **(8 marks)**

The network below shows the maximum rate of water flow (in litres per minute) through a system of water pipes from a source at A.




(a) What is the maximum amount of water that could be delivered to F, in litres per minute? (List each path used and the corresponding flow). (3 marks)

(b) Verify the maximum flow obtained in part (a) by showing a minimum cut on the given network. (1 mark)



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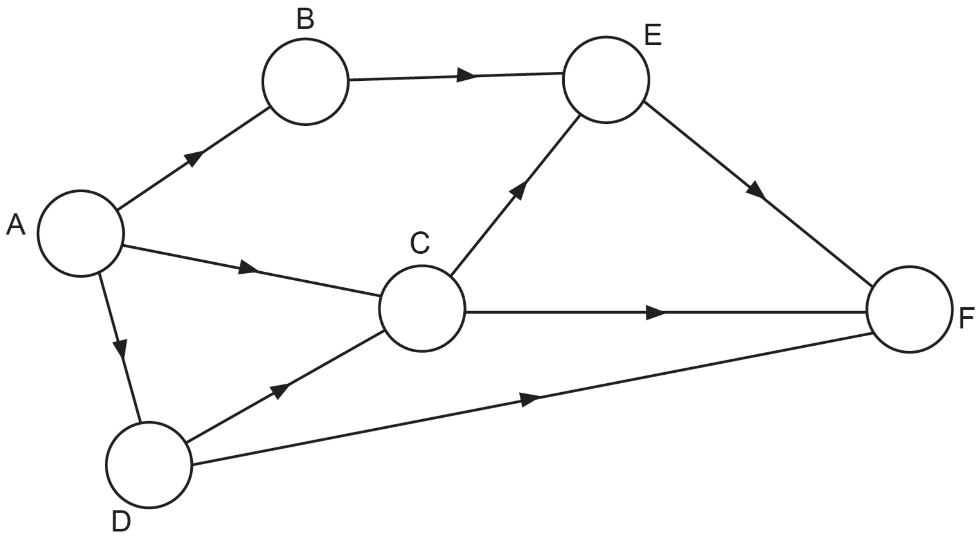
(c) Relabel the network below, showing the flow you would direct along each pipe in order to achieve the maximum flow found in part (a) to point F. (1 mark)




(d) When the maximum flow occurs from A to F, how much of the water, in litres per minute, passes through C? (1 mark)

(e) The water flow through C, as calculated in part (d), is reduced to a maximum of 480 litres per minute. In order to maintain the same maximum flow as that obtained in part (a), the capacity of a single pipe (arc) is to be increased by the least amount. Which pipe should be chosen, and by how much should its capacity be increased? (2 marks)

**Question 10** **(11 marks)**

A school canteen manager recorded the number of ice-creams sold for three weeks. The data are recorded in the table below, together with some calculations.

| | Sales day ($d$) | Ice-cream sales | Weekly mean | Percentage of weekly mean |
|---|---|---|---|---|
| Monday | 1 | 210 | $B$ | 132.9% |
| Tuesday | 2 | 230 | | 145.6% |
| Wednesday | 3 | 100 | | 63.3% |
| Thursday | 4 | 90 | | 57.0% |
| Friday | 5 | 160 | | 101.3% |
| Monday | 6 | 190 | 148 | 128.4% |
| Tuesday | 7 | 230 | | 155.4% |
| Wednesday | 8 | 90 | | 60.8% |
| Thursday | 9 | 80 | | 54.1% |
| Friday | 10 | 150 | | 101.4% |
| Monday | 11 | 180 | 142 | 126.8% |
| Tuesday | 12 | 220 | | 154.9% |
| Wednesday | 13 | $A$ | | $C$ |
| Thursday | 14 | 70 | | 49.3% |
| Friday | 15 | 150 | | 105.6% |

(a) Determine the values of $A$, $B$ and $C$, giving the value of $C$ correct to one decimal place. (4 marks)

(b) (i) Use the average percentage method to complete the table below by calculating the seasonal index for Wednesday. (1 mark)

| Day | Seasonal index |
|---|---|
| Monday | 129.4% = 1.294 |
| Tuesday | 152.0% = 1.520 |
| Wednesday | |
| Thursday | 56.8% = 0.568 |
| Friday | 102.8% = 1.028 |

