ATAR course examination, 2021
Question/Answer booklet

## MATHEMATICS APPLICATIONS

## Section Two: Calculator-assumed



In words

## Time allowed for this section

Reading time before commencing work: Working time:
ten minutes
one hundred minutes

Number of additional answer booklets used (if applicable):

## Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)
To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

## Important note to candidates

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

## Structure of this paper

| Section | Number of <br> questions <br> available | Number of <br> questions to <br> be answered | Working <br> time <br> (minutes) | Marks <br> available | Percentage <br> of <br> examination |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Section One: <br> Calculator-free | 7 | 7 | 50 | 52 | 35 |
| Section Two: <br> Calculator-assumed | 9 | 9 | 100 | 94 | 65 |
| Total |  |  |  |  | 100 |

## Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the Year 12 Information Handbook 2021: Part II Examinations. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specified to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

## Section Two：Calculator－assumed

This section has nine questions．Answer all questions．Write your answers in the spaces provided．

Supplementary pages for planning／continuing your answers to questions are provided at the end of this Question／Answer booklet．If you use these pages to continue an answer，indicate at the original answer where the answer is continued，i．e．give the page number．

Working time： 100 minutes．

## Question 8

The graph below shows the number of bookings at a dog grooming salon over its first few weeks of business.

(a) Which simple moving average would be the most suitable for the data displayed in this graph?

A more detailed view of the same data is given in the table below.

| Week | Day | $n$ | Number of bookings | Seasonal mean | Number of bookings as a percentage of the seasonal mean | Seasonally adjusted figures |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Tuesday | 1 | 12 | 17.6 | A | 17.7 |
|  | Wednesday | 2 | 14 |  | 79.55 | 16.9 |
|  | Thursday | 3 | 14 |  | 79.55 | 16.6 |
|  | Friday | 4 | 20 |  | 113.64 | 17.8 |
|  | Saturday | 5 | 28 |  | 159.09 | 18.3 |
| 2 | Tuesday | 6 | 14 | B | 67.31 | 20.6 |
|  | Wednesday | 7 | 19 |  | 91.35 | 23.0 |
|  | Thursday | 8 | 18 |  | 86.54 | 21.4 |
|  | Friday | 9 | 23 |  | 110.58 | 20.4 |
|  | Saturday | 10 | 30 |  | 144.23 | 19.7 |
| 3 | Tuesday | 11 | C | 22 | 68.18 | 22.1 |
|  | Wednesday | 12 | 17 |  | 77.27 | 20.6 |
|  | Thursday | 13 | 19 |  | 86.36 | 22.6 |
|  | Friday | 14 | 25 |  | 113.64 | 22.2 |
|  | Saturday | 15 | 34 |  | 154.55 | 22.3 |
| 4 | Tuesday | 16 | 18 | - | - | - |
|  | Wednesday | 17 | 21 |  | - | - |
|  | Thursday | 18 | 22 |  | - | - |

（b）Calculate the value of $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ in the table．
（c）Calculate the seasonal index for Saturday．
（d）The equation of the least－squares line using the seasonally adjusted figures is $y=0.40 n+16.94$ ．Draw this line on the graph．
（e）（i）Use the equation of the least－squares line given in part（d）to predict the number of bookings that will be made for the Saturday of Week 5.
（2 marks）
（ii）Comment on this prediction．
（2 marks）

## Question 9

A real estate agent is analysing data on the sale of houses over the last six months. The table shows the average sale price of houses, in thousands of dollars (\$'000), and their distance from the ocean, to the nearest kilometre.

| Distance from the <br> ocean (km) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Average sale price <br> $(\$ \mathbf{0 0 0})$ | 1758 | 1909 | 2265 | 1934 | 1228 | 1641 | 751 | 967 | 676 |

(a) State the explanatory variable.
(b) On the scatterplot below, plot the last two data points from the table.

