Government of **Western Australia**
School Curriculum and Standards Authority

Sample Assessment Tasks

Mathematics Specialist

ATAR Year 12

Acknowledgement of Country

Kaya. The School Curriculum and Standards Authority (the Authority) acknowledges that our offices are on Whadjuk Noongar boodjar and that we deliver our services on the country of many traditional custodians and language groups throughout Western Australia. The Authority acknowledges the traditional custodians throughout Western Australia and their continuing connection to land, waters and community. We offer our respect to Elders past and present.

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2015/3534[v8]

Sample assessment task

Mathematics Specialist – ATAR Year 12

Task 1 – Unit 3

Assessment type:	Response
Time allowed:	55 minutes Section one – calculator-free section – 35 minutes Section two – calculator assumed section – 20 minutes
Conditions:	In class, under supervised conditions Section two – Drawing instruments, templates, notes on two unfolded sheets of A4 paper and up to three calculators suitable for use in ATAR course examinations
Task weighting:	9% of the school mark for this pair of units

Section one – calculator-free section **(26 marks)**

Question 1 **(8 marks)**

(a) Given $z = \sqrt{3} + i$ evaluate z^6 giving the answer in Cartesian form. (2 marks)

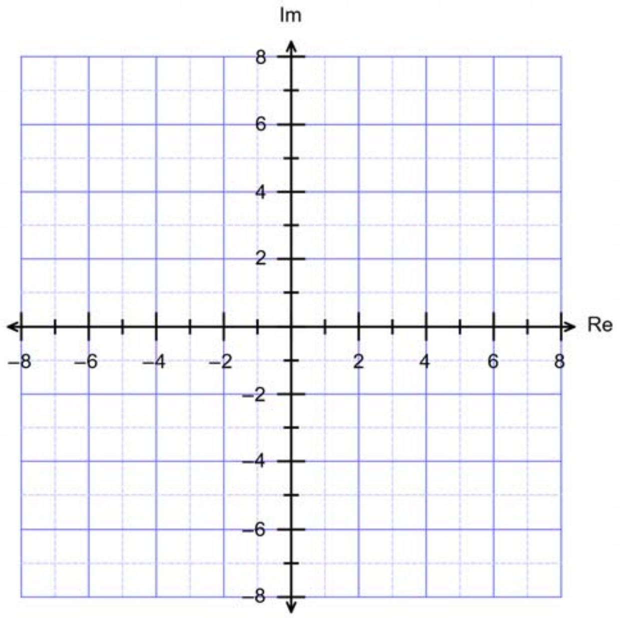
(b) Given $Z_1 = 2cis\left(\frac{\pi}{3}\right)$ and $Z_2 = cis\left(\frac{\pi}{4}\right)$ evaluate the following in exact Cartesian form (4 marks)

(i) $\overline{Z_1}$ (ii) iZ_2 (iii) $\left(\frac{Z_1}{Z_2}\right)^2$

(c) Solve $x^2 - 6x + 13 = 0$ for $x \in Im$ in exact form. (2 marks)

Question 2 **(6 marks)**

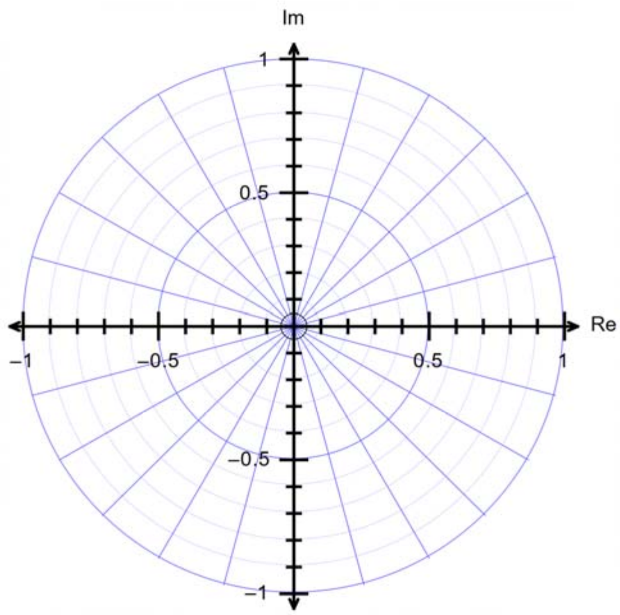
(a) Sketch the set of points defined by $|z - (2 + 3i)| \leq 4$. (4 marks)



