Government of **Western Australia**
**School Curriculum and Standards Authority**

## ATAR course examination, 2024

## Question/Answer booklet

# MATHEMATICS METHODS

## Section One: Calculator-free

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

WA student number: In figures

In words

### Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable):

### Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet
Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|---|---|---|---|---|---|
| Section One: Calculator-free | 7 | 7 | 50 | 51 | 35 |
| Section Two: Calculator-assumed | 10 | 10 | 100 | 100 | 65 |
| | | | | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2024: Part II Examinations*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

**See next page**

**Section One: Calculator-free** **35% (51 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

---

**Question 1** **(6 marks)**

(a) Differentiate the function $f(x) = x^2 \ln(4x + 3)$. (2 marks)

(b) Determine a fully simplified expression for $g(x)$, given that $g'(x) = \dfrac{3x}{3x^2 + 1}$ and $g(1) = \ln(6)$. (4 marks)

**Question 2** **(10 marks)**

An advertising graphic moves across the bottom of a television screen during a sporting game, changing direction to attract viewer attention. The position of the graphic is modelled by

$$x(t) = \frac{1}{3}t^3 - 7t^2 + 40t$$

where $x$, in centimetres, is the position of the graphic relative to the left side of the screen, and $t$, in seconds, is the time from when the graphic first appears on the screen.

The position of the graphic at integer time increments is given in the table below.

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $x(t)$ | 0 | $33\frac{1}{3}$ | $54\frac{2}{3}$ | 66 | $69\frac{1}{3}$ | $66\frac{2}{3}$ | 60 | $51\frac{1}{3}$ |

| $t$ | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|
| $x(t)$ | $42\frac{2}{3}$ | 36 | $33\frac{1}{3}$ | $36\frac{2}{3}$ | 48 | $69\frac{1}{3}$ | $102\frac{2}{3}$ | 150 |

(a) Determine the velocity of the graphic when it first appears on the screen. (2 marks)

(b) Is the graphic initially speeding up or slowing down? Justify your answer. (2 marks)

**See next page**

(c) Evaluate $\int_3^9 v(t)\,dt$ and explain what this integral represents. (3 marks)

(d) Calculate the total distance travelled by the graphic from the time it enters the screen to the time it leaves the screen 15 seconds later. (3 marks)



See next page

**Question 3** **(6 marks)**

The tables below display the partially completed probability distribution and cumulative distribution for a discrete random variable $X$.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\mathrm{P}(X = x)$ | 0.2 | 0.15 | | | 0.05 |

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\mathrm{P}(X \leq x)$ | 0.2 | | 0.6 | 0.95 | |

(a) Complete the missing probability entries in each of the tables above. (2 marks)

(b) Evaluate $\mathrm{P}(2 \leq X \leq 4)$. (2 marks)

(c) Evaluate $\mathrm{P}(X = 1 \mid X \leq 3)$. (2 marks)

**Question 4** **(6 marks)**

(a) The uniformly distributed continuous random variable $X$ has an expected value of 6 and a maximum value of 9. Determine the variance of $X$. (3 marks)

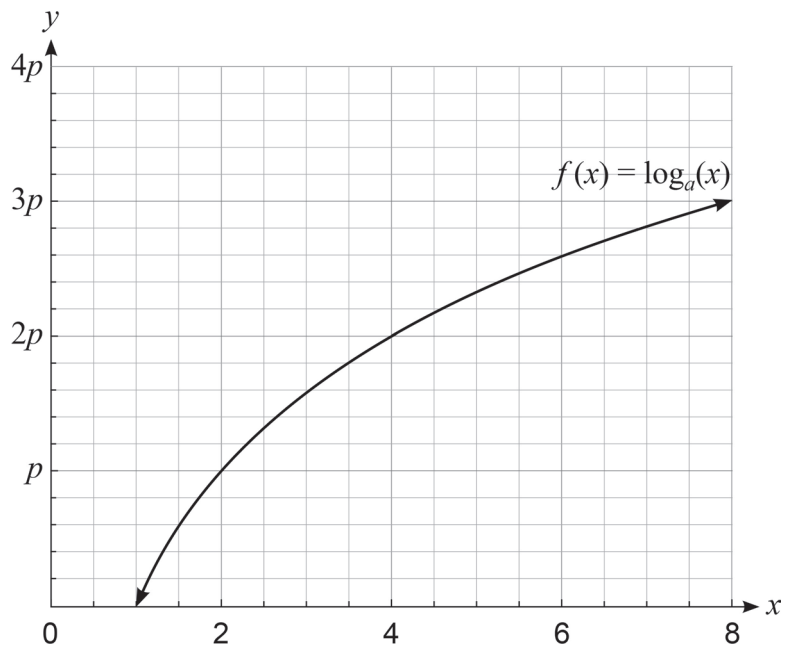
(b) The binomially distributed discrete random variable $W$ has a mean of $\frac{1}{2}$ and a variance of $\frac{5}{12}$. Evaluate $\mathrm{P}(W = 1)$. (3 marks)



**See next page**

**Question 5** **(8 marks)**

The function $f(x) = \log_a(x)$ is plotted below, where $a$ and $p$ are constants.