(ii) Use the seasonal index to determine the deseasonalised number of ice-cream sales for Tuesday of Week Three, correct to the nearest 10. (2 marks)

(c) The equation of the least-squares line used to forecast the deseasonalised number of ice-cream sales is

$$\text{deseasonalised number of ice-creams} = -1.695d + 161.16.$$

(i) Describe the trend in the number of ice-cream sales over time. (1 mark)

(ii) Predict the **actual** number of ice-cream sales for Friday of Week Four. (3 marks)

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**Question 11** **(10 marks)**

The data in the table below, taken from those surveyed by the Australian Bureau of Statistics, show estimates for the number of persons 15 years and over who participated in sport and physical recreation in Western Australia, Tasmania and Victoria.

**Participation in Sport and Physical Recreation, 2013–14**

| **Persons Participating** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Western Australia** | **15–17 years** | **18–24 years** | **25–34 years** | **35–44 years** | **45–54 years** | **55–64 years** | **65 years+** | **Total** |
| Males ('000) | 43.3 | 80.9 | 149.1 | 113.7 | 116.6 | 85.6 | 74.0 | 663.2 |
| % of males | 71.6 | 67.6 | 70.9 | 66.4 | 68.2 | 63.8 | 51.4 | 65.6 |
| Females ('000) | 30.3 | 84.3 | 131.8 | 111.4 | 107.8 | 82.8 | 76.5 | $A$ |
| % of females | 61.6 | 70.6 | 67.3 | 61.5 | 64.2 | 60.7 | 48.1 | 61.9 |
| Total ('000) | 73.6 | 165.2 | 280.9 | $B$ | 224.4 | 168.4 | 150.5 | 1288.1 |
| | | | | | | | | |
| **Tasmania** | **15–17 years** | **18–24 years** | **25–34 years** | **35–44 years** | **45–54 years** | **55–64 years** | **65 years+** | **Total** |
| Males ('000) | 7.6 | 16.5 | 23.2 | 23.6 | 25.4 | 19.5 | 23.5 | 139.3 |
| % of males | 73.4 | 71.4 | 81.3 | 77.7 | 72.7 | 55.7 | 57.4 | 68.5 |
| Females ('000) | 7.8 | 14.1 | 22.0 | 22.3 | 27.2 | 22.0 | 20.7 | 136.1 |
| % of females | 79.0 | 69.0 | 74.3 | 70.0 | 74.3 | 62.9 | 46.7 | 65.5 |
| Total ('000) | 15.4 | 30.6 | 45.2 | 45.9 | 52.6 | 41.5 | 44.2 | 275.4 |
| | | | | | | | | |
| **Victoria** | **15–17 years** | **18–24 years** | **25–34 years** | **35–44 years** | **45–54 years** | **55–64 years** | **65 years+** | **Total** |
| Males ('000) | 74.5 | 202.3 | 267.9 | 268.7 | 218.2 | 192.6 | 164.1 | 1388.3 |
| % of males | 69.4 | 65.1 | 62.4 | 67.9 | 59.7 | 60.1 | 44.0 | 61.7 |
| Females ('000) | 79.9 | 159.7 | 276.3 | 296.7 | 240.5 | 184.6 | 211.9 | 1449.6 |
| % of females | 68.4 | 62.5 | 63.7 | 71.9 | 62.6 | 55.8 | 49.8 | 61.4 |
| Total ('000) | 154.4 | 362.0 | 544.2 | 565.4 | 458.7 | 377.2 | 376.0 | 2837.9 |

Use the information in the table to answer the following questions.

(a) Determine the values of $A$ and $B$ for the Western Australian data. (2 marks)

(b) Which State, age and gender category had the highest rate of participation in sport and physical recreation? (3 marks)

(c) Which State had a higher percentage of females than males participating in sport and physical recreation in the *35–44 years* category? (1 mark)

(d) Compare and comment on the participation rates in the *55–64 years* category with those in the younger age groups. (2 marks)

(e) Determine the total number of females in Victoria who were surveyed. (2 marks)

**Question 12** **(11 marks)**

Thomas has borrowed \$16 000 from a bank at a reducible interest rate of 18% per annum with interest accrued and repayments made monthly. Standard repayments are set at \$500 per month.