(c) Determine the equation of the least-squares line for these data.
(d) Interpret the slope of the least-squares line from part (c) in the context of this question.
(2 marks)
（e）（i）State the value of the correlation coefficient for these data．
（ii）What does the correlation coefficient measure？
（iii）Describe the association between the variables in terms of direction and strength．
（2 marks）
（f）What percentage of the variation in average sale price can be explained by the variation in the distance from the ocean？
（g）In six months time，a homebuyer will have saved enough money for a deposit on a house．He would like to live about four kilometres from the ocean．
（i）Use the equation of the least－squares line from part（c）to predict the average sale price of houses four kilometres from the ocean．
（1 mark）
（ii）Explain why your prediction is different from the average sale price given in the table．
（h）Give a reason why extrapolation in the context of this question would not make sense．
（1 mark）
（i）The real estate agent was talking to some potential buyers and was heard to make the statement，＂Having property closer to the ocean causes higher selling prices＂．Comment on this statement．
（2 marks）

## Question 10

Wendy moved into an apartment and organised a loan of $\$ 16000$ to purchase new furniture. To pay off the loan Wendy makes repayments of $\$ 600$ at the end of each month. The spreadsheet below shows the progress of her loan.

| Month | Opening balance | Interest | Repayment | Closing balance |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 16000.00 | 98.67 | 600.00 | 15498.67 |
| 2 | 15498.67 | 95.58 | 600.00 | 14994.24 |
| 3 | 14994.24 | 92.46 | 600.00 | 14486.71 |
| 4 |  |  |  |  |

(a) Write a calculation to show that the yearly interest rate is approximately $7.4 \%$. (2 marks)
(b) Complete the fourth row of the spreadsheet.
(c) Write a recursive rule to determine the closing balance of the loan at the end of each month.
(d) Determine how many months it will take Wendy to pay off the loan.
(e) Calculate how much interest is paid over the duration of the loan.

On reflection，Wendy realised she could have repaid $\$ 800$ each month．
（f）Determine the maximum amount Wendy would have been able to borrow，if all other details of the loan and repayment time remained the same．

## Question 11

Judith monitors the water quality in her garden pond at the same time every day. She likes to maintain the concentration of algae at between 200 and 250 units per 100 litres (L). Her measurements show that the concentration increases daily according to the recursive rule $C_{n+1}=1.025 C_{n}$, where $C_{1}=200$ units per 100 L (the minimum concentration).

When the concentration gets above the 250 units per 100 L limit, she treats the water to bring the concentration back to the minimum 200 units per 100 L .
(a) If Judith treated the water on Sunday, 6 December 2020, determine
(i) the concentration on Wednesday, 9 December 2020.
(ii) the day and date when she next treated the water.
(b) During the first week of January 2021, Judith monitored the water and recorded the following readings.

| Day | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Concentration (C) | 200 | 206 | 212.18 | 218.55 | 225.10 | 231.85 | 238.81 |

(i) Determine the revised recursive rule.
(2 marks)
(ii) If she treated the water on 10 January and went on holiday until 20 January, when she next treated the water, calculate the concentration of the water on her return, assuming the recursive rule from part (b)(i) is used.
(2 marks)

## Question 12

Virat purchases a new motor vehicle for $\$ 24500$ ．For the first two years the vehicle depreciates at a rate of $13 \%$ per year and for the third year it depreciates at a lower rate of $9.5 \%$ per year．
（a）Calculate the value of the vehicle after one year．
（b）Calculate the value of the vehicle after the first three years．

For the next three years the rate of depreciation is constant at $r \%$ per year．The average rate of depreciation for the first six years is $11 \%$ per year．
（c）Calculate the value of $r$ as a percentage．

A kitchen renovation project consists of a number of tasks of different durations and completed in different orders. One such renovation has this information summarised in the table below.

| Task | Task description | Duration <br> (days) | Immediate <br> predecessor(s) |
| :---: | :--- | :---: | :---: |
| A | Prepare plans | 10 | None |
| B | Select contractor | 2 | None |
| C | Review plans | 4 | A, B |
| D | Purchase appliances | 1 | C |
| E | Remove old appliances and benches | 2 | C |
| F | Prepare electrics and plumbing | 2 | E |
| G | Construct new cupboards and benches | 12 | C, |
| H | Install cupboards and benches | 2 | F, G |
| J | Install appliances | 1 | see part (b) |
| K | Tiling and splashbacks | 5 | J |
| L | Flooring | 3 | H |
| M | Complete electrical and plumbing | 2 | K |
| N | Test and handover | 1 | L, M |

A partially-completed project network is shown below for this table.

