



# **MATHEMATICS SPECIALIST ATAR COURSE**

## **FORMULA SHEET**

**2023**

**Index**

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Differentiation and integration **3**

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Applications of calculus  
Functions  
Statistical inference **4**

---

Mensuration  
Vectors in 3D **5**

---

Complex numbers **6**

---

Trigonometry **7**

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**Differentiation and integration**

|   |  |   |     |   |
|---|--|---|-----|---|
| $\frac{d}{dx} x^n = nx^{n-1}$   | $\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$   |   |     |   |
| $\frac{d}{dx} e^{ax} = ae^{ax}$   | $\int e^{ax} dx = \frac{1}{a} e^{ax} + c$  |   |     |   |
| $\frac{d}{dx} \ln x = \frac{1}{x}$  | $\int \frac{1}{x} dx = \ln x  + c$   |   |     |   |
| $\frac{d}{dx} \ln f(x) = \frac{f'(x)}{f(x)}$  | $\int \frac{f'(x)}{f(x)} dx = \ln f(x)  + c$   |   |     |   |
| $\frac{d}{dx} \sin f(x) = f'(x) \cos f(x)$  | $\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$   |   |     |   |
| $\frac{d}{dx} \cos f(x) = -f'(x) \sin f(x)$   | $\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$  |   |     |   |
| $\frac{d}{dx} \tan f(x) = f'(x) \sec^2 f(x) = \frac{f'(x)}{\cos^2 f(x)}$  | $\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$  |   |     |   |
| Product rule  | <table style="width: 100%; border: none;"> <tr> <td style="width: 50%; border: none;">                     If <math>y = uv</math><br/>                     then<br/> <math display="block">\frac{d}{dx} (uv) = v \frac{du}{dx} + u \frac{dv}{dx}</math> </td> <td style="width: 10%; border: none; text-align: center;">or</td> <td style="width: 40%; border: none;">                     If <math>y = f(x) g(x)</math><br/>                     then<br/> <math display="block">y' = f'(x) g(x) + f(x) g'(x)</math> </td> </tr> </table>   | If $y = uv$<br>then<br>$\frac{d}{dx} (uv) = v \frac{du}{dx} + u \frac{dv}{dx}$  | or  | If $y = f(x) g(x)$<br>then<br>$y' = f'(x) g(x) + f(x) g'(x)$                          |
| If $y = uv$<br>then<br>$\frac{d}{dx} (uv) = v \frac{du}{dx} + u \frac{dv}{dx}$  | or   | If $y = f(x) g(x)$<br>then<br>$y' = f'(x) g(x) + f(x) g'(x)$  |     |   |
| Quotient rule   | <table style="width: 100%; border: none;"> <tr> <td style="width: 50%; border: none;">                     If <math>y = \frac{u}{v}</math><br/>                     then<br/> <math display="block">\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}</math> </td> <td style="width: 10%; border: none; text-align: center;">or</td> <td style="width: 40%; border: none;">                     If <math>y = \frac{f(x)}{g(x)}</math><br/>                     then<br/> <math display="block">y' = \frac{f'(x) g(x) - f(x) g'(x)}{(g(x))^2}</math> </td> </tr> </table> | If $y = \frac{u}{v}$<br>then<br>$\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ | or  | If $y = \frac{f(x)}{g(x)}$<br>then<br>$y' = \frac{f'(x) g(x) - f(x) g'(x)}{(g(x))^2}$ |
| If $y = \frac{u}{v}$<br>then<br>$\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ | or   | If $y = \frac{f(x)}{g(x)}$<br>then<br>$y' = \frac{f'(x) g(x) - f(x) g'(x)}{(g(x))^2}$                                     |     |   |
| Chain rule  | <table style="width: 100%; border: none;"> <tr> <td style="width: 50%; border: none;">                     If <math>y = f(u)</math> and <math>u = g(x)</math><br/>                     then<br/> <math display="block">\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}</math> </td> <td style="width: 10%; border: none; text-align: center;">or</td> <td style="width: 40%; border: none;">                     If <math>y = f(g(x))</math><br/>                     then<br/> <math display="block">y' = f'(g(x)) g'(x)</math> </td> </tr> </table>   | If $y = f(u)$ and $u = g(x)$<br>then<br>$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$                              | or  | If $y = f(g(x))$<br>then<br>$y' = f'(g(x)) g'(x)$                                     |
| If $y = f(u)$ and $u = g(x)$<br>then<br>$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$                              | or   | If $y = f(g(x))$<br>then<br>$y' = f'(g(x)) g'(x)$   |     |   |
| Fundamental theorem   | <table style="width: 100%; border: none;"> <tr> <td style="width: 50%; border: none;"> <math display="block">\frac{d}{dx} \left( \int_a^x f(t) dt \right) = f(x)</math> </td> <td style="width: 10%; border: none; text-align: center;">and</td> <td style="width: 40%; border: none;"> <math display="block">\int_a^b f'(x) dx = f(b) - f(a)</math> </td> </tr> </table>  | $\frac{d}{dx} \left( \int_a^x f(t) dt \right) = f(x)$   | and | $\int_a^b f'(x) dx = f(b) - f(a)$   |
| $\frac{d}{dx} \left( \int_a^x f(t) dt \right) = f(x)$   | and  | $\int_a^b f'(x) dx = f(b) - f(a)$   |     |   |