The table below shows the progress of the loan for the first six months. All values have been rounded to the nearest cent.

| Month | Amount owing at beginning of month | Interest for the month | Repayment | Amount owing at end of month |
|---|---|---|---|---|
| 1 | 16 000.00 | 240.00 | 500.00 | 15 740.00 |
| 2 | 15 740.00 | 236.10 | 500.00 | 15 476.10 |
| 3 | 15 476.10 | 232.14 | 500.00 | 15 208.24 |
| 4 | 15 208.24 | 228.13 | 500.00 | 14 936.37 |
| 5 | 14 936.37 | 224.04 | 500.00 | 14 660.41 |
| 6 | 14 660.41 | $A$ | 500.00 | $B$ |

(a) What is the monthly interest rate? (1 mark)

(b) Determine the values of $A$ and $B$. (2 marks)

(c) Determine the length of time it will take Thomas to pay off the loan. (1 mark)

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(d) Determine the total amount Thomas pays over the duration of the loan. (3 marks)

(e) The bank suggests that Thomas need only make repayments of $240 per month. Describe how this would affect the length of time and total amount he pays over the duration of the loan. (2 marks)

(f) After listening to advice, Thomas decides that he wants to pay off the loan completely in two years, making equal payments each month over that time. Determine the amount of each repayment he will need to make in order to make this happen (correct to the nearest cent). (2 marks)

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**Question 13** **(6 marks)**

Simon has $5000 that he wants to invest for a period of time without touching it.

(a) If he chooses to invest this money in an account earning compound interest at the rate of 6.5% per annum, determine the:

(i) value of his investment after three years, if interest is paid annually. (1 mark)

(ii) time required for him to double his investment, if interest is paid monthly. (2 marks)

(b) Simon is currently deciding between two options and wishes to compare them.

Option A: Invest the $5000 in an account earning compound interest at the rate of 5.5% per annum, with interest paid monthly.

Option B: Invest the $5000 in an account earning compound interest at the rate of 5.4% per annum, with interest paid daily.

He decides to calculate the effective annual rate of interest for each option, in order to compare the possible investments. He determines that Option A has an effective annual rate of interest of 5.64%, correct to two decimal places.

Calculate the effective annual rate of interest for Option B, correct to two decimal places, and hence decide on the better option for Simon. (3 marks)

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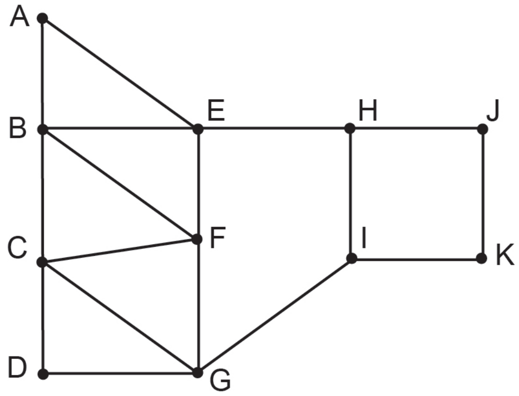
**Question 14** **(7 marks)**

Therese, a mathematics student at Trinity College, Dublin was employed as a guide for a cultural tour of Dublin. She decided to use graph theory to plan the walking tour.

Below is a network she constructed in which the:
- vertices represent the points of interest to be visited, and
- edges represent the most direct route between adjacent vertices.

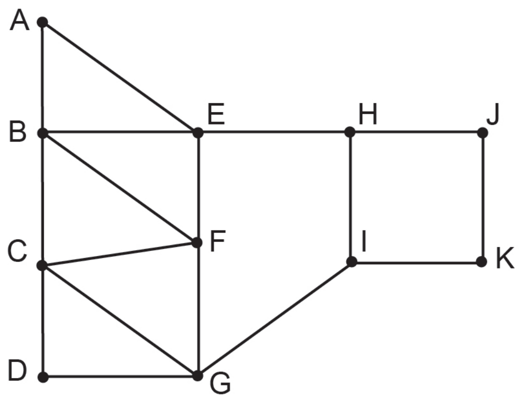



(a) Use Euler's formula to verify the network is connected. (2 marks)

(b) Therese planned to take the group on a closed walk. Explain the meaning of a closed walk. (1 mark)

