



MATHEMATICS METHODS

Calculator-assumed

ATAR course examination 2024

Marking key

Marking keys are an explicit statement about what the examining panel expect of candidates when they respond to particular examination items. They help ensure a consistent interpretation of the criteria that guide the awarding of marks.

Question 8

(11 marks)

- (a) Calculate the mass of medication remaining in John’s body 10 hours after taking a single tablet. (1 mark)

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|--|
| Solution |
| Setting $t = 10$ gives $A(10) = 5e^{-0.0173(10)}$ $\approx 4.21 \text{ mg}$ |
| Specific behaviours |
| ✓ obtains correct mass of medication including units |

- (b) After how many hours will the mass of medication remaining in John’s body have halved? (2 marks)

| |
|--|
| Solution |
| Setting $A = 2.5$ gives $2.5 = 5e^{-0.0173(t)}$ $\Rightarrow 0.5 = e^{-0.0173(t)}$ $\Rightarrow t = -\frac{1}{0.0173} \ln(0.5)$ $= 40.066$ $\approx 40 \text{ hours}$ |
| Specific behaviours |
| ✓ correctly substitutes $A = 2.5$ into the equation ✓ correctly solves for t |

- (c) Determine at what rate the mass of medication remaining in John’s body is decreasing 24 hours after taking a single tablet. (3 marks)

| |
|--|
| Solution |
| The derivative of A is $A'(t) = -0.0865 e^{-0.0173t}$ When $t = 24$ $A'(24) = -0.0865 e^{-0.0173(24)}$ $= -0.057 \text{ mg/hr}$ The mass is decreasing at a rate of 0.057 mg/hr. |
| Specific behaviours |
| ✓ correctly differentiates A ✓ substitutes $t = 24$ to obtain correct rate of increase ✓ converts to a rate of decrease |

- (d) How frequently should John take a tablet so that the mass of medication remaining in his body immediately after taking each tablet is 8.85 mg? (2 marks)

| Solution |
|--|
| <p>Setting $B = 8.85$ gives</p> $8.85 = \frac{5}{1 - e^{-0.0173T}}$ $\Rightarrow 1 - e^{-0.0173T} = \frac{5}{8.85}$ $\Rightarrow e^{-0.0173T} = \frac{3.85}{8.85}$ $\Rightarrow T = -\frac{1}{0.0173} \ln\left(\frac{3.85}{8.85}\right)$ $= 48.112$ $\approx 48 \text{ hours}$ <p>John should take one tablet every 48 hours.</p> |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ correctly substitutes $B = 8.85$ into the equation ✓ correctly solves for T |

- (e) Use the increments formula to approximate the change in B if the time between taking tablets increased by 30 minutes from the time determined in part (d). (3 marks)

| Solution |
|---|
| <p>The derivative of B with respect to T is given by</p> $\frac{dB}{dT} = -\frac{0.0865e^{-0.0173T}}{(1 - e^{-0.0173T})^2}$ <p>and when $T = 48$</p> $\frac{dB}{dT}(48) \approx -0.118$ <p>An increase of 30 minutes corresponds to $\delta T = 0.5$. Using the increments formula gives</p> $\delta B \approx -0.118 \times 0.5$ $= -0.059 \text{ mg}$ |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ correctly differentiates B with respect to T ✓ states that $\delta T = 0.5$ ✓ correctly uses the increments formula to estimate the change in B |

Question 9

(8 marks)

- (a) Calculate a 90% confidence interval for the proportion of heads obtained when the coin is flipped. (2 marks)

| |
|--|
| Solution |
| The sample proportion of heads is given by $\hat{p} = \frac{30}{50} = 0.6$ Hence, the 90% confidence interval is $0.6 - 1.645\sqrt{\frac{0.6 \times 0.4}{50}} \leq p \leq 0.6 + 1.645\sqrt{\frac{0.6 \times 0.4}{50}}$ $0.4860 \leq p \leq 0.7140$ |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ correctly calculates sample proportion of heads ✓ correctly calculates confidence interval |

