SAMPLE COURSE OUTLINE

MATHEMATICS METHODS
ATAR YEAR 11
Sample course outline

Mathematics Methods – ATAR Year 11

Unit 1

In Unit 1 students will be provided with opportunities to:

- understand the concepts and techniques in algebra, functions, graphs, trigonometric functions, counting and probability
- solve problems using algebra, functions, graphs, trigonometric functions, counting and probability
- apply reasoning skills in the context of algebra, functions, graphs, trigonometric functions, counting and probability
- interpret and evaluate mathematical information and ascertain the reasonableness of solutions to problems
- communicate their arguments and strategies when solving problems.

This course outline assumes an allocation of 4 hours contact time per week for the course. Each semester is based on a 15 week block.

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<th>Time placement (and allocation)</th>
<th>Topic/s</th>
<th>Key teaching points – Syllabus reference/s</th>
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<td><strong>Semester 1 (Unit 1)</strong></td>
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| **Weeks 1–2**                   | Topic 1.2: Trigonometric functions | **Cosine and sine rules** (1.2.1 – 1.2.4)
  • right-angled triangles and trigonometric ratios
  • unit circle definition of \( \cos \theta, \sin \theta \) and \( \tan \theta \) and periodicity using degrees
  • angle of inclination of a line and the gradient of that line
  • establish and use the cosine and sine rules, including consideration of the ambiguous case and the formula \( \text{Area} = \frac{1}{2}bc \sin A \) for the area of a triangle
  **Circular measure and radian measure** (1.2.5 – 1.2.6)
  • use radian measure and degree measure
  • calculate lengths of arcs and areas of sectors and segments in circles |
| **Week 2**                      | Topic 1.1: Functions and graphs | **Lines and linear relationships** (1.1.1 – 1.1.6)
  • coordinates of mid-points and end-point
  • direct proportion and linearly related variables
  • features of the graph of \( y = mx + c \)
  • equations of a straight lines given sufficient information, including parallel and perpendicular lines
  • solve linear equations, including those with algebraic fractions and variables on both sides |
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| **Weeks 3–4** (5 hours)       | Topic 1:1 Functions and graphs | **Quadratic relationships** (1.1.7 – 1.1.12)  
- examine examples of quadratically related variables  
- features of the graphs of $y = x^2$, $y = a(x - b)^2 + c$, and $y = a(x - b)(x - c)$, including their parabolic nature, turning points, axes of symmetry and intercepts  
- solve quadratic equations, including the use of quadratic formula and completing the square  
  - equation of a quadratic, turning points, zeros, discriminant graph of the general quadratic $y = ax^2 + bx + c$ |
| **Weeks 4–6** (7 hours)       | Topic 1.1: Functions and graphs | **Inverse proportion** (1.1.13 – 1.1.14)  
- examples of inverse proportion  
- equations of the graphs of $y = \frac{1}{x}$ and $y = \frac{a}{x-b}$, including their hyperbolic shapes and their asymptotes  
**Powers and polynomials** (1.1.15 – 1.1.20)  
- graphs of $y = x^n$ for $n \in \mathbb{N}, n = -1$ and $n = \frac{1}{2}$, shape, behaviour as $x \to \infty$ and $x \to -\infty$  
- coefficients and the degree of a polynomial  
- expand quadratic and cubic polynomials from factors  
- features and equations of the graphs of $y = x^3$, $y = a(x - b)^3 + c$ and $y = k(x - a)(x - b)(x - c)$; shape, intercepts and behaviour as $x \to \infty$ and $x \to -\infty$  
- factorise cubic polynomials (in cases where a linear factor is easily obtained)  
- solve cubic equations using technology, and algebraically in cases where a linear factor is easily obtained |
| **Weeks 7–8** (8 hours)       | Topic 1.1: Functions and graphs | **Graphs and relations** (1.1.21 – 1.1.22)  
- features and equations of the graphs of $x^2 + y^2 = r^2$ and $(x - a)^2 + (y - b)^2 = r^2$, their circular shapes, centres and radii  
- graph of $y^2 = x$, shape and axis of symmetry  
**Functions** (1.1.23 – 1.1.28)  
- the concept of a function as a mapping and as a rule or a formula that defines one variable quantity in terms of another  
- use function notation; determine domain and range; recognise independent and dependent variables  
- the graph of a function  
- translations and the graphs of $y = f(x) + a$ and $y = f(x - b)$  
- dilations and the graphs of $y = cf(x)$ and $y = f(dx)$  
- distinction between functions and relations and the vertical line test |
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| **Semester 1 (Unit 1)**       | **Trigonometric functions (1.2.7 – 1.2.16)** | - understand the unit circle definition of $\sin \theta$, $\cos \theta$ and $\tan \theta$ and periodicity using radians  
- recognise the exact values of $\sin \theta$, $\cos \theta$ and $\tan \theta$ at integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$  
- recognise the graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$ on extended domains  
- examine amplitude changes and the graphs of $y = a \sin x$ and $y = a \cos x$  
