## ATAR course examination, 2021

## Question/Answer booklet

## MATHEMATICS <br> METHODS

Section One: Calculator-free


In words

## Time allowed for this section

Reading time before commencing work: Working time:
five minutes fifty minutes

Number of additional answer booklets used (if applicable):

## Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet
To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

## Structure of this paper

| Section | Number of <br> questions <br> available | Number of <br> questions to <br> be answered | Working <br> time <br> (minutes) | Marks <br> available | Percentage <br> of <br> examination |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Section One: <br> Calculator-free | 7 | 7 | 50 | 51 | 35 |
| Section Two: <br> Calculator-assumed | 10 | 10 | 100 | 100 | 65 |
| Total |  |  |  |  | 100 |

## Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the Year 12 Information Handbook 2021: Part II Examinations. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

## Section One：Calculator－free

This section has seven questions．Answer all questions．Write your answers in the spaces provided．

Supplementary pages for planning／continuing your answers to questions are provided at the end of this Question／Answer booklet．If you use these pages to continue an answer，indicate at the original answer where the answer is continued，i．e．give the page number．

Working time： 50 minutes．

## Question 1

（a）Differentiate $\frac{3 x+1}{x^{3}}$ and simplify your answer．
（b）Let $f^{\prime}(x)=x \ln (2 x)$ ．Determine a simplified expression for the rate of change of $f^{\prime}(x)$ ．
（c）Given that $g^{\prime}(x)=4 e^{2 x}$ and $g(1)=0$ ，determine $g(5)$ ．

## Question 2

It takes Nahyun between 15 and 40 minutes to get to school each day, depending on traffic conditions. Nahyun leaves home for school at 8.00 am each school day. Let the random variable $X$ be the time, in minutes after 8:00 am, that Nahyun arrives at school. The probability density function of $X$ is shown below.

(a) What is the name of this type of distribution?
(b) Determine:
(i) the values of $p, q$ and $k$
(ii) the expected value of $X$
(iii) the probability that Nahyun arrives at school before 8:25 am.
(2 marks)

Nahyun will be late for her first class if she arrives at school after 8:28 am. Otherwise, she will not be late.
(c) If Nahyun is not late for her first class, what is the probability that she arrives after 8:25 am?
(d) If Nahyun only wants to be late for her first class at most 4\% of the time, what time should she leave home, assuming the 15 to 40 minute travel time remains the same?

## Question 3

Given that $\ln (2) \approx 0.693$, use the increments formula to determine an approximation for $\ln (2.02)$.

## Question 4

Determine the following:
(a) $\int\left(2 x^{2}-x^{3}\right) d x$
(b) $\int_{0}^{\frac{\pi}{2}} \frac{\sin (x)}{3-\cos (x)} d x$
(c) $\quad \frac{d}{d y} \int_{-1}^{y} 3 x^{2} \cos (2 x) d x$

## Question 5

(a) Determine the area between the parabola $y=x^{2}-x+3$ and the straight line $y=x+3$.
(4 marks)
(b) The area between the parabola $y=x^{2}-x-2$ and the straight line $y=x-2$ is the same as the area determined in part (a). Explain why this is the case.
(2 marks)
(a) The graphs of three normal distributions are displayed below. The distributions have been labelled $A, B$ and $C$.

(i) What is the mean of distribution A?
(ii) Which of the distributions has the largest standard deviation? Justify your answer. (1 mark)
(b) A random variable $X$ is normally distributed. The distribution of $X$ is graphed below.

(i) Shade the region with area corresponding to $\mathrm{P}(6 \leq X \leq 9)$.
(1 mark)
(ii) Is $\mathrm{P}(6 \leq X \leq 9) \geq 0.5$ ? Justify your answer.
(2 marks)
(c) A random variable $Y$ has probability $\mathrm{P}(Y \geq 2)>\mathrm{P}(Y>2)$. Explain whether it is possible for the distribution of $Y$ to be normal or binomial.
(2 marks)

## Question 7

(a) Consider the function $f(x)=\frac{1}{x}$, graphed twice below.

(i) Shade two different regions (one on each graph above) each with area exactly $\ln (2)$.
(ii) Given that

$$
\int_{a}^{b} \frac{1}{x} d x=\ln (3)
$$

what is the relationship between $a$ and $b$ ?
(2 marks)
（b）Another graph of $f(x)=\frac{1}{x}$ is shown below．

（i）By considering the areas of the rectangles shown，demonstrate and explain why

$$
\frac{11}{30}<\int_{2}^{3} \frac{1}{x} d x<\frac{9}{20} .
$$

（ii）Hence show that $\frac{11}{30}<\ln (1.5)<\frac{9}{20}$ ． （2 marks）

Supplementary page
Question number:

Supplementary page
Question number：

Supplementary page
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Question number：

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