MATHEMATICS SPECIALIST

Unit 1 and Unit 2

Formula Sheet

(For use with Year 11 examinations and response tasks)
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This document is valid for teaching and examining from 1 July 2015.
Measurement

Circle: \[ C = 2\pi r = \pi D, \text{ where } C \text{ is the circumference, } \]
\[ r \text{ is the radius and } D \text{ is the diameter} \]
\[ A = \pi r^2, \text{ where } A \text{ is the area} \]

Triangle: \[ A = \frac{1}{2} bh, \text{ where } b \text{ is the base and } h \text{ is the perpendicular height} \]

Parallelogram: \[ A = bh \]

Trapezium: \[ A = \frac{1}{2} (a + b)h, \text{ where } a \text{ and } b \text{ are the lengths of the parallel sides} \]

Prism: \[ V = Ah, \text{ where } V \text{ is the volume and } A \text{ is the area of the base} \]

Pyramid: \[ V = \frac{1}{3} Ah \]

Cylinder: \[ S = 2\pi rh + 2\pi r^2, \text{ where } S \text{ is the total surface area} \]
\[ V = \pi r^2h \]

Cone: \[ S = \pi rs + \pi r^2, \text{ where } s \text{ is the slant height} \]
\[ V = \frac{1}{3}\pi r^2h \]

Sphere: \[ S = 4\pi r^2 \]
\[ V = \frac{4}{3}\pi r^3 \]
Combinatorics

Combinations

Number of arrangements: (of \( n \) different objects in an ordered list)
\[
n(n-1)(n-2) \times \ldots \times 3 \times 2 \times 1 = n!
\]

Number of combinations: (of \( r \) objects taken from a set of \( n \) distinct objects)
\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} ; \quad \binom{n}{n} = 1 ; \quad \binom{n}{0} = 1
\]

Number of permutations: (of \( r \) objects taken from a set of \( n \) distinct objects)
\[
nP_r = n(n-1)(n-2) \ldots (n-r+1) = \frac{n!}{(n-r)!}
\]

Number of permutations with some identical objects:
\[
\frac{n!}{r_1!r_2!r_3! \ldots}
\]

Inclusion – exclusion principle:
\[
|A \cup B| = |A| + |B| - |A \cap B|
\]
\[
|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|
\]

Vectors in the Plane

Representing vectors

Magnitude of a vector:
\[
|\mathbf{a}| = \left|\left(a_1, a_2\right)\right| = \sqrt{a_1^2 + a_2^2}
\]

Algebra of vectors

Unit vector:
\[
\hat{a} = \frac{a}{|a|}
\]

Scalar product:
\[
\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta \quad \text{or} \quad \mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2
\]

Vector projection (of \( \mathbf{a} \) on \( \mathbf{b} \)):
\[
\mathbf{p} = (\mathbf{a} \cdot \mathbf{b}) \mathbf{b} = |\mathbf{a}| \cos \theta \mathbf{b}
\]
Trigonometry

Basic trigonometric functions

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\sin(\theta + \frac{\pi}{2}) = \cos \theta$$

$$\cos(\theta - \frac{\pi}{2}) = \sin \theta$$

Cosine and sine rules

For any triangle $ABC$ with corresponding length of sides $a,b,c$

Cosine rule:  
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Sine rule:  
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Area of $\Delta$:  
$$A = \frac{1}{2}ab \sin C$$

$$= \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{1}{2}(a+b+c)$$

Circular measure and radian measure

In a circle of radius $r$ for an arc subtending angle $\theta$ (radians) at the centre

Length of arc:  
$$\ell = r\theta$$

Length of chord:  
$$l = 2r \sin \frac{1}{2}\theta$$

Area of sector:  
$$A = \frac{1}{2}r^2\theta$$

Area of segment:  
$$A = \frac{1}{2}r^2(\theta - \sin \theta)$$

Compound angles

Angle sum and difference identities:  
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Double angle identities:  
$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$
Reciprocal trigonometric functions

\[
\sec \theta = \frac{1}{\cos \theta}, \quad \cos \theta \neq 0 \\
\cosec \theta = \frac{1}{\sin \theta}, \quad \sin \theta \neq 0 \\
\cot \theta = \frac{1}{\tan \theta}, \quad \tan \theta \neq 0
\]

Trigonometric identities

Pythagorean identities:

\[
\sin^2 \theta + \cos^2 \theta = 1 \\
1 + \tan^2 \theta = \sec^2 \theta \\
\cot^2 \theta + 1 = \cosec^2 \theta
\]

Product identities:

\[
\cos A \cos B = \frac{1}{2} \left[ \cos(A - B) + \cos(A + B) \right] \\
\sin A \sin B = \frac{1}{2} \left[ \cos(A - B) - \cos(A + B) \right] \\
\sin A \cos B = \frac{1}{2} \left[ \sin(A + B) + \sin(A - B) \right] \\
\cos A \sin B = \frac{1}{2} \left[ \sin(A + B) - \sin(A - B) \right]
\]