- examine period changes and the graphs of $y = b \sin x$, $y = b \cos x$ and $y = \tan bx$  
- examine phase changes and the graphs of $y = \sin(x - c)$, $y = \cos(x - c)$ and $y = \tan(x - c)$  
- examine the relationships $\sin\left(x + \frac{\pi}{2}\right) = \cos x$ and $\cos\left(x - \frac{\pi}{2}\right) = \sin x$  
- prove and apply the angle sum and difference identities  
- identify contexts suitable for modelling by trigonometric functions and use them to solve practical problems  
- solve equations involving trigonometric functions using technology, and algebraically in simple cases |
| **Weeks 9–10 (10 hours)**    | **Topic 1.2: Trigonometric functions** |  |
| **Week 11 (4 hours)**        | **Topic 1.3: Counting and probability** | **Combinations (1.3.1 – 1.3.5)**  
- understand the notion of a combination as a set of $r$ objects taken from a set of $n$ distinct objects  
- use the notation $\binom{n}{r}$ and the formula $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ for the number of combinations of $r$ objects taken from a set of $n$ distinct objects  
- expand $(x + y)^n$ for small positive integers $n$  
- recognise the numbers $\binom{n}{r}$ as binomial coefficients (as coefficients in the expansion of $(x + y)^n$)  
- use Pascal’s triangle and its properties |
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<td><strong>Semester 1 (Unit 1)</strong></td>
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<td><strong>Language of events and sets (1.3.6 – 1.3.8)</strong></td>
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<td>• review the concepts and language of outcomes, sample spaces, and events, as sets of outcomes</td>
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<td>• use set language and notation for events, including:</td>
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<td></td>
<td>a. $\overline{A}$ (or $A'$) for the complement of an event $A$</td>
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<td>b. $A \cap B$ and $A \cup B$ for the intersection and union of events $A$ and $B$ respectively</td>
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<td>c. $A \cap B \cap C$ and $A \cup B \cup C$ for the intersection and union of the three events $A, B$ and $C$ respectively</td>
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<td>d. recognise mutually exclusive events</td>
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<td>• use everyday occurrences to illustrate set descriptions and representations of events and set operations</td>
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<td>Week 12 (4 hours)</td>
<td>Topic 1.3: Counting and probability</td>
<td><strong>Review of the fundamentals of probability (1.3.9 – 1.3.12)</strong></td>
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<td>• review probability as a measure of ‘the likelihood of occurrence’ of an event</td>
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<td>• review the probability scale: $0 \leq P(A) \leq 1$ for each event $A$ with $P(A) = 0$ if $A$ is an impossibility and $P(A) = 1$ if $A$ is a certainty</td>
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<td>• review the rules: $P(\overline{A}) = 1 - P(A)$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$</td>
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<td>• use relative frequencies from data as estimates of probabilities</td>
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<td>Week 13 (4 hours)</td>
<td>Topic 1.3: Counting and probability</td>
<td><strong>Conditional probability and independence (1.3.13 – 1.3.17)</strong></td>
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<td>• understand the notion of a conditional probability and recognise and use language that indicates conditionality</td>
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<td>• use the notation $P(A</td>
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<td>• understand the notion of independence of an event $A$ from an event $B$, as defined by $P(A</td>
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<td>• establish and use the formula $P(A \cap B) = P(A)P(B)$ for independent events $A$ and $B$, and recognise the symmetry of independence</td>
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<td>• use relative frequencies obtained from data as estimates of conditional probabilities and as indications of possible independence of events</td>
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<td>Weeks 14–15 (6 hours)</td>
<td>Topic 1.3: Counting and probability</td>
<td><strong>Revision and end of Unit 1 assessment</strong></td>
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Sample course outline
Mathematics Methods – ATAR Year 11

Unit 2

In Unit 2 students will be provided with opportunities to:

- understand the concepts and techniques used in algebra, sequences and series, functions, graphs, and calculus
- solve problems in algebra, sequences and series, functions, graphs, and calculus
- apply reasoning skills in algebra, sequences and series, functions, graphs, and calculus
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems
- communicate arguments and strategies when solving problems.

This course outline assumes an allocation of 4 hours contact time per week for the course.