- (b) State the distribution for X . (2 marks)

| |
|--|
| Solution |
| $X \sim \text{Bin}(20, 0.9)$ |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ states that the distribution is binomial ✓ states correct distribution parameters |

- (c) Determine the expected value and variance of X . (2 marks)

| |
|--|
| Solution |
| The expected value of X is given by $E(X) = 20 \times 0.9$ $= 18$ The variance of X is given by $\text{Var}(X) = 20 \times 0.9 \times 0.1$ $= 1.8$ |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ correctly calculates expected value ✓ correctly calculates variance |

- (d) Calculate the probability that the confidence intervals of three students do not contain the true proportion. (2 marks)

| |
|---|
| Solution |
| If three confidence intervals did not contain the true proportion, then 17 did contain the true proportion. $P(X = 17) = 0.1901$ |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ identifies that they are considering 17 confidence intervals containing the true proportion or defines the distribution for the complementary event ✓ calculates the correct probability |

Question 10

(13 marks)

- (a) (i) Identify and explain **one** possible source of bias in the proposed sampling procedure. (2 marks)

| Solution |
|--|
| Answers could include: <ul style="list-style-type: none"> • the sample only includes books from the newest printing press, which may perform differently to the other three presses • the sample was gathered over a single narrow time period, so books printed later in the week are not included (this may involve different operators and the performance of the presses might change during their use over the week). |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ identifies a source of bias ✓ provides a correct explanation for that source |

- (ii) Identify **two** changes to the sampling procedure that would reduce bias. (2 marks)

| Solution |
|---|
| The sample could be randomly selected: <ul style="list-style-type: none"> • from books printed across the entire duration of the print run • from all printing presses. |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ suggests a change to improve randomisation across the entire print run ✓ suggests a second change to improve randomisation across the entire print run |

- (b) Use the approximate normality of the distribution of sample proportions to determine the probability that the sample proportion of books with errors is less than 0.04. (2 marks)

| Solution |
|---|
| The distribution of \hat{p} can be approximated as $\hat{p} \sim N(0.05, 0.0002375)$ Hence, $P(\hat{p} < 0.04) = 0.2582$ |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ calculates the correct distribution parameters (mean and standard deviation/variance) ✓ calculates the correct probability |

- (c) Determine a 95% confidence interval for the proportion of books that will have printing errors. (1 mark)

| Solution |
|---|
| A 95% confidence interval is given by $95\% \text{ CI} = (0.1 - 0.024, 0.1 + 0.024)$ $= (0.0760, 0.1240)$ |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ correctly calculates confidence interval |

Question 10 (continued)

- (d) On the basis of the confidence interval determined in part (c), is the proportion of books with printing errors different from what was claimed by the publisher? (2 marks)

| |
|--|
| Solution |
| The proportion of books with errors claimed by the publisher was $\frac{10}{200} = 0.05$. This proportion is not within the confidence interval and so there is sufficient evidence to conclude that the claim of the publisher is incorrect at the 95% confidence level. |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ states that the claimed proportion is not within the confidence interval ✓ states that there is sufficient evidence at the above confidence level to conclude that the claimed proportion is incorrect |

- (e) Suggest **two** changes that could be made in order to decrease the margin of error of the confidence interval. (2 marks)

| |
|---|
| Solution |
| The margin of error could be decreased by <ul style="list-style-type: none"> • increasing the size of the sample • decreasing the confidence level. |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ states one possible change ✓ states a second possible change |

- (f) Determine the minimum sample size that would be necessary to guarantee that the margin of error of the resulting 95% confidence interval was at most 0.02. (2 marks)

| |
|--|
| Solution |
| For the worst-case scenario set $\hat{p} = 0.5$. Solving for a margin of error equal to 0.02 gives |
| $0.02 = 1.96\sqrt{\frac{0.5(1-0.5)}{n}}$ $\Rightarrow n = 2401$ |
| A sample size of at least 2401 would ensure the margin of error is at most 0.02. |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ sets $\hat{p} = 0.5$ and provides a correct expression for the sample size ✓ correctly solves for the minimum sample size. |