(b) Determine the maximum value of $|z|$. (2 marks)

Question 3 **(6 marks)**

(a) Determine and locate all solutions in the Argand plane to the equation $z^5 = 1$. (4 marks)



(b) The solution located in the fourth quadrant above is also a solution to the equation $z^4 = k$. Determine the solution to the equation $z^4 = k$ that lies in the first quadrant. (2 marks)

Question 4 **(6 marks)**

Given $H(z) = z^5 - 2z^4 + 5z^3 - 10z^2 + 4z - 8$

(a) Evaluate $H(i)$ and $H(2)$. (2 marks)

(b) Hence, find all roots of the equation $z^5 - 2z^4 + 5z^3 - 10z^2 + 4z - 8 = 0$. (4 marks)

Section two – calculator assumed section **(27 marks)**

Question 5 **(10 marks)**

(a) Expand and simplify the expression $F(\theta) = (\cos\theta + i\sin\theta)^5$. (2 marks)

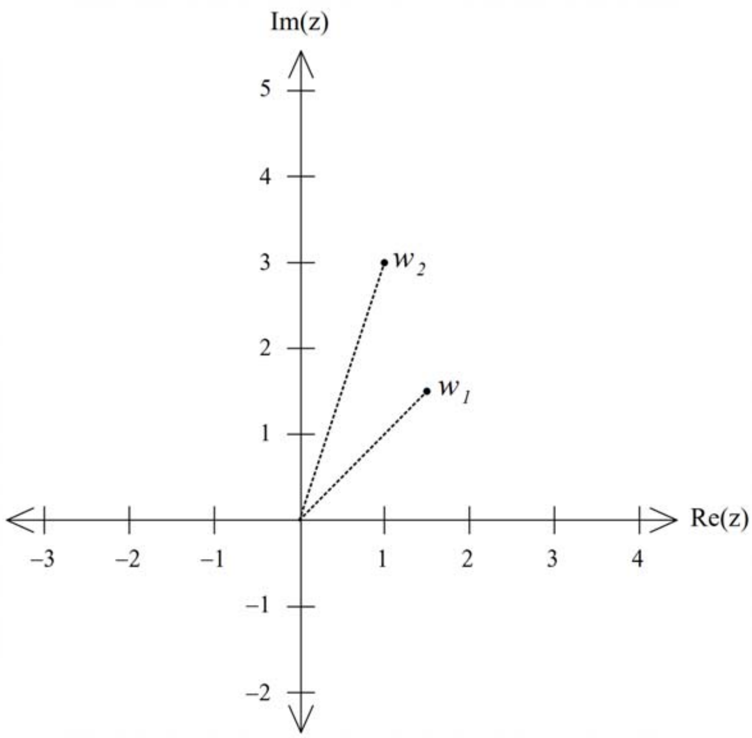
(b) Hence, express the $Re(F)$ in terms of $cos\,\theta$. (3 marks)

(c) Use $Re(F)$ to solve the equation $16x^5 - 20x^3 + 5x - 1 = 0$ and express the solutions in polar form.

(5 marks)

Question 6 **(7 marks)**

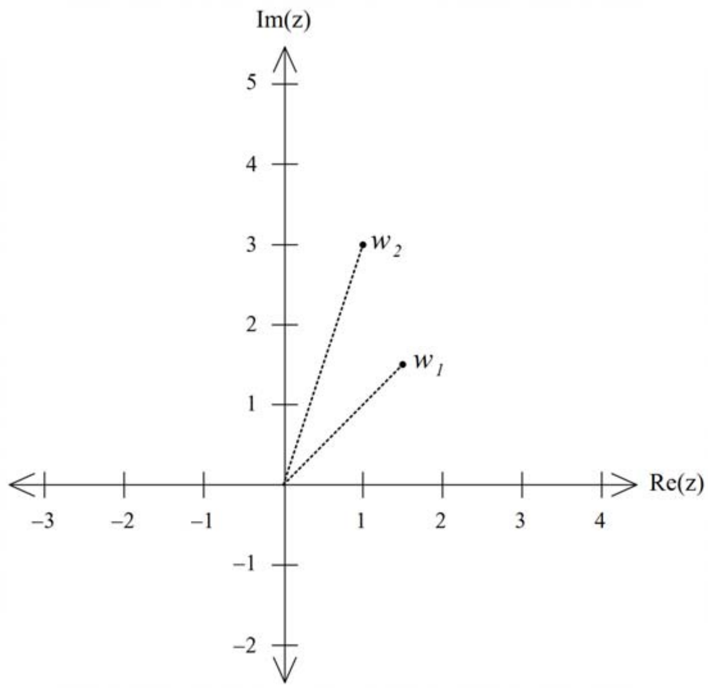
The complex numbers w_1 and w_2 are shown on the Argand diagram below.