(a) Express $\log_a(0.5)$ in terms of $p$. (2 marks)

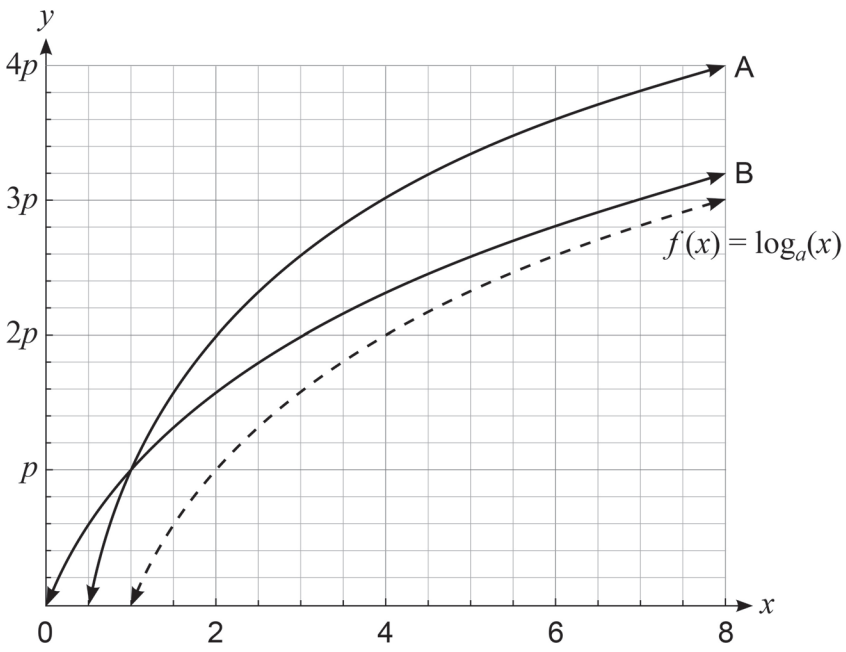
(b) Evaluate $a^{5p}$. (2 marks)

**See next page**

(c) Solve $\log_a(x-3) = 3p$ for $x$. (2 marks)



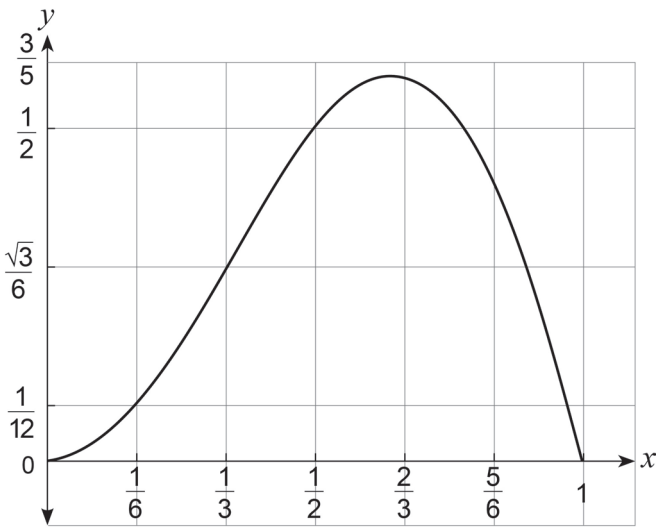
The function $f(x) = \log_a(x)$ has been transformed to give two other logarithmic functions with base $a$ on the axes as shown below.




(d) Determine an equation for each of the **two** functions, A and B. (2 marks)

**Question 6** **(5 marks)**

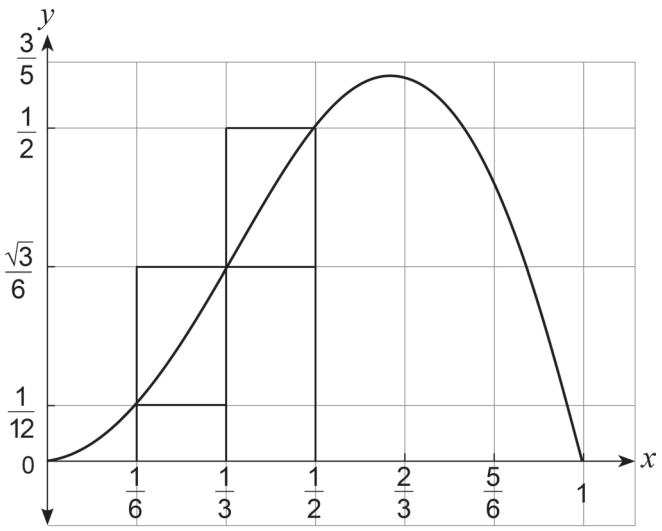
The graph of $y = x \sin(\pi x)$ for $0 \leq x \leq 1$ is shown below.




