Government of Western Australia
School Curriculum and Standards Authority

Sample Assessment Tasks

Mathematics Applications
ATAR Year 12

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Sample assessment task

Mathematics Applications – ATAR Year 12

Test 3 – Unit 3

Assessment type: Response

Conditions:
Time for the task: Up to 50 minutes, in class, under test conditions

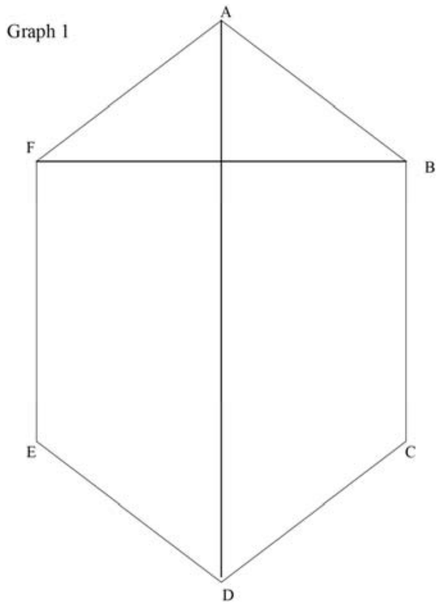
Materials required:

Section One: Calculator-free	Standard writing equipment
Section Two: Calculator-assumed	Calculator (to be provided by the student)
Other materials allowed:	Drawing templates, one page of notes in Section Two
Marks available:	52 marks
Section One: Calculator-free	(28 marks)
Section Two: Calculator-assumed	(24 marks)
Task weighting:	8% for the pair of units

Section One: Calculator-free (28 marks)

Suggested time: 20 minutes

Question 1 (3.3.1) (4 marks)



(a) Define the set of vertices and a list of edges for Graph 1. (2 marks)

(b) Is Graph 1 planar? Explain (2 marks)

Question 2 (3.3.2) (9 marks)

A Friday night darts competition has five competitors: Alex (A), Bob (B), Colin (C), Dave (D) and Evan (E), who are due to play each other in a round-robin competition over a number of weeks. On the first Friday, rounds 1 and 2 were played where:

Round 1 – Alex played Bob; Colin played Dave; and Evan had a bye

Round 2 – Alex played Evan; Colin played Bob; and Dave had a bye.

(a) Draw a graph to represent the games played on the first Friday night. (3 marks)

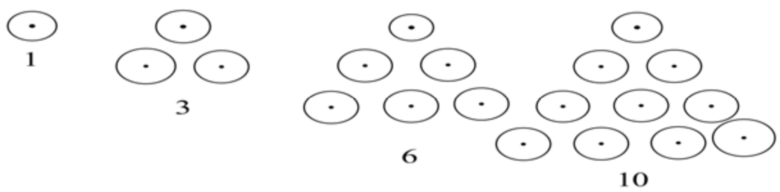
(b) Draw the complement to the graph in (a) and explain what it represents. (3 marks)

(c) Draw a graph to represent all the games to be played in the whole competition. (3 marks)

Question 3 (3.2.9) **(6 marks)**

The set of numbers 1, 3, 6, 10 is named triangular because the units (objects) can be arranged to form triangles, as in the diagram below.

(a) Draw the next pattern in the diagram below and give its value. (2 marks)



(b) Given $T_{10}=55$ evaluate T_{11}. (1 mark)

(c) Give the first order recurrence for the sequence. (3 marks)

Question 4 (3.2.4, 3.2.3) **(9 marks)**

A wedding photographer is quoting the following price for producing a wedding album for the newlyweds:

A fixed minimum cost of $150, with 80 photos in a hard-backed album. Further photos may also be added in lots of 10 photos at $0.70 per photo, up to a maximum of 200 photos.

He wants to set up a table below, showing:

- the type of album where T_1 is the basic album, with 80 photos at a cost of $150
- the number of photos in each of the possible album sizes
- the cost in dollars of each of the different albums.

