



Western Australian Certificate of Education Examination, 2015

Question/Answer Booklet

MATHEMATICS: SPECIALIST 3A/3B

Section Two: Calculator-assumed

Please place your student identification label in this box

Student Number: In figures

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In words

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

Number of additional answer booklets used (if applicable):

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	50	33 $\frac{1}{3}$
Section Two: Calculator-assumed	13	13	100	100	66 $\frac{2}{3}$
Total					100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section Two: Calculator-assumed

66²/₃% (100 Marks)

This section has **13** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 9**(6 marks)**

For this question, assume that the earth is spherical, with a radius of 6370 km.

Consider a point P on the surface of the earth that has latitude and longitude $(L^\circ S, 116^\circ E)$, where L is an unknown.

If you travel x kilometres due south from P the latitude changes by 5° .

If you travel y kilometres due east from P the longitude increases by 5° .

(a) If $L = 29$, determine the values of x and y .

(4 marks)

(b) If $x = y$ what can you say about the value of L ?

(2 marks)

Question 10

(5 marks)

A surveyor records measurements for the sides a , b and c of a triangular field as follows:

Side	Length in metres
a	275
b	180
c	X

- (a) From the information given, what are you able to say about the value of X ?
Justify your answer. (2 marks)
- (b) If $X = 132$, calculate the cosine of the angle of the vertex C which lies opposite side c . (1 mark)
- (c) Deduce the value of $\sin C$. (1 mark)
- (d) Hence, or otherwise, determine the area of the field. (1 mark)

Question 11

(8 marks)

(a) The function $f(x)$ is given by

(5 marks)

$$f(x) = \begin{cases} x + 2 & \text{when } x < 0 \\ ae^{bx} & \text{when } 0 \leq x \leq 2 \\ cx + d & \text{when } x > 2 \end{cases}$$

If $f(x)$ is both continuous and differentiable for all real values of x , determine the values of the constants a , b , c and d .

(b) Let the function $g(x)$ be defined as follows:

(3 marks)

$$\begin{aligned} g(x) &= 2|x| \quad \text{when } x \geq 0, \\ g(x) &= k|x| \quad \text{when } x < 0. \end{aligned}$$

For what value(s) of k is $g(x)$ differentiable at $x = 0$?

Question 12

(5 marks)

Suppose that two variables x and y are related by $y = 6^x$.

(a) Use the definition of a logarithm to express x in terms of y . (1 mark)

(b) Given that $\log_6 2 = q$, write the following in terms of q .

(i) $\log_6 6$ (1 mark)

(ii) $\log_6 3$ (1 mark)

(iii) $\log_6 24$ (1 mark)

(iv) $\log_6 0.5$ (1 mark)

Question 13

(7 marks)

A model used for determining the number n of tests required to ensure the quality of S_n products is given in the following table.

n	Expression for S_n	S_n
1	1^2	1
2	$2^2 - 1^2$	3
3	$3^2 - 2^2 + 1^2$	6
4	$4^2 - 3^2 + 2^2 - 1^2$	10
5	$5^2 - 4^2 + 3^2 - 2^2 + 1^2$?
...		
10	?	?

- (a) Evaluate S_5 . (1 mark)
- (b) Write an expression for S_{10} and determine its value. (2 marks)
- (c) Suggest the general value for S_n in terms of n . (1 mark)
- (d) Given that there is a total of 500 products to be checked, calculate the minimum number of tests required. (3 marks)

Question 14

(6 marks)

A bottle is filled with a pure fluid whose concentration is 100%.

A small fraction, $\left(\frac{1}{n}\right)^{th}$, of the fluid is now removed and is replaced by water. The bottle is shaken so that its contents are well mixed.