(a) On the Argand diagram, draw and clearly label the position of (4 marks)

(i) $w_1 - w_2$ (ii) ${w_1}^2$ (iii) $\frac{w_1}{w_2}$

(b) On the diagram below, sketch the set of complex numbers such that $|z - w_2| < |z - w_1|$. (3 marks)



Question 7 **(10 marks)**

Given $z = \cos\theta + i\sin\theta$:

(a) Express $\frac{(z-\overline{z})}{i(z+\overline{z})}$ in simplified polar form. (4 marks)

(b) Show $z^2 + \frac{1}{z^2} = 2\cos 2\theta$ and hence prove $cos\, 2\theta = 2\cos^2\theta - 1$. (6 marks)

Marking key for sample assessment task 1

Section one – calculator-free section **(26 marks)**

Question 1 **(8 marks)**

(a) Given $z = \sqrt{3} + i$ evaluate z^6 giving the answer in Cartesian form. (2 marks)

Solution	
Given $z = \sqrt{3} + i$ evaluate z^6 $z^6 = \left(\sqrt{3} + i\right)^6$ $= \sqrt{3}^6 + 6.\left(\sqrt{3}\right)^5.(i)^1 + 15.\left(\sqrt{3}\right)^4.(i)^2 + 20.\left(\sqrt{3}\right)^3.(i)^3 + 15.\left(\sqrt{3}\right)^2.(i)^4 + 6.\left(\sqrt{3}\right)^1.(i)^5 + (i)^6$ $= 27 - 135 + 45 - 1 + \left(54\sqrt{3} - 60\sqrt{3} + 6\sqrt{3}\right)i$ $= 27 - 135 + 45 - 1 + \left(54\sqrt{3} - 60\sqrt{3} + 6\sqrt{3}\right)i$ $= -64$ or $\mathrm{z} = 2cis\left(\frac{\pi}{6}\right) \Rightarrow z^6 = 64cis(\pi) = -6$	
Behaviours	**Marks**
Expands the Cartesian form of z^6 Simplifies correctly	1 1
or	
Expresses z^6 in polar form Expresses the answer in Cartesian form	1 1
Subtotal	**/2**

(b) Given $Z_1 = 2cis\left(\frac{\pi}{3}\right)$ and $Z_2 = cis\left(\frac{\pi}{4}\right)$ evaluate the following in exact Cartesian form (4 marks)

(i) $\overline{Z_1}$ (ii) iZ_2 (iii) $\left(\frac{Z_1}{Z_2}\right)^2$

Solution		
(i) $\overline{Z_1} = 2cis\left(-\frac{\pi}{3}\right)$ $= 1 - \sqrt{3}i$	(ii) $iZ_2 = cis\left(\frac{\pi}{2}\right) cis\left(\frac{\pi}{4}\right)$ $= cis\left(\frac{3\pi}{4}\right)$ $= \frac{-\sqrt{2}+\sqrt{2}i}{2}$	(iii) $\left(\frac{Z_1}{Z_2}\right)^2 = \left(\frac{2cis\left(\frac{\pi}{3}\right)}{cis\left(\frac{\pi}{4}\right)}\right)^2$ $= \left(2cis\left(\frac{\pi}{12}\right)\right)^2$ $= 4cis\left(\frac{\pi}{6}\right)$ $= 2\sqrt{3} + 2i$

Behaviours	Marks
(i) Writes the Cartesian form of $\overline{Z_1}$ correctly	1
(ii) Writes the Cartesian form of iZ_2 correctly	1
(iii) Expresses polar term for required term correctly	1
Writes the Cartesian form correctly	1
Subtotal	**/4**

(c) Solve $x^2 - 6x + 13 = 0$ for $x \in Im$ in exact form. (2 marks)