Question 11

(10 marks)

(a) Given that the width of the 95% confidence interval for p is 0.096

(i) determine the 95% confidence interval for p . (2 marks)

| Solution |
|--|
| The margin of error of the 95% confidence interval is $E = \frac{0.096}{2} = 0.048$ The sample proportion is $\hat{p} = 0.7$, and so the confidence interval is $\hat{p} - E \leq p \leq \hat{p} + E$ $\Rightarrow 0.7 - 0.048 \leq p \leq 0.7 + 0.048$ $\Rightarrow 0.652 \leq p \leq 0.748$ |
| Specific behaviours |
| ✓ correctly determines the margin of error of the confidence interval ✓ correctly states the confidence interval |

(ii) determine the number of people surveyed. (2 marks)

| Solution |
|--|
| For a 95% confidence interval $E = 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $\Rightarrow 0.048 = 1.96\sqrt{\frac{0.7(1-0.7)}{n}}$ $\Rightarrow n = 350.14$ Therefore, there were 350 people surveyed. |
| Specific behaviours |
| ✓ correctly solves for n ✓ correctly rounds to the nearest integer |

(b) What does the data scientist's confidence interval suggest about the protest group's claim? (2 marks)

| Solution |
|--|
| The proportion claimed by the protest group falls within the 95% confidence interval. Hence, the claim cannot be rejected at the 95% confidence level. |
| Specific behaviours |
| ✓ states that the claimed proportion is within the confidence interval ✓ concludes that the claimed proportion cannot be rejected at the 95% confidence level |

Question 11 (continued)

(c) For each of the following city pairs, identify which had the widest 95% confidence interval. Justify your answer.

(i) Brisbane and Sydney (2 marks)

| |
|---|
| Solution |
| Brisbane has the widest confidence interval as it has a smaller sample size (and the same sample proportion). |
| Specific behaviours |
| ✓ states that Brisbane has the widest confidence interval ✓ provides correct justification |

(ii) Brisbane and Hobart (2 marks)

| |
|---|
| Solution |
| Brisbane has the widest confidence interval as its sample proportion is closest to 0.5 (and the sample size is the same). |
| Specific behaviours |
| ✓ states that Brisbane has the widest confidence interval ✓ provides correct justification |

Question 12

(9 marks)

(a) Determine the exact radius of the log.

(2 marks)

| Solution | |
|---|---|
| By Pythagoras' theorem | $(2r)^2 = 40^2 + 40^2$ $= 3200$ $\Rightarrow 2r = \sqrt{3200}$ $= 40\sqrt{2}$ $\Rightarrow r = 20\sqrt{2} \text{ cm}$ |
| Or | $r^2 = 20^2 + 20^2$ $= 800$ $\Rightarrow r = \sqrt{800}$ $r = 20\sqrt{2} \text{ cm}$ |
| Specific behaviours | |
| <ul style="list-style-type: none"> ✓ writes a correct expression for the radius using Pythagoras' theorem ✓ calculates the exact radius | |

(b) Using the variables defined in the diagram, show that the cross-sectional area, in cm^2 , of a single sideboard is $A(x) = 2x\sqrt{400 - 40x - x^2}$.

(3 marks)

| Solution | |
|---|---|
| Using Pythagoras' theorem to relate x and y gives | |
| | $r^2 = \left(\frac{y}{2}\right)^2 + (x + 20)^2$ $\Rightarrow 800 = \left(\frac{y}{2}\right)^2 + (x + 20)^2$ $\Rightarrow \left(\frac{y}{2}\right)^2 = 800 - (x + 20)^2$ $\Rightarrow \frac{y}{2} = \sqrt{800 - (x + 20)^2}$ $\Rightarrow y = 2\sqrt{400 - 40x - x^2}$ |
| Hence, the area is given by | |
| | $A = xy$ $= 2x\sqrt{400 - 40x - x^2}$ |
| Specific behaviours | |
| <ul style="list-style-type: none"> ✓ uses Pythagoras' theorem to write a correct expression relating x and y ✓ solves expression for y (i.e. expresses y in terms of x) ✓ combines expression for y with $A = xy$ to obtain required result | |