(a) Complete the network diagram.
(b) Identify the immediate predecessor(s) of Task J .
(c) What does the dotted line on the network indicate?
(d) Determine the critical path and the minimum completion time for the project.
(e) Determine the float time for Task D and explain its meaning in terms of the renovation.
(f) Once Task E (removal of old appliances and benches) begins, the kitchen cannot be used. What is the least amount of time the occupants of the house will be without a working kitchen?
(g) If Task G was actually completed in nine days, how would this affect the critical path and minimum completion time?
(2 marks)

Tasks E and F are both delayed.
(h) What is the maximum possible delay that does not affect the original minimum completion time?

Patrick has retired and invested his lump sum superannuation payout of $\$ 717850$ at a rate of $5.7 \%$ per annum compounded monthly. He begins the investment strategy from 1 January.
(a) Patrick will receive $\$ 4500$ at the end of each month for general living expenses and will also receive a further $\$ 4000$ at the end of each year for an annual holiday.
(i) Identify this type of investment account.
(ii) Determine the balance in the account at the end of the first year.
(iii) Determine the balance in the account at the end of the second year. (3 marks)
（b）When Patrick retired，he also considered the option of setting up a perpetuity with his superannuation payout still at $5.7 \%$ per annum compounded monthly．Calculate the quarterly payments Patrick would have received with this perpetuity in place．（3 marks）

## Question 15

A number of streets connecting locations A to $G$ inclusive have been identified as needing lighting upgrades with energy-efficient LED lamps. The council has decided that each location must have at least one connecting street that receives the lighting upgrade. The table below shows the cost, in dollars, of completing the upgrade in each street.

|  | A | B | C | D | E | F | G |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | - | 7900 | 10300 | - | 7100 | - | - |
| B | 7900 | - | 7600 | 6200 | 6500 | - | 15400 |
| C | 10300 | 7600 | - | 8500 | - | 12200 | 9200 |
| D | - | 6200 | 8500 | - | 8000 | 4700 | 9800 |
| E | 7100 | 6500 | - | 8000 | - | 4800 | - |
| F | - | - | 12200 | 4700 | 4800 | - | 10100 |
| G | - | 15400 | 9200 | 9800 | - | 10100 | - |

The council has a limited budget, so it needs to complete the upgrades at minimum cost.
(a) Demonstrate the use of Prim's algorithm on the table above to determine the minimum spanning tree and draw it on the diagram below.
C

- ${ }^{G}$
- ${ }^{\text {D }}$
${ }^{\bullet} \mathrm{F}$
(b) The council has set aside $\$ 42000$ to complete the lighting upgrades. Does it have enough in its budget to make the necessary upgrades? Justify your answer.
(2 marks)
(c) Due to the location of the police station, the upgrade from D to C must be included. What effect will this have on the minimum cost and the spanning tree?
(2 marks)
- A


E
${ }^{\bullet}$ F

## Question 16

The graph below shows a network of water pipes. The water source and main pumping station are located at S . The distribution centre is at T and the other vertices are intermediate pumping stations. The weights on the edges show the capacities in kilolitres per hour that can flow through each pipe.

(a) (i) Determine the value of the cut shown in Diagram 1.
(ii) Using your answer to part (a)(i), what can be said about the maximum flow of water through the network?
(b) State the maximum possible flows along the paths SABT and SCDT.
(2 marks)
(c) Determine the maximum flow from S to T , listing each path and the corresponding flow.
(3 marks)
(d) Using Diagram 2 below, indicate a possible flow along each pipe corresponding to the maximum flow calculated in part (c).

(e) Determine the minimum cut that corresponds to the maximum flow and illustrate this on the copy of Diagram 1 shown below.
(1 mark)


Supplementary page
Question number:

Supplementary page
Question number:

Supplementary page
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Supplementary page
Question number：

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