- (a) If this process is repeated a second time, explain why the concentration of the original fluid is now (2 marks)

$$100 \left(1 - \frac{1}{n}\right)^2 \%$$

- (b) Write an expression for the concentration C_n of the original fluid in the bottle if this process is repeated n times. (2 marks)

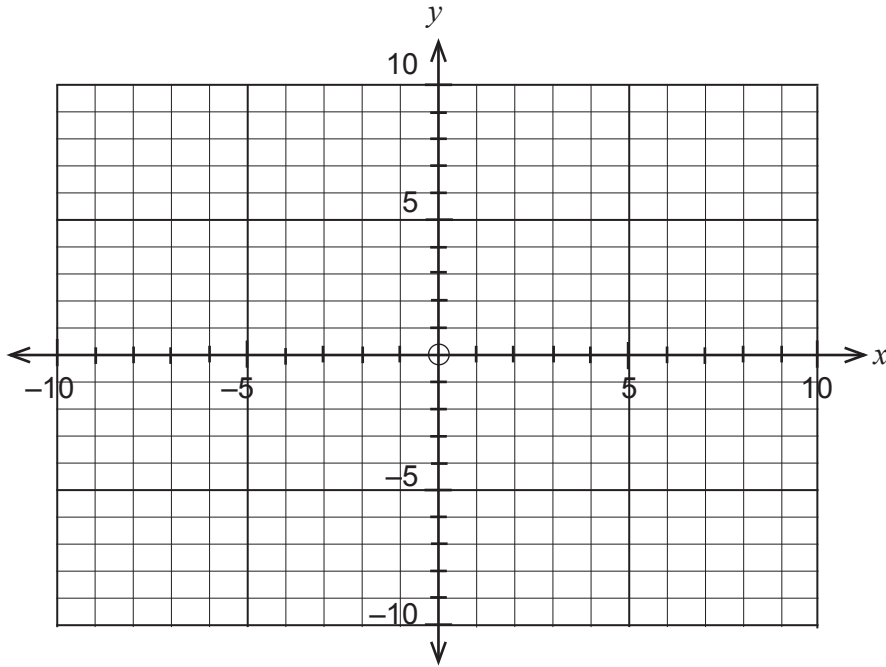
- (c) What happens to C_n as $n \rightarrow \infty$? (2 marks)

Question 15

(10 marks)

Consider the function $f(x) = (x - 3)^2 - 1$ defined for the domain $x \leq 3$.

- (a) On the axes provided, sketch the graph of $f(x)$. (2 marks)



- (b) Determine the inverse function $f^{-1}(x)$ and then graph it on the axes above. (4 marks)

- (c) State the domain and range of $f^{-1}(x)$. (2 marks)

Consider the function $g(x) = (x - a)^2 - b$ defined for the domain $x \geq a$.

- (d) What are the domain and range of $g^{-1}(x)$? (2 marks)

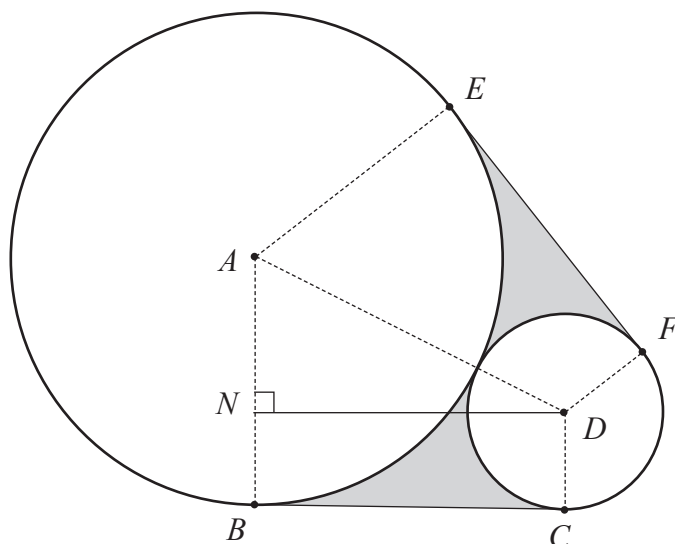
Question 16

(8 marks)

In the diagram below, a large circle with centre A touches a smaller circle, centre D , at one point.

The radius of the smaller circle is p cm, and the radius of the larger circle is three times the radius of the smaller circle.

The line AD joining the centres of the two circles passes through their point of contact. The two circles, and two tangents drawn to touch the two circles at B, C and E, F enclose a shaded region, as indicated in the diagram below.