Question 12 (continued)

- (c) Use calculus techniques to determine the dimensions x and y that maximise the cross-sectional area of one sideboard. (4 marks)

| Solution | |
|--|---|
| <p>The derivative of A is</p> $A'(x) = 2\sqrt{400 - 40x - x^2} - \frac{x(40 + 2x)}{\sqrt{400 - 40x - x^2}}$ $= -\frac{4x^2 + 120x - 800}{\sqrt{400 - 40x - x^2}}$ <p>Solving $A'(x) = 0$ gives</p> $0 = -\frac{4x^2 + 120x - 800}{\sqrt{400 - 40x - x^2}}$ $0 = 4x^2 + 120x - 800$ $x = -15 \pm 5\sqrt{17}$ $\approx 5.6155, -35.6155$ <p>Hence, $x = 5.6155$ cm as the lengths cannot be negative. Since $A''(5.6155) = -13.75 < 0$ it follows that $x = 5.6155$ is a local maximum. The corresponding value of y is given by</p> $y = 2\sqrt{400 - 40(5.6155) - 5.6155^2}$ $= 23.9872 \text{ cm}$ | <p style="text-align: center;">Specific behaviours</p> <ul style="list-style-type: none"> ✓ states correct derivative of A ✓ solves $A'(x) = 0$ to obtain $x = 5.62$ cm ✓ verifies that $x = 5.62$ is a local maximum using either the second derivative test or sign test ✓ calculates the correct value for y |

Question 13

(12 marks)

- (a) Determine the standard deviation of
- X
- . (2 marks)

| Solution | |
|---|--------------------------------------|
| From CAS | $P(Z > 0.6666) = 0.2525$ |
| Hence, | $0.6666 = \frac{400 - 350}{\sigma}$ |
| | $\Rightarrow \sigma = 75 \text{ km}$ |
| Specific behaviours | |
| <ul style="list-style-type: none"> ✓ calculates correct z-value ✓ correctly calculates the standard deviation | |

- (b) Calculate the probability that on any given day she will be able to drive to Albany without recharging the vehicle. (1 mark)

| Solution | |
|--|-----------------------|
| Given that $X \sim N(350, 75^2)$ it follows that | $P(X > 420) = 0.1753$ |
| Specific behaviours | |
| ✓ correctly calculates the probability | |

- (c) Determine the expected value and variance of
- Y
- . (3 marks)

| Solution | |
|---|----------------------------------|
| Given that $Y = \frac{1}{1.6}X$ it follows that | $E(Y) = \frac{1}{1.6}E(X)$ |
| | $= \frac{350}{1.6}$ |
| | $= 218.75 \text{ miles}$ |
| and | $Var(Y) = \frac{1}{1.6^2}Var(X)$ |
| | $= \frac{75^2}{1.6^2}$ |
| | $\approx 2197 \text{ miles}^2$ |
| Specific behaviours | |
| <ul style="list-style-type: none"> ✓ determines correct relationship between X and Y ✓ calculates the correct expected value for Y ✓ calculates the correct variance for Y | |

Question 13 (continued)

- (d) On the basis of the histogram, is it appropriate to use a normal distribution to model the distance a Spruky Cars vehicle will travel between recharges? Justify your answer. (2 marks)

| |
|--|
| Solution |
| No. The graph is skewed/not symmetrical. |
| Specific behaviours |
| ✓ concludes that the normal distribution is not an appropriate model ✓ provides appropriate justification |

- (e) Assuming the distances are uniformly distributed within each interval, use the histogram to estimate the expected distance that a Spruky Cars vehicle will be able to travel before needing to recharge. (2 marks)

| |
|---|
| Solution |
| Using the interval mid-points the expected distance can be estimated as follows |
| $E(W) = 270 \times \frac{4}{200} + 290 \times \frac{8}{200} + 310 \times \frac{10}{200} + 330 \times \frac{12}{200} + 350 \times \frac{18}{200} + 370 \times \frac{40}{200}$ $+ 390 \times \frac{54}{200} + 410 \times \frac{34}{200} + 430 \times \frac{20}{200}$ $= 375.8 \text{ km}$ |
| Specific behaviours |
| ✓ correctly converts frequencies to probabilities ✓ correctly calculates expected value |