(a) Complete each of the blank cells of the table. (3 marks)

Type	T_1	T_2	T_3	T_4	T_5		T_n
Number of pictures	80						200
$ cost of album	$150						

(b) Write a rule that will calculate the number of pictures in album type = T_n. (3 marks)

(c) Write a rule that will calculate the cost of album type = C_n. (3 marks)

Section Two: Calculator-assumed (24 marks)

Suggested time: 30 minutes

Question 5 (3.2.9) **(3 marks)**

(a) Complete the table below for the first five terms of the sequence, defined by
$a_{n+1} = a_n + (n+1),\ a_1 = 1$. (2 marks)

n	1	2	3	4	5
a_n					

(b) Evaluate a_{50}. (1 mark)

Question 6 (3.2.10, 3.2.11) **(12 marks)**

A fish farm operates a fish breeding pond in which the population of a particular fish increases by 3% per month.

(a) Give the recurrence formula to calculate the fish population at the end of each month, assuming the rate does not change and the initial population is 1000 fish. (2 marks)

(b) Calculate the population numbers at the end of the first six months of its operation, given the initial population is 1000 fish. (2 marks)

(c) After the initial six months, 40 fish per month are removed at the end of each month. Assuming the population growth is maintained at 3%, how many fish are expected to be in the tank at the end of 12 months? (3 marks)

(d) Describe what is happening to the population. (1 mark)

(e) Estimate, to the nearest whole number, the maximum number of fish that may be removed from the tank per month without the numbers of fish decreasing. (4 marks)

Question 7 (3.2.7) **(9 marks)**

For the recurrence relation $a_{n+1} = a_n + 0.6$ and $a_0 = 3.1$

(a) Deduce the rule for the n[th] term of the relation. (3 marks)

(b) Check the truth of the following proposition $a_{10} = 2 \times a_9 - a_8$. (2 marks)

(c) Prove the above proposition can be generalised to $a_{n+2} = 2 \times a_{n+1} - a_n$. (4 marks)

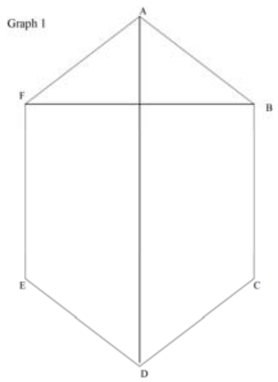
Marking key for Test 3 – Unit 3

Section One: Calculator-free **(28 marks)**

Question 1

(a) Define the set of vertices and a list of edges for Graph 1.

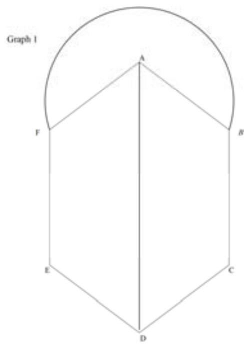
Solution
$V(G_1)=\{A,B,C,D,E,F\}$ and $E(G_1)=AB,AD,AF,BF,BC,CD,DE,EF$



Behaviours	Marks	Item* (S/C)
Defines the vertices of Graph 1	1	simple
Defines the edges of Graph 1	1	simple

(b) Is Graph 1 planar? Explain

Solution
Yes the graph is planar since it can be drawn such that none of the edges intersect other than at the vertices.



Behaviours	Marks	Item* (S/C)
States that Graph 1 is planar	1	complex
States that Graph 1 can be drawn where no edges intersect.	1	complex

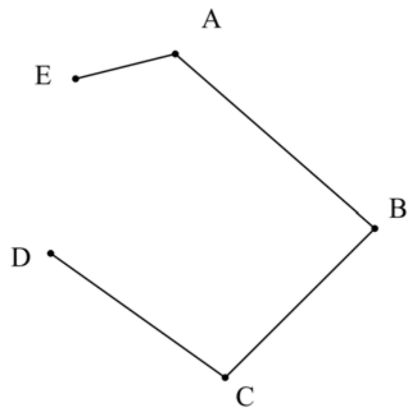
Question 2

A Friday night darts competition has five competitors: Alex (A), Bob (B), Colin (C), Dave (D) and Evan (E) who are due to play each other in a round-robin competition over a number of weeks. On the first Friday, rounds 1 and 2 were played where:

Round 1 – Alex played Bob; Colin played Dave; and Evan had a bye
Round 2 – Alex played Evan; Colin played Bob; and Dave had a bye.