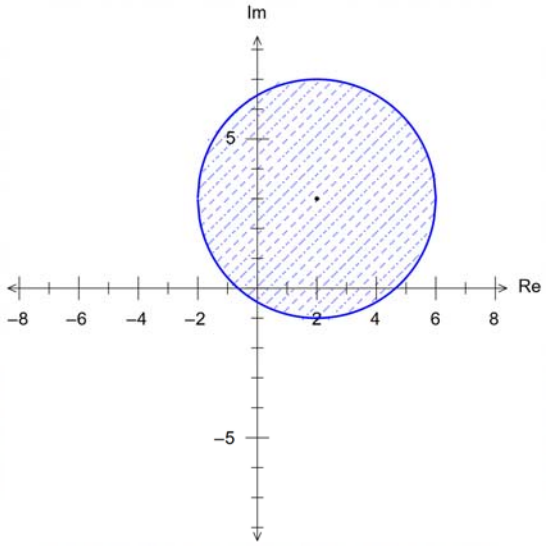
Solution	
Solve $x^2 - 6x + 13 = 0$ $\Leftrightarrow x^2 - 6x + 9 = -4$ $\Leftrightarrow (x-3)^2 = -4$ $\Leftrightarrow (x-3)^2 = 4i^2$ $\Leftrightarrow x = 3 \pm 2i$	Or Solve $x^2 - 6x + 13 = 0$ $\Leftrightarrow a = 1, b = -6$ and $c = 13$ $\Leftrightarrow x = \frac{6 \pm \sqrt{36-52}}{2}$ $\Leftrightarrow x = \frac{6 \pm \sqrt{-16}}{2} = 3 \pm 2i$

Behaviours	Marks
Completes the square correctly Solves the equation using the exact form	1 1
or	
Substitutes into the quadratic formula correctly Simplifies the expressions to the correct exact form	1 1
Subtotal	**/2**

Question 2 **(6 marks)**

(a) Sketch the set of points defined by $|z-(2+3i)| \leq 4$. (4 marks)

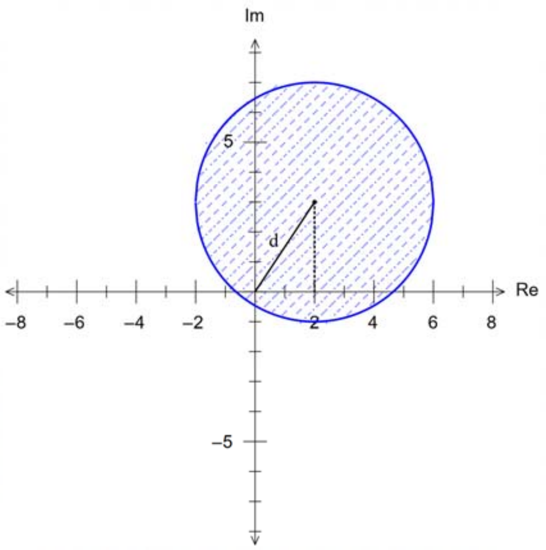
Solution



Behaviours	Marks
Draws a circle with a solid line	1
Has the correct centre, $(2+3i)$	1
Has the correct radius	1
Shades the interior of the circle	1
Subtotal	**/4**

(b) Determine the maximum value of $|z|$. (2 marks)

Solution



$d = \sqrt{2^2+3^2}$

$= \sqrt{13}$

Maximum distance $= \sqrt{13}+4$

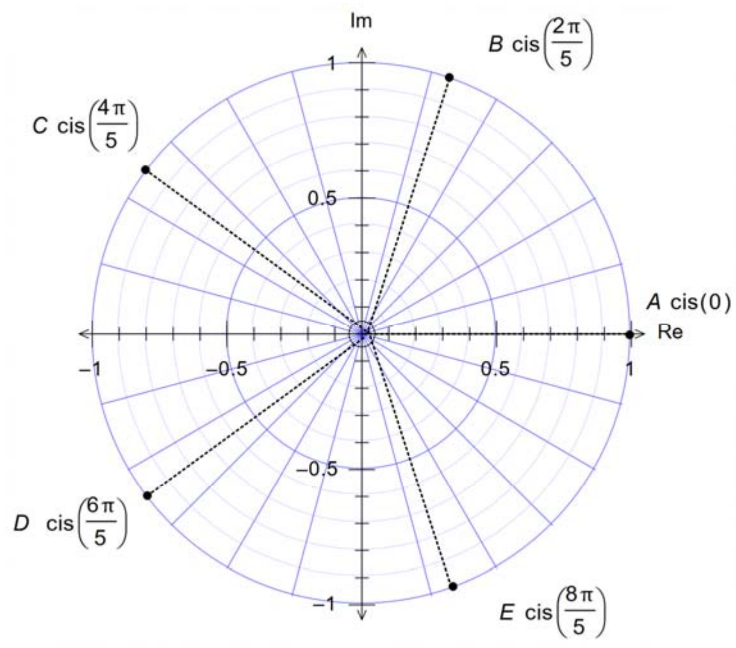
Behaviours	Marks
Determines distance from centre of circle to (0, 0) correctly	1
Adds the length of the radius	1