- (f) In which company's vehicle (Zaprer or Spruky) would Brianna be more likely to drive to Albany without recharging? Justify your answer. (2 marks)

| |
|--|
| Solution |
| The vehicle needs to travel 420 km in order to arrive in Albany from Brianna's house. Given that $P(X > 420) = 0.1753$ is greater than $P(W > 420) = 0.1$, Brianna would be most likely to drive to Albany without recharging using a Zaprer vehicle. |
| Specific behaviours |
| ✓ determines that Zaprer vehicle is most likely ✓ correct mathematical justification provided |

Question 14

(14 marks)

(a) Using the experimental data above, estimate the probability of

(i) winning in exactly two rolls. (1 mark)

| |
|--|
| Solution |
| $P(X = 2) = \frac{113}{500}$ $= 0.226$ |
| Specific behaviours |
| ✓ calculates correct probability |

(ii) not winning in two or less rolls. (2 marks)

| |
|--|
| Solution |
| Probability of winning in two or less rolls is |
| $P(X \leq 2) = \frac{66 + 113}{500}$ $= 0.358$ |
| So, the probability of not winning in 2 or less rolls is |
| $P(X > 2) = 1 - P(X \leq 2)$ $= 1 - 0.358$ $= 0.642$ |
| Specific behaviours |
| ✓ correctly calculates $P(X \leq 2)$ |
| ✓ correctly calculates $P(X > 2)$ |

(b) State **two** reasons why the game cannot be modelled using a binomial distribution. (2 marks)

| |
|---|
| Solution |
| Answers could include: |
| <ul style="list-style-type: none"> • the number of rolls/trials is not fixed. Rolls will continue until the winning condition is reached • the probability of success (achieving at least two winning dice) is not fixed (i.e. if a single winning dice is removed) • trials/rolls are not independent as the number of dice in a roll depends on the outcome of previous rolls. |
| Specific behaviours |
| ✓ one valid reason is provided |
| ✓ a second valid reason is provided |

Question 14 (continued)

- (c) Using the data above, complete the probability distribution table for Y . (3 marks)

| Solution | | | |
|---|-----------------------------|-------|-------------------------|
| Y | -1 | 0 | 1 |
| $P(Y = y)$ | $1 - 0.349 - 0.208 = 0.443$ | 0.208 | $0.134 + 0.215 = 0.349$ |
| Specific behaviours | | | |
| <ul style="list-style-type: none"> ✓ correctly lists possible values for Y (first row of table) ✓ correctly determines one probability ✓ correctly determines remaining two probabilities | | | |

- (d) Calculate the

- (i) expected value of Y . (2 marks)

| Solution |
|---|
| $E(Y) = 1 \times 0.349 + 0 \times 0.208 - 1 \times 0.443$ $= -\$0.094$ |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ states correct expression for the expected value ✓ correctly calculates the expected value |

- (ii) variance of Y . (2 marks)

| Solution |
|---|
| $Var(Y) = (1 - (-0.094))^2 \times 0.349 + (0 - (-0.094))^2 \times 0.208$ $+ (-1 - (-0.094))^2 \times 0.443$ $= \$^2 0.783$ |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ states correct expression for the variance ✓ correctly calculates the variance |

- (e) In the long run, do you expect that the game will be profitable for the charity? Justify your answer. (2 marks)

| Solution |
|---|
| Yes. The expected profit to the player is $-\$0.094$, so the expected profit to the charity is $\$0.094$. |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ states that it is expected to be profitable ✓ provides a correct justification with reference to the answer from part (d)(i) |

Question 15

(9 marks)