(a) Draw a graph to represent the games played on the first Friday night.

Solution

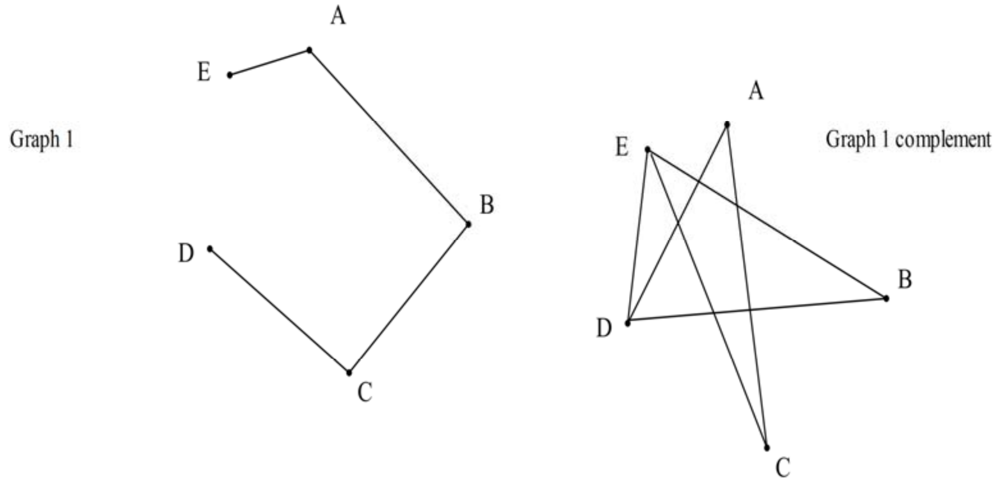


Behaviours	Marks	Rating
Draws and labels all five vertices	1	simple
Draws only the four correct edges required	2	simple

(b) Draw the complement to the graph in (a) and explain what it represents.

Solution

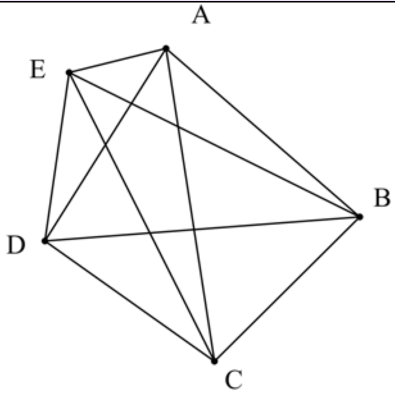
a)



These represent the games still to be played.

Behaviours	Marks	Rating
Includes all five vertices in the graph	1	simple
Draws only the six correct edges required	1	simple
Gives the correct interpretation of the complement	1	simple

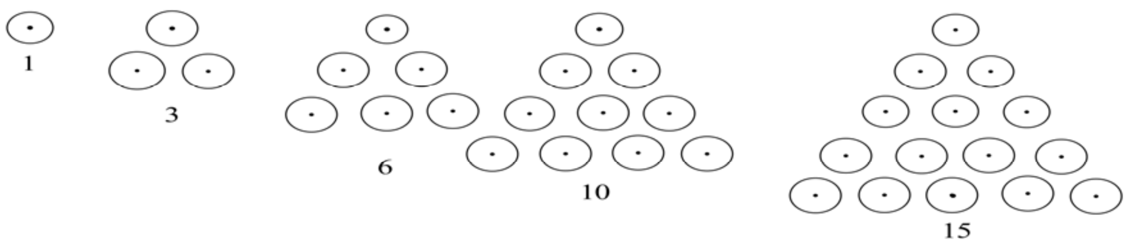
(c) Draw a graph to represent all the games that will be played in the whole competition.