Subtotal	**/2**

Question 3 **(6 marks)**

(a) Determine and locate all solutions in the Argand plane to the equation $z^5 = 1$. (4 marks)

Solution



$$z_0 = 1 \times cis\left(0 + \frac{2\pi \times 0}{5}\right) = 1 + 0i$$
$$z_1 = 1 \times cis\left(0 + \frac{2\pi \times 1}{5}\right) = cis\left(\frac{2\pi}{5}\right)$$
$$z_2 = 1 \times cis\left(0 + \frac{2\pi \times 2}{5}\right) = cis\left(\frac{4\pi}{5}\right)$$
$$z_3 = 1 \times cis\left(0 + \frac{2\pi \times 3}{5}\right) = cis\left(\frac{6\pi}{5}\right)$$
$$z_4 = 1 \times cis\left(0 + \frac{2\pi \times 4}{5}\right) = cis\left(\frac{8\pi}{5}\right)$$

Behaviours	Marks
Expresses one solution correctly and locates it on the polar graph	1
Identifies that there are five solutions, with angular separation of $\frac{2\pi}{5}$	1
Expresses all five solutions correctly	1
Locates the solutions on the polar graph	1
Subtotal	**/4**

(b) The solution located in the fourth quadrant above is also a solution to the equation $z^4 = k$. Determine the solution to the equation $z^4 = k$ that lies in the first quadrant. (2 marks)

Solution	
Using E: $cis\left(\frac{8\pi}{5}\right)$ Solution in first quadrant is $cis\left(\frac{8\pi}{5} + \frac{\pi}{2}\right)$ $= cis\left(\frac{\pi}{10}\right)$	
Behaviours	**Marks**
Identifies the solution in the fourth quadrant from (a)	1
Determines the first quadrant solution correctly	1
Subtotal	**/2**

Question 4 **(6 marks)**

Given $H(z) = z^5 - 2z^4 + 5z^3 - 10z^2 + 4z - 8$

(a) Evaluate $H(i)$ and $H(2)$. (2 marks)

Solution	
$H(z) = z^5 - 2z^4 + 5z^3 - 10z^2 + 4z - 8$ $H(i) = i - 2 - 5i + 10 + 4i - 8 = 0$ $H(2) = 32 - 32 + 40 - 40 + 8 - 8 = 0$	
Behaviours	**Marks**
Evaluates $H(i)$ correctly	1
Evaluates $H(2)$ correctly	1
Subtotal	**/2**

(b) Hence, find all roots of the equation $z^5 - 2z^4 + 5z^3 - 10z^2 + 4z - 8 = 0$. (4 marks)

Solution	
Given $H(z) = z^5 - 2z^4 + 5z^3 - 10z^2 + 4z - 8$ from part (a) $H(i) = 0 \iff (z - i)$ is a factor of $H(z)$ and $(z + i)$ is a factor of $H(z)$ $H(2) = 0 \iff (z - 2)$ is a factor of $H(z)$ $z^5 - 2z^4 + 5z^3 - 10z^2 + 4z - 8 = (z - i)(z + i)(z - 2)(z^2 + bz + c)$ $= (z^3 - 2z^2 + z - 2)(z^2 + bz + c)$ Then equating the constant: $-2c = -8,\ c = 4$ And equating coefficients of z: $-2b = 4,\ b = 0$ $z^5 - 2z^4 + 5z^3 - 10z^2 + 4z - 8 = (z - i)(z + i)(z - 2)(z^2 + 4)$ $= (z + i)(z - i)(z - 2)(z + 2i)(z - 2i)$ Hence, the roots to $z^5 - 2z^4 + 5z^3 - 10z^2 + 4z - 8 = 0$ are $z = \{\pm i, \pm 2i, 2\}$	
Behaviours	**Marks**
Uses the factor theorem to give factors $(z + i)(z - 2)$	1
Identifies that the conjugate $(z - i)$ is a factor	1
Determines the remaining factors $(z + 2i)(z - 2i)$	1
Correctly writes all the roots	1
Subtotal	**/4**