- (a) Use the graph to approximate the moment magnitude M_w of an earthquake with a seismic moment of 3.16×10^{13} Nm. You must show clearly how you have used the graph. (2 marks)

| Solution | |
|--|--|
| <p>Firstly, note that $M_0 = 3.16 \times 10^{13}$ then</p> $\log_{10}(M_0) = \log_{10}(3.16 \times 10^{13}) = 13.5$ <p>Using the graph as shown below, $M_w = 3$.</p> | |
| | |
| Specific behaviours | |
| <ul style="list-style-type: none"> ✓ correctly calculates the value of $\log_{10}(3.16 \times 10^{13})$ ✓ clearly demonstrates the use of the graph to approximate the moment magnitude | |

- (b) The relationship between M_w and M_0 can be expressed in the form

$$M_w = a \log_{10}(M_0) + b.$$

Determine the values of a and b .

(2 marks)

| Solution | |
|--|--|
| <p>The vertical intercept of the graph is $b = -6$. The gradient is</p> $a = \frac{6}{9} = \frac{2}{3}$ | |
| Specific behaviours | |
| <ul style="list-style-type: none"> ✓ correctly determines the value of b ✓ clearly calculates the value of a | |

Question 15 (continued)

(c) Hence, or otherwise, express the relationship between M_w and M_0 in the form

$$M_w = a \log_{10} \left(\frac{M_0}{c} \right). \quad (3 \text{ marks})$$

| Solution | |
|---|--|
| From part (b) | $M_w = \frac{2}{3} \log_{10} (M_0) - 6$ $= \frac{2}{3} (\log_{10} (M_0) - 9)$ $= \frac{2}{3} (\log_{10} (M_0) - \log_{10} (10^9))$ $= \frac{2}{3} \log_{10} \left(\frac{M_0}{10^9} \right)$ |
| Specific behaviours | |
| <ul style="list-style-type: none"> ✓ factors out $\frac{2}{3}$ in the expression from part (b) ✓ expresses 9 in the form $\log_{10} (10^9)$ ✓ applies logarithm law to obtain the correct expression | |

(d) Determine the seismic moment, M_0 , of an earthquake with moment magnitude $M_w = 4$.
(2 marks)

| Solution | |
|--|--|
| Or | $4 = \frac{2}{3} \log_{10} \left(\frac{M_0}{10^9} \right)$ $\Rightarrow 6 = \log_{10} \left(\frac{M_0}{10^9} \right)$ $\Rightarrow 10^6 = \frac{M_0}{10^9}$ $\Rightarrow M_0 = 10^{15} \text{ Nm}$ |
| | $4 = \frac{2}{3} \log_{10} (M_0) - 6$ $\Rightarrow 15 = \log_{10} (M_0)$ $\Rightarrow M_0 = 10^{15} \text{ Nm}$ |
| Or could obtain answer from graph. | |
| Specific behaviours | |
| <ul style="list-style-type: none"> ✓ substitutes $M_w = 4$ into the equation from part (b) or (c), or uses the graph to determine $\log_{10} (M_0) = 15$ ✓ correctly solves for the seismic moment | |

Question 16

(8 marks)

- (a) Calculate the volume V of shampoo in the bottle, if it is partially filled to a height of 10 cm. (4 marks)

| Solution | |
|---|--|
| <p>The shampoo level meets the edge of the bottle when</p> $10 = 20 - \frac{4}{5}x^2$ $\Rightarrow x = \pm \frac{5}{\sqrt{2}}$ <p>Hence, the volume of shampoo is given by</p> $V = 4 \left(\frac{10}{\sqrt{2}} \times 10 + 2 \int_{\frac{5}{\sqrt{2}}}^5 20 - \frac{4}{5}x^2 dx \right)$ $= \frac{400}{\sqrt{2}} + 8 \left[20x - \frac{4}{15}x^3 \right]_{\frac{5}{\sqrt{2}}}^5$ $= \frac{400}{\sqrt{2}} + 8 \left(\frac{200}{3} - \frac{250}{3\sqrt{2}} \right)$ $= \frac{1600}{3} - \frac{800}{3\sqrt{2}}$ $= \frac{1600 - 400\sqrt{2}}{3} \text{ cm}^3 (\approx 344.77 \text{ cm}^3)$ <p>Or</p> $V = 4 \left(\int_{-5}^5 20 - \frac{4}{5}x^2 dx - \int_{-\frac{5}{\sqrt{2}}}^{\frac{5}{\sqrt{2}}} 20 - \frac{4}{5}x^2 - 10 dx \right)$ $= 4 \left(\frac{400}{3} - \frac{100\sqrt{2}}{3} \right)$ $= \frac{1600 - 400\sqrt{2}}{3} \text{ cm}^3$ | |
| Specific behaviours | |
| <ul style="list-style-type: none"> ✓ correctly calculates value/s of x where shampoo level meets the edge of the bottle ✓ states a correct expression for the cross-sectional area ✓ multiplies by the width to obtain a volume expression ✓ correctly evaluates the volume | |