(a) Calculate the exact size, in radians, of the angle BAD . (2 marks)

(b) Determine the exact size of the angle CDA , justifying your answer. (1 mark)

(c) Calculate the length of BC . (2 marks)

(d) Hence, or otherwise, prove that the area of the shaded region is (3 marks)

$$p^2 \left(8\sqrt{3} - \frac{11\pi}{3} \right) \text{cm}^2 .$$

Question 17

(9 marks)

The polynomial $P(z) = z^3 - 8$ can be factorised to give

$$P(z) = (z - 2)Q(z) = (z - 2)(z^2 + az + b),$$

where a and b are real numbers.

(a) Verify that $a = 2$ and $b = 4$ by expanding the product $(z - 2)(z^2 + 2z + 4)$. (1 mark)

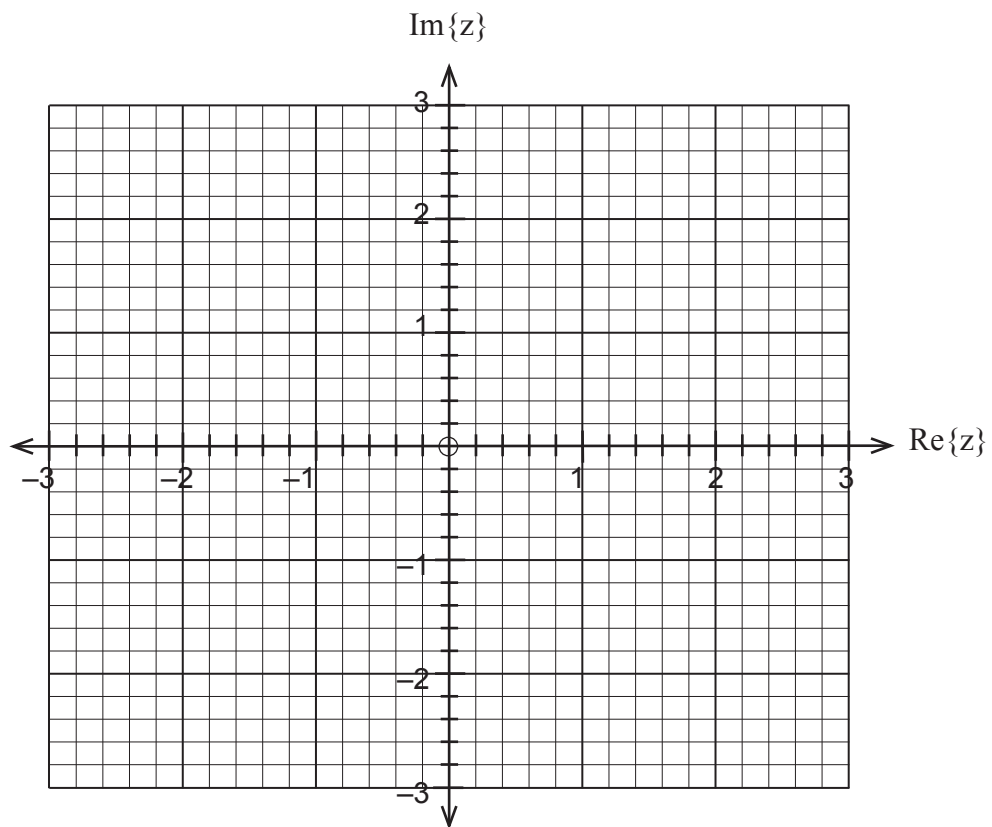
(b) Use the quadratic formula to determine the solutions ω_1 and ω_2 of $Q(z) = 0$. (2 marks)

(c) Hence write $P(z)$ as a product of three linear factors. (1 mark)

(d) How is ω_1 related to ω_2 ? (1 mark)

- (e) Evaluate $\frac{\omega_1^2}{\omega_2}$. (2 marks)

- (f) Plot the three roots of $P(z)$ in the complex plane. (1 mark)



