Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS SPECIALIST
ATAR COURSE

FORMULA SHEET

2021

Index

Differentiation and integration

$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$
$\frac{d}{dx}e^{ax} = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\ln x = \frac{1}{x}$	$\int \frac{1}{x}dx = \ln\lvert x\rvert + c$
$\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$	$\int \frac{f'(x)}{f(x)}dx = \ln\lvert f(x)\rvert + c$
$\frac{d}{dx}\sin f(x) = f'(x)\cos f(x)$	$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\cos f(x) = -f'(x)\sin f(x)$	$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\tan f(x) = f'(x)\sec^2 f(x) = \frac{f'(x)}{\cos^2 f(x)}$	$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$

Product rule	If $y = uv$ then $\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$	or	If $y = f(x)\,g(x)$ then $y' = f'(x)\,g(x) + f(x)\,g'(x)$
Quotient rule	If $y = \frac{u}{v}$ then $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	or	If $y = \frac{f(x)}{g(x)}$ then $y' = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{(g(x))^2}$
Chain rule	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$	or	If $y = f(g(x))$ then $y' = f'(g(x))\,g'(x)$
Fundamental theorem	$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$	and	$\int_a^b f'(x)\,dx = f(b) - f(a)$

Applications of calculus

Growth and decay	
Exponential equation	$\frac{dP}{dt} = kP \Leftrightarrow P = P_0 e^{kt}$
Logistic equation	$\frac{dP}{dt} = rP(k - P) \Leftrightarrow P = \frac{kP_0}{P_0 + (k - P_0)e^{-rkt}}$
Volumes of solids of revolution	
About the x-axis	$V = \pi \int_a^b [f(x)]^2 dx$
About the y-axis	$V = \pi \int_c^d [f(y)]^2 dy$
Simple harmonic motion	
If $\frac{d^2x}{dt^2} = -k^2 x$ then $x = A \sin(kt + \alpha)$ or $x = A \cos(kt + \beta)$ where A is the amplitude, α and β are phase angles, v is the velocity and x is the displacement	
$v^2 = k^2(A^2 - x^2)$ Period: $T = \frac{2\pi}{k}$ Frequency: $f = \frac{1}{T}$	
Increments formula	$\delta y \approx \frac{dy}{dx} \times \delta x$
Acceleration	$\frac{dv}{dt}$ or $v\frac{dv}{dx}$ or $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$

Functions

Quadratic function	If $f(x) = ax^2 + bx + c$ and $f(x) = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Absolute value function	$\lvert x \rvert = \begin{cases} x, & \text{for } x \geq 0 \\ -x, & \text{for } x < 0 \end{cases}$

Statistical inference

Confidence interval for the mean of the population	$\overline{X} - z\frac{s}{\sqrt{n}} \leq \mu \leq \overline{X} + z\frac{s}{\sqrt{n}}$
Sample size	$n = \left(\frac{z \times s}{d}\right)^2$

Mensuration

Parallelogram	$A = bh$
Triangle	$A = \frac{1}{2} bh$ or $A = \frac{1}{2} ab \sin C$
Trapezium	$A = \frac{1}{2}(a + b)h$
Circle	$A = \pi r^2$ and $C = 2\pi r = \pi d$

Prism	$V = Ah$, where A is the area of the cross section	
Pyramid	$V = \frac{1}{3} Ah$, where A is the area of the base	
Cylinder	$V = \pi r^2 h$	$TSA = 2\pi rh + 2\pi r^2$
Cone	$V = \frac{1}{3}\pi r^2 h$	$TSA = \pi rs + \pi r^2$, where s is the slant height
Sphere	$V = \frac{4}{3}\pi r^3$	$TSA = 4\pi r^2$