Solution		
		
Behaviours	**Marks**	**Item* (S/C)**
Includes all five vertices in the graph	1	simple
Draws all 10 correct edges	2	simple

Question 3

The set of numbers 1, 3, 6, 10 is named triangular because the units (objects) can be arranged to form triangles, as in the diagram below.

(a) Draw the next pattern in the diagram above and give its value.

Solution		
		
Behaviours	**Marks**	**Item* (S/C)**
Draws the correct pattern	1	simple
Correctly counts the units of the fifth term	1	simple

(b) Given T_{10}=55 evaluate T_{11}.

Solution		
$T_{10} = 55 \Rightarrow T_{11} = 55 + 11 = 66$		
Behaviours	**Marks**	**Item* (S/C)**
Calculates the correct term (T_{11}) value	1	simple

(c) Give the first order recurrence for the sequence.

Solution
$T_1 = 1$ $T_2 = 1 + 2 = 3 \quad \Rightarrow T_2 = T_1 + 2$ $T_3 = 3 + 3 = 6 \quad \Rightarrow T_3 = T_2 + 3$ $T_4 = 6 + 4 = 10 \quad \Rightarrow T_4 = T_3 + 4$ $\Rightarrow T_n = T_{n-1} + n, \text{ given } T_1 = 1$

Behaviours	Marks	Item* (S/C)
States correctly the first term T_1	1	simple
Shows the recursive operation was addition	1	complex
Indicates the term to be added equals the term number *n*	1	complex

Question 4

The photographer wants to set up a table below showing:

- the type of album where T_1 is the basic album with 80 photos at a cost of $150
- the number of photos in each of the possible album sizes
- the cost in dollars of each of the different albums.

(a) Complete each of the blank cells of the table.

Solution

Type	T_1	T_2	T_3	T_4	T_5		T_n
Number of pictures	80	90	100	110	120		200
$ cost of album	$150	157	164	171	178		234

Behaviours	Marks	Item* (S/C)
Completes the numbers of pictures in each cell correctly	1	complex
Completes the cost of each type correctly	1	complex
Completes T_n correctly	1	complex

(b) Write a rule that will calculate the number of pictures in album type = T_n.

Solution
$T_n = a + (n-1)d$ given $a = 80$ and $T_2 = 90$ $\Rightarrow T_2 = 80 + d = 90$ $\Rightarrow d = 10$ $\Rightarrow T_n = 80 + (n-1)10$ $\Rightarrow T_n = 70 + 10n$

Behaviours	Marks	Item* (S/C)
Applies the linear rule with correct common difference *d*	1	simple
Calculates *a* from the table of values	1	simple
States the rule in terms of *n*	1	simple

(c) Write a rule that will calculate the cost of album type = C_n.

Solution
$C_n = a + (n-1)d$ given $a = 150$ and $C_2 = 157$ $\Rightarrow C_2 = 150 + d = 157$ $\Rightarrow d = 7$ $\Rightarrow C_n = 150 + (n-1)7$ $\Rightarrow C_n = 143 + 7n$

Behaviours	Marks	Item* (S/C)
Applies the linear rule with correct common difference *d*	1	simple
Calculates *a* from the table of values	1	simple
States the rule in terms of *n*	1	simple

Section Two: Calculator-assumed section (24 marks)

Question 5

(a) Complete the table below for the first five terms of the sequence, defined by $a_{n+1} = a_n + (n+1)$, $a_1 = 1$.