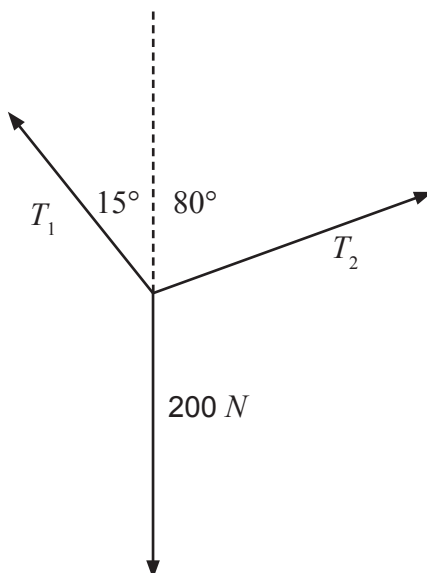
- (g) What geometrical shape is formed by the vertices of the points plotted in part (f)? (1 mark)

Question 18

(10 marks)

In the diagram below, a weight of 200 N is suspended on a wire.

The tensions in the two parts of the wire on either side of the weight are T_1 and T_2 and the system is in equilibrium.



- (a) Determine the magnitudes of T_1 and T_2 , giving your answers correct to one decimal place. (4 marks)

- (b) Determine whether the line $\mathbf{r} = 3\mathbf{i} + \mathbf{j} + \lambda(-\mathbf{i} + \mathbf{j})$ meets the circle $|\mathbf{r} - (\mathbf{i} + \mathbf{j})| = \sqrt{2}$. If they do meet, find the point(s) of intersection. Interpret your answer geometrically. (6 marks)

Question 19

(9 marks)

Assume that the orbits of the planets Earth and Venus around the sun are circular and occupy the same plane. Let the radius of the earth's orbit be one astronomical unit (AU), and its period of motion be one year. Furthermore, when time $t = 0$ suppose that the two planets are aligned in the same direction out from the sun.

The vector equation $\mathbf{P} = R \cos \theta \mathbf{i} + R \sin \theta \mathbf{j}$ defines a circle.

- (a) Explain why this equation must be that of a circle. (1 mark)
- (b) Given that the earth is at $R=1$ and $\theta=2\pi t$, write the vector equation for the earth's circular orbit around the sun. (2 marks)
- (c) Given that Venus orbits the sun every 0.615 years, and that its radius of orbit is 0.723 AU, explain why $\mathbf{V} = 0.723 \cos\left(\frac{2\pi t}{0.615}\right)\mathbf{i} + 0.723 \sin\left(\frac{2\pi t}{0.615}\right)\mathbf{j}$, is the vector equation of Venus's circular orbit around the sun. (2 marks)
- (d) How far apart are the planets after one year? (2 marks)
- (e) What is the first time $t > 0$ when Venus and the earth are on diametrically opposite sides of the sun? (2 marks)

Question 20

(8 marks)

A medical treatment uses small hollow cylindrical titanium rods filled with radioactive Iodine I-125.

The radioactivity of I-125 decays at a rate proportional to the mass R of I-125 present.

- (a) If the initial mass of I-125 inserted is R_0 , write an expression for R in terms of R_0 , the time t months, and some exponential decay constant $k > 0$. (2 marks)

- (b) If the half-life of I-125 is two months, determine the value of k . (2 marks)

- (c) A patient undergoing this treatment is warned not to be in the vicinity of a young child until the mass of I-125 has dropped below 5% of its initial amount.

How long will this take? (2 marks)

- (d) A second isotope Z has a decay rate twice as large as I-125. If the initial mass of Z is $10R_0$, how many months will pass before the mass of Z is the same as that of I-125? (2 marks)

Question 21**(9 marks)**

A cruise ship loses power to its engines at 8:40 am when it is 30 km east and 15 km north of the nearest harbour. The ship drifts at a speed of $10\sqrt{2}$ km/h in a south-westerly direction.

Let \mathbf{i} and \mathbf{j} denote vectors of length one km in the due east and due north directions respectively.

(a) Write a vector form for the velocity of the ship. (1 mark)

(b) What is the position vector of the cruise ship at 8:55 am? (2 marks)

At 8:55 a rescue vessel is 50 km east of the harbour, and is dispatched at its full speed of 45 km/h to try to intercept the cruise ship.

(c) When does the rescue vessel reach the cruise ship? (6 marks)

End of questions

Additional working space

Question number: _____

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