**Applications of calculus**

|  |   |
|--|---|
| Growth and decay   |   |
| Exponential equation   | $\frac{dP}{dt} = kP \Leftrightarrow P = P_0 e^{kt}$                                   |
| Logistic equation  | $\frac{dP}{dt} = rP(k - P) \Leftrightarrow P = \frac{kP_0}{P_0 + (k - P_0)e^{-rkt}}$  |
| Volumes of solids of revolution  |   |
| About the $x$ -axis  | $V = \pi \int_a^b [f(x)]^2 dx$  |
| About the $y$ -axis  | $V = \pi \int_c^d [f(y)]^2 dy$  |
| Simple harmonic motion   |   |
| <p>If <math>\frac{d^2x}{dt^2} = -k^2x</math> then <math>x = A \sin(kt + \alpha)</math> or <math>x = A \cos(kt + \beta)</math><br/>                     where <math>A</math> is the amplitude, <math>\alpha</math> and <math>\beta</math> are phase angles, <math>v</math> is the velocity and <math>x</math> is the displacement</p> |   |
| <p><math>v^2 = k^2(A^2 - x^2)</math>      Period: <math>T = \frac{2\pi}{k}</math>      Frequency: <math>f = \frac{1}{T}</math></p>   |   |
|  |   |
| Increments formula   | $\delta y \approx \frac{dy}{dx} \times \delta x$                                      |
| Acceleration   | $\frac{dv}{dt}$ or $v \frac{dv}{dx}$ or $\frac{d}{dx} \left( \frac{1}{2} v^2 \right)$ |

**Functions**

|                         |  |
|-------------------------|--|
| Quadratic function      | If $f(x) = ax^2 + bx + c$ and $f(x) = 0$ , then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ |
| Absolute value function | $ x  = \begin{cases} x, & \text{for } x \geq 0 \\ -x, & \text{for } x < 0 \end{cases}$   |

**Statistical inference**

|  |   |
|--|---|
| Confidence interval for the mean of the population | $\bar{X} - z \frac{s}{\sqrt{n}} \leq \mu \leq \bar{X} + z \frac{s}{\sqrt{n}}$ |
| Sample size  | $n = \left( \frac{z \times s}{d} \right)^2$                                   |

**Mensuration**

|               |   |
|---------------|---|
| Parallelogram | $A = bh$  |
| Triangle      | $A = \frac{1}{2}bh$ or $A = \frac{1}{2}ab \sin C$ |
| Trapezium     | $A = \frac{1}{2}(a + b)h$                         |
| Circle        | $A = \pi r^2$ and $C = 2\pi r = \pi d$            |

|          |   |   |
|----------|---|---|
| Prism    | $V = Ah$ , where $A$ is the area of the cross section   |   |
| Pyramid  | $V = \frac{1}{3}Ah$ , where $A$ is the area of the base |   |
| Cylinder | $V = \pi r^2 h$   | $TSA = 2\pi r h + 2\pi r^2$                               |
| Cone     | $V = \frac{1}{3}\pi r^2 h$                              | $TSA = \pi r s + \pi r^2$ , where $s$ is the slant height |
| Sphere   | $V = \frac{4}{3}\pi r^3$                                | $TSA = 4\pi r^2$  |