Solution

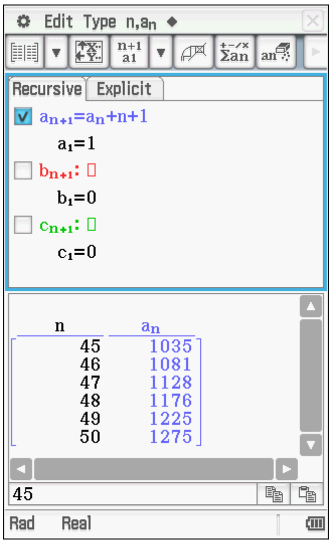
n	1	2	3	4	5
a_n	1	3	6	10	15

Behaviours	Marks	Item* (S/C)
Calculates at least two terms correctly	1	simple
Calculates all terms correctly	1	simple

(b) Evaluate a_{50}.

Solution

$a_{50} = 1275$



Behaviours	Marks	Item* (S/C)
Evaluates $a_{50} = 1275$	1	simple

Question 6

A fish farm operates a fish breeding pond in which the population of a particular fish increases by 3% per month.

(a) Give the recurrence formula to calculate the fish population at the end of each month, assuming the rate does not change and the initial population is 1000 fish.

Solution		
$P_{n+1} = 1.03 \times P_n$ where $P_0 = 1000$		
Behaviours	**Marks**	**Item* (S/C)**
Defines the recurrence relationship	1	simple
States $P_0 = 1000$	1	simple

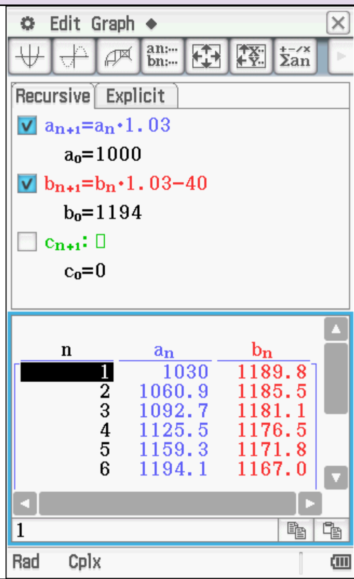
(b) Calculate the population numbers at the end of the first six months of its operation, given the initial population is 1000 fish.

Solution

n	0	1	2	3	4	5	6
P_n	1000	1030	1060	1092	1125	1159	1194

Behaviours	Marks	Item* (S/C)
Starts the sequence at P_1 = 1030	1	simple
Completes the sequence at P_6=1194	1	simple

(c) After the initial six months, 40 fish per month are removed at the end of each month. Assuming the population growth is maintained at 3%, how many fish are expected to be in the tank at the end of 12 months?

Solution	
Given $Q_n = Q_0 \times 1.03 - 40$, and $\Rightarrow Q_0 = 1194$ $\Rightarrow Q_6 = 1167$	

Behaviours	Marks	Item* (S/C)
States the new recurrence relation	1	complex
States the initial condition Q_0	1	complex
Calculates the correct population	1	complex

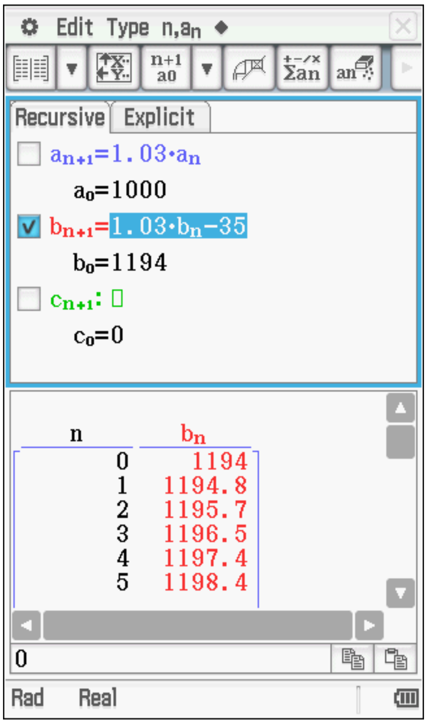
(d) Describe what is happening to the population.