Vectors in 3D

Magnitude	$\lvert(a_1, a_2, a_3)\rvert = \sqrt{a_1^2 + a_2^2 + a_3^2}$
Dot product	$\mathbf{a} \bullet \mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert \cos\theta = a_1b_1 + a_2b_2 + a_3b_3$
Cross product	$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$
Equation of a line	One point and direction $\mathbf{r} = \mathbf{a} + \lambda\mathbf{u}$ Two points A and B $\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$
Equation of a plane	$\mathbf{r} = \mathbf{a} + \lambda\mathbf{u}_1 + \mu\mathbf{u}_2$ or $\mathbf{r} \bullet \mathbf{n} = \mathbf{a} \bullet \mathbf{n}$
Equation of a sphere	$\lvert\mathbf{r} - \mathbf{d}\rvert = r$ or $(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$
Cartesian equation of a line	$\frac{x - a_1}{u_1} = \frac{y - a_2}{u_2} = \frac{z - a_3}{u_3}$
Cartesian equation of a plane	$ax + by + cz = d$
Parametric equation of a line	$x = a_1 + \lambda u_1 \ldots \ldots (1)$ $y = a_2 + \lambda u_2 \ldots \ldots (2)$ $z = a_3 + \lambda u_3 \ldots \ldots (3)$

Complex numbers

Cartesian form	
$z = a + bi$	$\bar{z} = a - bi$
$\text{Mod}(z) = \lvert z \rvert = \sqrt{a^2 + b^2} = r$	$\text{Arg}(z) = \theta, \quad \tan\theta = \dfrac{b}{a}, \quad -\pi < \theta \le \pi$
$\lvert z_1 z_2 \rvert = \lvert z_1 \rvert \lvert z_2 \rvert$	$\left\lvert \dfrac{z_1}{z_2} \right\rvert = \dfrac{\lvert z_1 \rvert}{\lvert z_2 \rvert}$
$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$	$\arg\left(\dfrac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$
$z\bar{z} = \lvert z \rvert^2$	$z^{-1} = \dfrac{1}{z} = \dfrac{\bar{z}}{\lvert z \rvert^2}$
$\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$	$\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$
Polar form	
$z = a + bi = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$	$\bar{z} = r\,\text{cis}(-\theta)$
$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$	$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$
$\text{cis}(\theta_1 + \theta_2) = \text{cis}\,\theta_1\,\text{cis}\,\theta_2$	$\text{cis}(-\theta) = \dfrac{1}{\text{cis}\,\theta}$
De Moivre's theorem	
$z^n = \lvert z \rvert^n\,\text{cis}(n\theta)$	$(\text{cis}\,\theta)^n = \cos n\theta + i\sin n\theta$
$z^{\frac{1}{q}} = r^{\frac{1}{q}}\left(\cos\dfrac{\theta + 2\pi k}{q} + i\sin\dfrac{\theta + 2\pi k}{q}\right)$, for k an integer	

Trigonometry

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$	Length of arc $= r\theta$
$a^2 = b^2 + c^2 - 2bc\cos A$	Area of segment $= \dfrac{1}{2} r^2 (\theta - \sin\theta)$
$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$	Area of sector $= \dfrac{1}{2} r^2 \theta$
Identities	
$\cos^2 x + \sin^2 x = 1$	$1 + \tan^2 x = \sec^2 x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\cos 2x = \cos^2 x - \sin^2 x$ $= 2\cos^2 x - 1$ $= 1 - 2\sin^2 x$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\sin 2x = 2\sin x \cos x$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$
$\cos A \cos B = \dfrac{1}{2}(\cos(A - B) + \cos(A + B))$	$\sin A \cos B = \dfrac{1}{2}(\sin(A + B) + \sin(A - B))$
$\sin A \sin B = \dfrac{1}{2}(\cos(A - B) - \cos(A + B))$	$\cos A \sin B = \dfrac{1}{2}(\sin(A + B) - \sin(A - B))$

Note: Any additional formulas identified by the examination panel as necessary will be included in the body of the particular question.

This document is valid for teaching and examining until 31 December 2021.

Published by the School Curriculum and Standards Authority of Western Australia
303 Sevenoaks Street
CANNINGTON WA 6107