**Vectors in 3D**

|                               |   |
|-------------------------------|---|
| Magnitude                     | $ (a_1, a_2, a_3)  = \sqrt{a_1^2 + a_2^2 + a_3^2}$  |
| Dot product                   | $\mathbf{a} \cdot \mathbf{b} =  \mathbf{a}   \mathbf{b}  \cos \theta = a_1 b_1 + a_2 b_2 + a_3 b_3$   |
| Cross product                 | $\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$ |
| Equation of a line            | One point and direction $\mathbf{r} = \mathbf{a} + \lambda \mathbf{u}$  |
|                               | Two points A and B $\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$   |
| Equation of a plane           | $\mathbf{r} = \mathbf{a} + \lambda \mathbf{u}_1 + \mu \mathbf{u}_2$ or $\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$  |
| Equation of a sphere          | $ \mathbf{r} - \mathbf{d}  = r$<br>or<br>$(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$  |
| Cartesian equation of a line  | $\frac{x - a_1}{u_1} = \frac{y - a_2}{u_2} = \frac{z - a_3}{u_3}$   |
| Cartesian equation of a plane | $ax + by + cz = d$  |
| Parametric equation of a line | $x = a_1 + \lambda u_1 \dots \dots (1)$<br>$y = a_2 + \lambda u_2 \dots \dots (2)$<br>$z = a_3 + \lambda u_3 \dots \dots (3)$   |

**Complex numbers**

| Cartesian form   |   |
|--|---|
| $z = a + bi$   | $\bar{z} = a - bi$  |
| $\text{Mod}(z) =  z  = \sqrt{a^2 + b^2} = r$   | $\text{Arg}(z) = \theta, \quad \tan \theta = \frac{b}{a}, \quad -\pi < \theta \leq \pi$ |
| $ z_1 z_2  =  z_1   z_2 $  | $\left  \frac{z_1}{z_2} \right  = \frac{ z_1 }{ z_2 }$                                  |
| $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$  | $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$                              |
| $z\bar{z} =  z ^2$   | $z^{-1} = \frac{1}{z} = \frac{\bar{z}}{ z ^2}$  |
| $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$   | $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$  |
| Polar form   |   |
| $z = a + bi = r(\cos \theta + i \sin \theta) = r \text{ cis } \theta$  | $\bar{z} = r \text{ cis } (-\theta)$  |
| $z_1 z_2 = r_1 r_2 \text{ cis } (\theta_1 + \theta_2)$   | $\frac{z_1}{z_2} = \frac{r_1}{r_2} \text{ cis } (\theta_1 - \theta_2)$                  |
| $\text{cis}(\theta_1 + \theta_2) = \text{cis } \theta_1 \text{ cis } \theta_2$   | $\text{cis}(-\theta) = \frac{1}{\text{cis } \theta}$                                    |
| De Moivre's theorem  |   |
| $z^n =  z ^n \text{ cis } (n\theta)$   | $(\text{cis } \theta)^n = \cos n\theta + i \sin n\theta$                                |
| $z^{\frac{1}{q}} = r^{\frac{1}{q}} \left( \cos \frac{\theta + 2\pi k}{q} + i \sin \frac{\theta + 2\pi k}{q} \right), \quad \text{for } k \text{ an integer}$ |   |

Trigonometry

|  |   |
|--|---|
| $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$         | Length of arc = $r\theta$   |
| $a^2 = b^2 + c^2 - 2bc \cos A$                                   | Area of segment = $\frac{1}{2} r^2 (\theta - \sin \theta)$                  |
| $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$                           | Area of sector = $\frac{1}{2} r^2 \theta$                                   |
| <b>Identities</b>  |   |
| $\cos^2 x + \sin^2 x = 1$  | $1 + \tan^2 x = \sec^2 x$   |
| $\cos (x \pm y) = \cos x \cos y \mp \sin x \sin y$               | $\cos 2x = \cos^2 x - \sin^2 x$<br>$= 2 \cos^2 x - 1$<br>$= 1 - 2 \sin^2 x$ |
| $\sin (x \pm y) = \sin x \cos y \pm \cos x \sin y$               | $\sin 2x = 2 \sin x \cos x$   |
| $\tan (x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$ | $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$                                   |
| $\cos A \cos B = \frac{1}{2} (\cos(A - B) + \cos(A + B))$        | $\sin A \cos B = \frac{1}{2} (\sin(A + B) + \sin(A - B))$                   |
| $\sin A \sin B = \frac{1}{2} (\cos(A - B) - \cos(A + B))$        | $\cos A \sin B = \frac{1}{2} (\sin(A + B) - \sin(A - B))$                   |

*Note: Any additional formulas identified by the examination panel as necessary will be included in the body of the particular question.*

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