Question 16 (continued)

(b) Determine the shampoo level h .

(4 marks)

| Solution | |
|---|--|
| <p>The shampoo level meets the edge of the bottle when</p> $h = \frac{4}{5}x^2$ $\Rightarrow x = \pm \frac{\sqrt{5h}}{2}$ <p>Using the volume calculated in part (a) it follows that</p> $\frac{1600 - 400\sqrt{2}}{3} = 4 \left(2 \int_0^{\frac{\sqrt{5h}}{2}} h - \frac{4}{5}x^2 dx \right)$ $= 8 \left[hx - \frac{4}{15}x^3 \right]_0^{\frac{\sqrt{5h}}{2}}$ $= 4\sqrt{5}h^{\frac{3}{2}} - \frac{4\sqrt{5}}{3}h^{\frac{3}{2}}$ $= \frac{8\sqrt{5}}{3}h^{\frac{3}{2}}$ $\Rightarrow h^{\frac{3}{2}} = \frac{1600 - 400\sqrt{2}}{8\sqrt{5}}$ $\Rightarrow h = \left(\frac{1600 - 400\sqrt{2}}{8\sqrt{5}} \right)^{\frac{2}{3}}$ $= (40\sqrt{5} - 10\sqrt{10})^{\frac{2}{3}} \text{ cm } (\approx 14.95 \text{ cm})$ | |
| Specific behaviours | |
| <ul style="list-style-type: none"> ✓ correctly determines an expression for x in terms of h where the shampoo level meets the edge of the bottle ✓ states a correct expression relating the shampoo volume/cross-sectional area to h ✓ correctly evaluates the volume integral ✓ correctly solves for h | |

Question 17

(6 marks)

- (a) Determine the parameters b and c , given that the speed cuber already knows 21 of the ZBLL algorithms (at $t = 0$) and learnt an additional 32 algorithms by the end of the first week. (3 marks)

| Solution |
|--|
| <p>We are told that $A(0) = 21$, and $A(1) = 21 + 32 = 53$. Hence,</p> $21 = b \log_4(1) + c$ $53 = b \log_4(2) + c$ <p>From the first equation $c = 21$. Substituting into the second equation yields</p> $53 = b \log_4(2) + 21$ $\Rightarrow b = 64$ |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ determines that $A(1) = 53$ ✓ correctly solves for c ✓ correctly solves for b |

- (b) Determine how many of the ZBLL algorithms the speed cuber will have learnt after 26 weeks. (1 mark)

| Solution |
|---|
| $A(26) = 64 \log_4(27) + 21$ $= 173.16$ <p>The speed cuber will know 173 algorithms after 26 weeks.</p> |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ correctly determines the number of algorithms, rounding down to the nearest integer |

- (c) Based on the assumed model, will the speed cuber learn the entire ZBLL algorithm set within their lifetime? Justify your answer. (2 marks)

| Solution |
|--|
| <p>The time to learn the entire set is given by</p> $493 = 64 \log_4(t+1) + 21$ $t = 27553 \text{ weeks}$ <p>This equates to approximately 528 years, which is far longer than a human lifetime. Hence, the speed cuber will not learn the entire algorithm set.</p> |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ correctly determines the number of weeks to learn the full algorithm set ✓ concludes, with justification, that the speed cuber will not learn the full set |

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