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<td><strong>Semester 2 (Unit 2)</strong></td>
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| **Weeks 16–18** (10 hours)    | Topic 2.1: Exponential functions | **Indices and the index laws** (2.1.1 – 2.1.3)
  • review indices (including fractional and negative indices) and the index laws
  • use radicals and convert to and from fractional indices
  • understand and use scientific notation and significant figures

**Exponential functions** (2.1.4 – 2.1.7)
  • establish and use the algebraic properties of exponential functions
  • recognise the qualitative features of the graph of \( y = a^x \) (\( a > 0 \)), including asymptotes, and of its translations (\( y = a^x + b \) and \( y = a^{x-c} \))
  • identify contexts suitable for modelling by exponential functions and use them to solve practical problems
  • solve equations involving exponential functions using technology, and algebraically in simple cases

| **Weeks 18–19** (6 hours)  | Topic 2.2: Arithmetic and geometric sequences and series | **Arithmetic sequences** (2.2.1 – 2.2.4)
  • recognise and use the recursive definition of an arithmetic sequence \( t_{n+1} = t_n + d \)
  • develop and use the formula \( t_n = t_1 + (n - 1)d \) for the general term of an arithmetic sequence and recognise its linear nature
  • use arithmetic sequences in contexts involving discrete linear growth or decay, such as simple interest
  • establish and use the formula for the sum of the first \( n \) terms of an arithmetic sequence
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| **Weeks 20–22**              | Topic 2.2: Arithmetic and geometric sequences and series | **Geometric sequences** (2.2.5 – 2.2.9)  
- recognise and use the recursive definition of a geometric sequence  
  \( t_{n+1} = t_n r \)  
- develop and use the formula  
  \( t_n = t_1 r^{n-1} \)  
- understand the limiting behaviour as  \( n \to \infty \) of the terms  \( t_n \) in a geometric sequence and its dependence on the value of the common ratio \( r \)  
- establish and use the formula  
  \( S_n = t_1 \frac{r^n - 1}{r - 1} \)  
- use geometric sequences in contexts involving geometric growth or decay, such as compound interest |
| (9 hours)                    |         |                                          |
| **Weeks 22–24**              | Topic 2.3: Introduction to differential calculus | **Rates of change and the concept of the derivative** (2.3.1 – 2.3.9)  
- interpret the difference quotient  
  \( \frac{f(x+h)-f(x)}{h} \)  
- use the Leibniz notation  
  \( \delta x \) and  \( \delta y \) for changes or increments in the variables  \( x \) and  \( y \)  
- use the notation  
  \( \frac{\delta y}{\delta x} \) for the difference quotient  
  \( \frac{f(x+h)-f(x)}{h} \)  
- interpret the ratios  
  \( \frac{f(x+h)-f(x)}{h} \) and  \( \frac{\delta y}{\delta x} \) as the slope or gradient of a chord or secant of the graph of  \( y = f(x) \)  
- examine the behaviour of the difference quotient  
  \( \frac{f(x+h)-f(x)}{h} \) as  \( h \to 0 \)  
- define the derivative  
  \( f'(x) \) as  
  \( \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \)  
- use the Leibniz notation for the derivative:  
  \( \frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} \) and the correspondence  
  \( \frac{dy}{dx} = f'(x) \)  
- interpret the derivative as the instantaneous rate of change  
- interpret the derivative as the slope or gradient of a tangent line of the graph of  \( y = f(x) \) |
| (9 hours)                    |         |                                          |
| **Weeks 24–26**              | Topic 2.3: Introduction to differential calculus | **Computation and properties of derivatives** (2.3.10 – 2.3.15)  
- estimate numerically the value of a derivative for simple power functions  
- examine examples of variable rates of change of non-linear functions  
- establish the formula  
  \( \frac{d}{dx}(x^n) = nx^{n-1} \)  
- understand the concept of the derivative as a function  
- identify and use linearity properties of the derivative  
- calculate derivatives of polynomials |

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<td><strong>Semester 2 (Unit 2)</strong></td>
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<td><strong>Applications of derivatives and anti-derivatives (2.3.16 – 2.3.22)</strong></td>
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| **Weeks 26–29** (12 hours)    | Topic 2.3: Introduction to differential calculus | • determine instantaneous rates of change  
• determine the slope of a tangent and the equation of the tangent  
• construct and interpret position-time graphs with velocity as the slope of the tangent  
• recognise velocity as the first derivative of displacement with respect to time  
• sketch curves associated with simple polynomials, determine stationary points, and local and global maxima and minima, and examine behaviour as \( x \to \infty \) and \( x \to -\infty \)  
• solve optimisation problems arising in a variety of contexts involving polynomials on finite interval domains  
• calculate anti-derivatives of polynomial functions |
| **Week 29–30**                |         | **Revision and end of course assessment** |