(a) On the diagram above, shade a region whose area is equal to $\int_{\frac{1}{6}}^{\frac{1}{2}} x \sin(\pi x)\, dx$. (1 mark)

A spare diagram is provided at the end of this Question/Answer booklet. If you need to use it, cross out this attempt and indicate that you have redrawn it on the spare diagram.

**See next page**



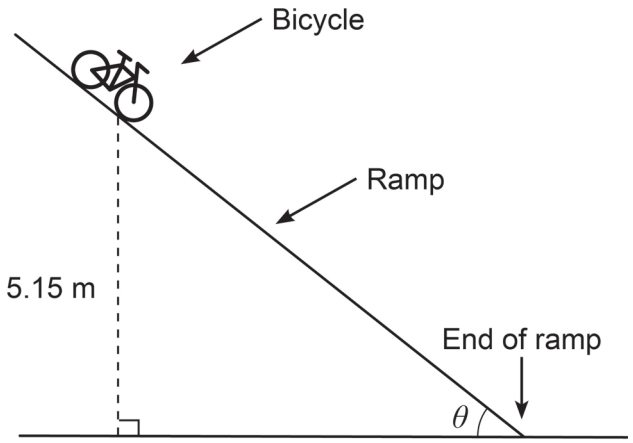

(b) (i) By considering the areas of the rectangles shown in the graph of $y = x\sin(\pi x)$ above, demonstrate and explain why

$$\frac{1+2\sqrt{3}}{72} < \int_{\frac{1}{6}}^{\frac{1}{2}} x\sin(\pi x)\,dx < \frac{3+\sqrt{3}}{36}.$$ (3 marks)

(ii) State **one** suggestion as to how the approximation from part (b)(i) could be improved. (1 mark)

**Question 7** **(10 marks)**

A bicycle rolls down a ramp from an initial height of 5.15 m as shown in the diagram below.




The speed, $s$, of the bicycle (in metres per second) when it reaches the end of the ramp is given by

$$s(\theta) = \sqrt{\frac{101 \sin(\theta) - \cos(\theta)}{\sin(\theta)}}$$

where $\theta$ (in radians) is the ramp angle shown in the diagram.

(a) Determine the speed of the bicycle at the end of the ramp, if the ramp angle is $45°$.

(2 marks)

(b) Determine $\frac{d}{d\theta}\left(\frac{101\sin(\theta)-\cos(\theta)}{\sin(\theta)}\right)$. Simplify your answer. (3 marks)

(c) Hence, show that $\frac{ds}{d\theta}=\frac{1}{2\sin^2(\theta)}\sqrt{\frac{\sin(\theta)}{101\sin(\theta)-\cos(\theta)}}$. (2 marks)



See next page

**Question 7** (continued)

(d) Use the increments formula to estimate the change in $s$ if the ramp angle is changed from $45°$ to $46°$. (3 marks)

**End of section**

Supplementary page

Question number: _____

Supplementary page

Question number: _____

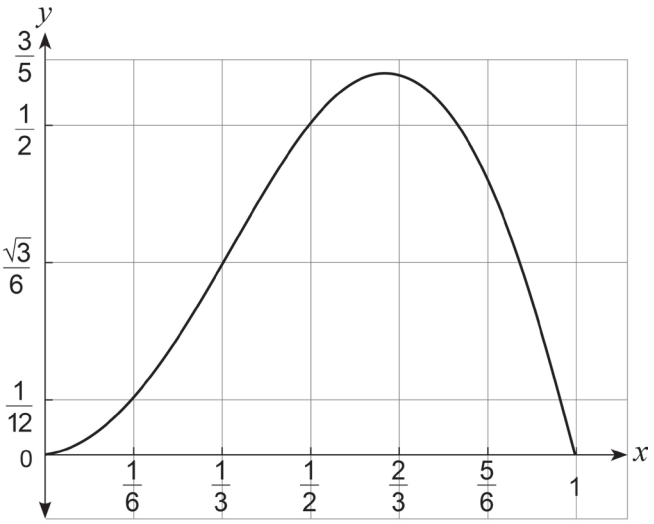
Supplementary page

Question number: _____

Supplementary page

Question number: _____

Spare diagram for Question 6(a)



*Published by the School Curriculum and Standards Authority of Western Australia*
*303 Sevenoaks Street*
*CANNINGTON WA 6107*