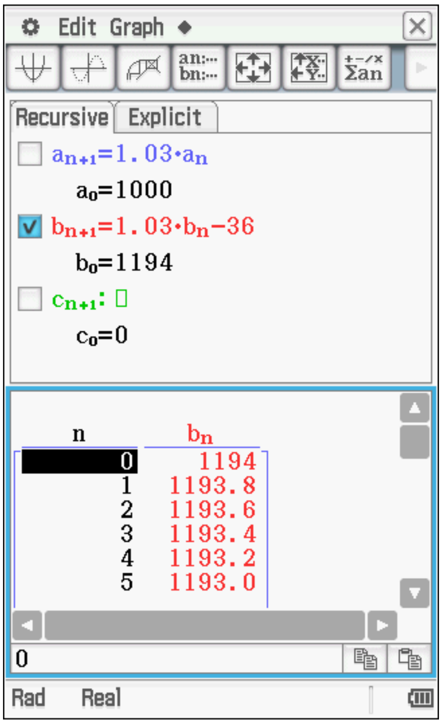
Solution
The population of fish is declining slowly.

Behaviours	Marks	Item* (S/C)
States the correct change in population	1	simple

(e) Estimate, to the nearest whole number, the maximum number of fish that may be removed from the tank per month without the numbers of fish decreasing.

Solution





Given $Q_n = Q_0 \times 1.03 - k$, where k is the number of fish taken from the pond

$Q_0 = 1194$ and $k = 36$ Or	$Q_0 = 1194$ and $k = 35$
$Q_5 = Q_0 \times 1.03 - 36$,	$Q_5 = Q_0 \times 1.03 - 35$,
$\Rightarrow Q_5 \approx 1193$	$\Rightarrow Q_5 \approx 1198$
$\Rightarrow Q$ is decreasing	$\Rightarrow Q$ is increasing

Hence k = 35 is the maximum number of fish to be taken per month without depleting the population.

Behaviours	Marks	Item* (S/C)
Uses recurrence formula $Q_n = Q_0 \times 1.03 - k$ to check values of k	1	complex
Shows that $k \geq 36$ causes a decrease in Q	1	complex
Shows that $k = 35$ causes an increase in Q	1	complex
States k=35 is the maximum value for which Q increases	1	complex

Question 7

For the recurrence relation $a_{n+1} = a_n + 0.6$ and $a_0 = 3.1$

(a) Deduce the rule for the n[th] term of the relation.

Solution
$a_{n+1} = a_n + 0.6$ and $a_0 = 3.1$ $\Leftrightarrow T_n = a + (n-1)d$ $\Leftrightarrow T_n = 3.1 + (n-1)0.6$ $T_n = 0.6n + 2.5$

Behaviours	Marks	Item* (S/C)
Uses a linear relation such as $T_n = a + (n-1)d$	1	simple
Defines *a* and *d* correctly	1	simple
Simplifies the expression	1	simple

(b) Check the truth of the following proposition $a_{10} = 2 \times a_9 - a_8$.

Solution
$a_{10} = 2 \times a_9 - a_8 \Leftrightarrow 8.5 = 2 \times 7.9 - 7.3$ $\Leftrightarrow 8.5 = 15.8 - 7.3$ True

Behaviours	Marks	Item* (S/C)
Substitutes the correct values into the equation	1	simple
Demonstrates the LHS of the equation = RHS	1	simple

(c) Prove the above proposition can be generalised to $a_{n+2} = 2 \times a_{n+1} - a_n$.

Solution
$a_{n+1} = a_n + 0.60 + \ldots\ldots..(1)$ $\Leftrightarrow a_{n+2} = a_{n+1} + 0.6\ldots\ldots(2)$ subtracting equations $\Leftrightarrow a_{n+2} - a_{n+1} = a_{n+1} - a_n \ldots\ldots\ldots(2)-(1)$ rearranging $\Leftrightarrow a_{n+2} = 2 \times a_{n+1} - a_n \ldots\ldots,(3)$

Behaviours	Marks	Item* (S/C)
Sets up equations (1) and (2)	2	complex
Eliminates the constant by subtraction	1	complex
Rearranges the equation to give the result	1	complex