Section two – calculator assumed section **(27 marks)**

Question 5 **(10 marks)**

(a) Expand and simplify the expression $F(\theta) = (\cos\theta + i\sin\theta)^5$. (2 marks)

Solution	
$F(\theta) = (\cos\theta + i\sin\theta)^5$ $= (\cos^5\theta - 10\cos^3\theta\,.\sin^2\theta + 5\cos\theta\,.\sin^4\theta) + (\sin^5\theta + 5\cos^4\theta\,.\sin\theta - 10\cos^2\theta\,.\sin^3\theta)i$	
Behaviours	**Marks**
Shows the real and imaginary terms correctly	2

(b) Hence, express the $Re(F)$ in terms of $cos\,\theta$. (3 marks)

Solution	
$Re(F) = \cos^5\theta - 10\cos^3\theta\,.\sin^2\theta + 5\cos\theta\,.\sin^4\theta$ $= \cos^5\theta - 10\cos^3\theta\,(1-\cos^2\theta) + 5\cos\theta\,(1-\cos^2\theta)^2$ $= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos\theta\,(1 - 2\cos^2\theta + \cos^4\theta)$ $= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$	
Behaviours	**Marks**
Writes the real part of $F(\theta)$	1
Substitutes for $\sin^2\theta = 1 - \cos^2\theta$	1
Simplifies to give the correct expression for $Re(F)$	1
Subtotal	**/3**

(c) Use $Re(F)$ to solve the equation $16x^5 - 20x^3 + 5x - 1 = 0$ and express the solutions in polar form. (5 marks)

Solution
$F(\theta) = (\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$ $Re(F) = \cos 5\theta$ $Re(F) = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ from part (b) Hence $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ To solve $16x^5 - 20x^3 + 5x - 1 = 0 \quad \{\text{Let } x = \cos\theta\}$ $16\cos^5\theta - 20\cos^3\theta + 5\cos\theta = 1$ $\cos 5\theta = 1$ $5\theta = 0, 2\pi, \ldots$ $\theta = \frac{2\pi n}{5}, n = 0,1,2,3,4.$ $x = \cos\left(\frac{2\pi n}{5}\right), n = 0,1,2,3,4.$

Behaviours	Marks
Uses de Moivre's theorem to state $\text{Re}(F) = \cos 5\theta$	1
Makes the substitution $x = \cos\theta$ in polynomial	1
Replaces the polynomial with $\cos 5\theta$	1
Solves $\cos 5\theta = 1$ in terms of θ	1
Gives all five solutions in terms of x	1
Subtotal	**/5**

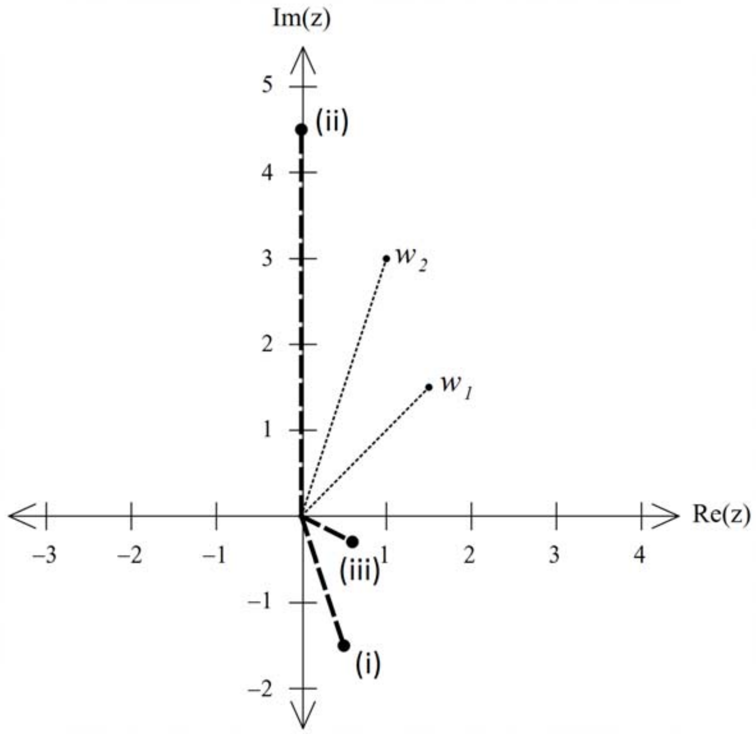
Question 6 **(7 marks)**

The complex numbers w_1 and w_2 are shown on the Argand diagram below.