Auxiliary angle formulae:

\[
a \sin x \pm b \cos x = R \sin(x \pm \alpha) \quad \text{for} \quad 0 < \alpha < \frac{\pi}{2}, \quad \text{where} \quad R^2 = a^2 + b^2, \tan \alpha = \frac{b}{a}
\]

Triple angle identities:

\[
\sin(3A) = 3 \sin A - 4 \sin^3 A \\
\cos(3A) = 4 \cos^3 A - 3 \cos A \\
\tan(3A) = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}
\]
Matrices

Matrix arithmetic

Identity matrix: If \( A \) is invertible, \( AA^{-1} = I \) where \( I \) is the identity matrix

Inverse matrix: \[
\begin{bmatrix}
a & b \\
c & d
\end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix}
d & -b \\
-c & a
\end{bmatrix}
\]

Determinant: If \( A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \), then \( \text{det} \ A = ad - bc \)

Transformation Matrices

Dilation: \[
\begin{bmatrix}
a & 0 \\
0 & b
\end{bmatrix}
\]

Rotation: \[
\begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}
\]

where \( \theta \) is an anti-clockwise rotation about the origin

Reflection: \[
\begin{bmatrix}
\cos 2\theta & \sin 2\theta \\
\sin 2\theta & -\cos 2\theta
\end{bmatrix}
\]

where the reflection is in the line \( y = x \tan \theta \)

Real and Complex numbers

Number Sets

Natural Numbers: \( \mathbb{N} := \{1, 2, 3, \ldots \} \)

Integer Numbers: \( \mathbb{Z} := \{\ldots, -2, -1, 0, 1, 2, \ldots \} \)

Rational Numbers: \( \mathbb{Q} := \left\{ q : q = \frac{a}{b}, \text{ where } a \text{ and } b \text{ are integers, } b \neq 0 \right\} \)

Irrational Numbers: Numbers that cannot be expressed as the ratio of two integers

Real Numbers: The set of all rational and irrational numbers \( (\mathbb{R}) \)

Complex Numbers: \( \mathbb{C} := \{ z : z = a + bi, \text{ where } a, b \in \mathbb{R}, i^2 = -1 \} \)

Complex Numbers

For \( z = a + ib \), where \( a, b \in \mathbb{R}, i^2 = -1 \)

Modulus: \( \text{mod } z = |z| = |a + ib| = \sqrt{a^2 + b^2} \)

Product: \( |z_1z_2| = |z_1||z_2| \)

Conjugate: \( \overline{z} = a - ib, \overline{z^2} = \overline{z}, \overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}, \overline{z_1z_2} = \overline{z_1}\overline{z_2} \)
Other useful results

Binomial expansion:

\[(x + y)^n = x^n + \binom{n}{1}x^{n-1}y + \cdots + \binom{n}{r}x^{n-r}y^r + \cdots + y^n\]

Binomial coefficients:

\[\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\cdots(n-r+1)}{r(r-1)\cdots2\cdot1}\]

Index laws:

For \(a, b > 0\) and \(m, n\) real,

\[a^m b^m = (ab)^m\]
\[a^m a^n = a^{m+n}\]
\[(a^m)^n = a^{mn}\]
\[a^{-m} = \frac{1}{a^m}\]
\[\frac{a^m}{a^n} = a^{m-n}\]
\[a^0 = 1\]

For \(a > 0\), \(m\) an integer and \(n\) a positive integer, \(a^\frac{m}{n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m\)

Arithmetic sequences

For initial term \(a\) and common difference \(d\):

\[T_n = a + (n-1)d, n \geq 1\]
\[T_{n+1} = T_n + d, \text{ where } T_1 = a\]
\[S_n = \frac{n}{2}(2a + (n-1)d)\]

Geometric sequences

For initial term \(a\) and common difference \(r\):

\[T_{n+1} = rT_n, \text{ where } T_1 = a\]
\[T_n = ar^{n-1}, n \geq 1\]
\[S_n = \frac{a(1-r^n)}{1-r}\]
\[S_{\infty} = \frac{a}{1-r}, \text{ for } |r| < 1\]

Lines and Linear relationships

For points \(P(x_1, y_1)\) and \(Q(x_2, y_2)\)

Mid-point of \(P\) and \(Q:\)

\[M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\]

Gradient of the line through \(P\) and \(Q:\)

\[m = \frac{y_2 - y_1}{x_2 - x_1}\]

Equation of the line through \(P\) with slope \(m:\)

\[y - y_1 = m(x - x_1)\]

Parallel lines:

\[m_1 = m_2\]

Perpendicular lines:

\[m_1 m_2 = -1\]

General equation of a line:

\[ax + by + c = 0 \text{ or } y = mx + c\]
Quadratic relationships

For the general quadratic equation \( ax^2 + bx + c = 0, \ a \neq 0 \)

Completing the square: \( ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right) \)

Discriminant: \( \Delta = b^2 - 4ac \)

Quadratic formula: \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

Graphs and Relations

Equation of a circle: \( (x-a)^2 + (y-b)^2 = r^2 \)
where, \( (a,b) \) is the centre and \( r \) is the radius

Note: Any additional formulas identified by the examination writers as necessary will be included in the body of the particular question.