(c) She also stated that the walk would qualify as a Hamiltonian cycle. State the **two** properties that makes the walk a Hamiltonian cycle. (2 marks)

(d) Given that the walk started at G (Trinity College, Dublin), mark the Hamiltonian cycle on the network below. (2 marks)

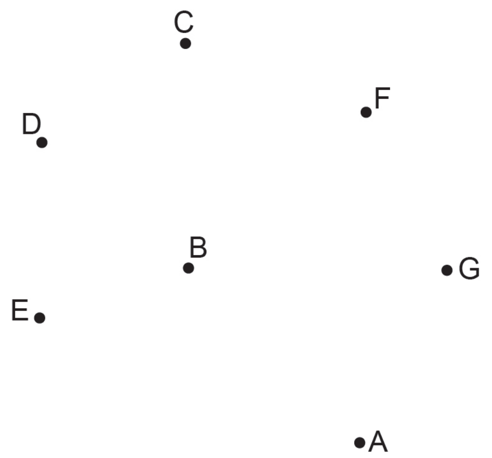


**Question 15** **(8 marks)**

An express bus service runs between seven adjacent shopping centres in the city. Below is an adjacency matrix of the seven shopping centres, A to G.

$$\begin{array}{c} \\ \mathbf{A} \\ \mathbf{B} \\ \mathbf{C} \\ \mathbf{D} \\ \mathbf{E} \\ \mathbf{F} \\ \mathbf{G} \end{array}
\begin{array}{c} \begin{array}{ccccccc} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} & \mathbf{E} & \mathbf{F} & \mathbf{G} \end{array} \\
\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

(a) Draw the network diagram associated with the adjacency matrix, assuming the arcs are undirected. (3 marks)



(b) The buses only run between adjacent shopping centres. However, a passenger can buy a multi-stage ticket at any shopping centre. A one-stage ticket means a passenger can travel from one shopping centre to an adjacent shopping centre, such as:

- $A \rightarrow B$ or $A \rightarrow E$ etc.

Similarly for a two-stage ticket:

- $A \rightarrow B \rightarrow A$ which is a return journey
- $A \rightarrow B \rightarrow C$ which is a one-way journey.

(i) What feature on the adjacency matrix tells us that the buses run in both directions between adjacent shopping centres? (2 marks)

(ii) How many different one-stage journeys are available from shopping centre B? (1 mark)

(iii) List **all** the different two-stage, one-way journeys available from shopping centre B. (2 marks)

**Question 16** **(13 marks)**

(a) Alex is about to retire and is planning to take an annuity from his pension fund. He sets up the pension fund on his 65th birthday with $500 000 and he estimates the fund can generate a growth rate of 6% per year. He plans to start withdrawing an annuity of $40 000 starting on his following birthday.

(i) Write a recurrence relation to calculate the total amount in the fund directly after each withdrawal. (3 marks)

(ii) For how many years will Alex be able to receive his annuity of $40 000? (2 marks)

(iii) Assuming that all other conditions are the same, explain what would happen if Alex decided to withdraw $30 000 per year instead of $40 000 per year. (2 marks)

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(b) Abbey sets up her pension fund on July 1 2016 with a principal of $850 000. The fund guarantees an annual growth rate of 7.5% compounded monthly and she plans to take an annuity of $75 000 each year on July 1, starting in 2017.

(i) Calculate the balance in the fund after the annuity is withdrawn in July 2020. (2 marks)

The investment fund revised its annual interest rate to 9% compounded monthly on July 1 2020 guaranteed for the period to July 2025 and Abbey continued withdrawing $75 000 as usual.

(ii) Calculate the balance in the fund after a withdrawal is made on July 1 2025. (2 marks)

(iii) Calculate, to the nearest $100, the maximum amount Abbey could withdraw annually, starting in 2020, without decreasing her balance. (2 marks)

**End of questions**

Additional working space

Question number: ________________

Additional working space

Question number: ____________________

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Question number: ______________

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Question number: ________________

**ACKNOWLEDGEMENTS**

**Section Two**

**Question 11** Data source: Australian Bureau of Statistics. (2015, February 18). *4177.0—Participation in sport and physical recreation, Australia, 2013–14*. Retrieved April, 2016, from www.abs.gov.au/AUSSTATS/abs@.nsf/DetailsPage/4177.02013-14?OpenDocument
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