(a) On the Argand diagram, draw and clearly label the position of (4 marks)

(ii) $w_1 - w_2$ (ii) ${w_1}^2$ (iii) $\frac{w_1}{w_2}$

Solution

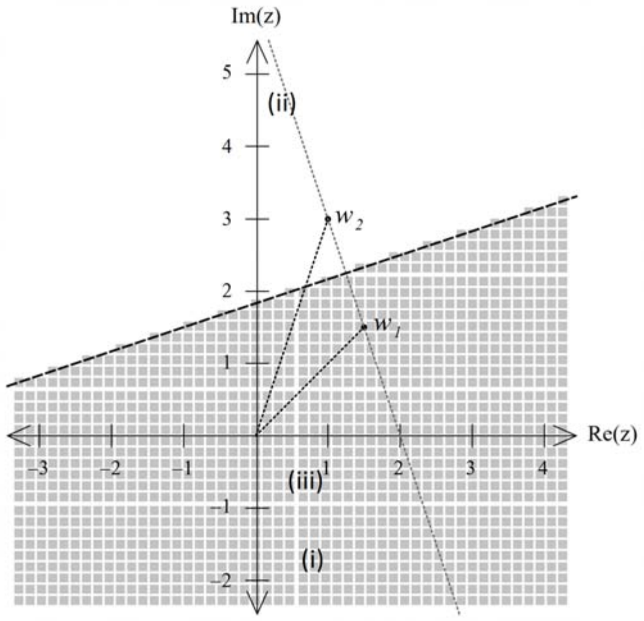


Behaviours	**Marks**
Shows the location of $w_1 - w_2$ on the Argand diagram	1
Shows the location of ${w_1}^2$ on the Imaginary axis	1
Locates ${w_1}^2$ with correct modulus	1
Shows the location of $\frac{w_1}{w_2}$ on the Argand diagram	1
Subtotal	**/4**

(b) On the diagram below, sketch the set of complex numbers such that $|z - w_2| < |z - w_1|$.

(3 marks)

Solution



Behaviours	Marks
Draws the perpendicular bisector of the line joining the w_1 and w_2	1
Draws a dotted line	1
Shades the diagram correctly	1
Subtotal	**/3**

Question 7 **(10 marks)**

Given $z = \cos\theta + i\sin\theta$:

(a) Express $\frac{(z-\bar{z})}{i(z+\bar{z})}$ in simplified polar form. (4 marks)

Solution

$$\frac{(z-\bar{z})}{i(z+\bar{z})} = \frac{(\cos\theta + i\sin\theta) - (\cos\theta - i\sin\theta)}{i\big((\cos\theta + i\sin\theta) + (\cos\theta - i\sin\theta)\big)}$$

$$= \frac{2i\sin\theta}{2i\cos\theta}$$

$$= \tan\theta$$

Behaviours	Marks
Rewrites the expression with z and $\bar{z}$ in polar form	1
Simplifies both numerator and denominator correctly	2
Writes the correct final term	1
Subtotal	**/4**

(b) Show $z^2 + \frac{1}{z^2} = 2\cos 2\,\theta$ and hence prove $cos\, 2\,\theta = 2\cos^2\theta - 1$. (6 marks)

Solution
$z^2 + \dfrac{1}{z^2} = (\cos\theta + i\sin\theta)^2 + (\cos\theta - i\sin\theta)^2$ $= \cos 2\theta + i\sin 2\theta + \cos 2\theta - i\sin 2\theta$ $= 2\cos 2\theta$ (1) $z^2 + \dfrac{1}{z^2} = (\cos\theta + i\sin\theta)^2 + (\cos\theta - i\sin\theta)^2$ $= (\cos^2\theta + 2i\sin\theta\cos\theta - \sin^2\theta) + (\cos^2\theta - 2i\sin\theta\cos\theta - \sin^2\theta)$ $= 2(\cos^2\theta - \sin^2\theta)$ (2) (1) = (2) $\Leftrightarrow 2\cos 2\theta = 2(\cos^2\theta - \sin^2\theta)$ $\Leftrightarrow \cos 2\theta = 2\cos^2\theta - 1$

Behaviours	Marks
Rewrites z^2 and $\frac{1}{z^2}$ using double angle form 2θ	1
Gathers terms and simplifies	1
Rewrites z^2 and $\frac{1}{z^2}$ using double angle form θ	1
Gathers terms and simplifies	1
Equates both equations	1
Writes correct final expression	1
Subtotal	**/6**

Sample assessment task

Mathematics Specialist – ATAR Year 12

Task 6 – Unit 4

Assessment type:	Investigation
Conditions:	The investigation will be completed over one week. Students will be encouraged to work independently to complete the task and may use any appropriate technology. Note: while the Authority provides sample assessment tasks for guidance, it is the expectation of the Authority that teachers will develop tasks customised to reflect their school's context and the needs of the student cohort. This resource is available on a public website and use of the resource without modification may affect the integrity of the assessment.
Task weighting:	12% of the school mark for this pair of units

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Centrepiece **(40 marks)**

Your task is to design a mould for the production of a unique shaped candle that will be used as the centre piece on each table at a graduation dinner.

The curve(s) defining the shape for the candle must pass through the points (2, 2) and (3, 5) and the mould must hold 1L of molten wax.

Write a report to present your design to the graduation committee. As you write your report, take care to clearly identify the underlying mathematics used throughout the process.

Your report should include the following:

- An **introduction**, that clearly defines the purpose of the task, identifies key information, any assumptions made and an outline of your strategy (10 marks)
- **Evidence of the application of mathematical model and strategies**, including calculations and results using appropriate representations (graphs, tables, formulae etc.) (10 marks)
- Your final design communicated in a systematic and concise manner, including **analysis and interpretation** in the context of the problem and consideration of the reasonableness and limitations of the results. (15 marks)
- Use of correct mathematical conventions, symbols and terminology. (5 marks)

The format of the report may be written or digital.

Note: the creativity, complexity and quality of your solution, along with attention to mathematical detail will be assessed in this task.

Marking key for sample assessment task 6 – Unit 4

This marking key may be adjusted based on the conditions of the task

Introduction

Behaviours	Marks
Succinctly writes a general introduction that accurately summarises all aspects of the investigation	1–2
Identifies and documents the need to determine volumes of revolution	1
States a suitable scale to represent the parameters provided	1
Describes or sketches a proposed design	1
Produces a proposed design that: • uses curves • includes more than one function • is rotationally symmetrical.	3
Identifies assumptions made, e.g. rotation can be about either axis **or** mould will be filled to be level with the top of the container **or** any function or relation can be used to model the container **or** the mould design needs to be rotationally symmetrical	2
Subtotal	**/10**

Application of the mathematical model and strategy

Behaviours	Marks
Clearly identifies the axis of rotation used	1
Selects more than one type of function to model the proposed curve	1
Includes a function other than a polynomial	1
Accurately determines points of intersection as boundaries of each function	2
Demonstrates accurate use of integration to determine the volume of revolution for each curved surface	2
Determines total volume of proposed shape	1
Shows refinement of solution to meet volume requirements and justifies any rounding of the solution	2
Subtotal	**/10**

Analysis and interpretation

Behaviours	Marks
Presents an accurate graph of the final function/s	1–3
Clearly labels all key points on the graph (intersections, end points)	1–3
Presents an accurate sketch of the final product formed by the rotation about the axis	1–2
Clearly demonstrates that the proposed model meets the requirements for volume	1–2
Clearly demonstrates that the proposed model passes through the prescribed points	1–2
Discusses the reasonableness of the model within the given context and with reference to assumptions made	1–3
Subtotal	**/15**

Use of mathematical conventions, symbols and terminology

Behaviours	Marks
Graphs are correctly labelled and displayed appropriately (sometimes = 1 mark, consistently = 2 marks)	1–2
Uses mathematical language throughout the investigation (sometimes = 1 mark, consistently = 2 marks)	1–2
Presents investigation in a systematic and concise way	1
Subtotal	**/5**
Total	**/40**