Government of **Western Australia**
School Curriculum and Standards Authority

ATAR course examination, 2023

Question/Answer booklet

MATHEMATICS APPLICATIONS

Section Two:
Calculator-assumed

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

WA student number: In figures

In words

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	6	6	50	52	35
Section Two: Calculator-assumed	8	8	100	97	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2023: Part II Examinations*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Section Two: Calculator-assumed 65% (97 Marks)

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 7 **(9 marks)**

The heights of players and the average number of rebounds per game were recorded for a basketball team over the course of a 30 game season. The data collected is shown in the table below.

Player	Height (x) (cm)	Average number of rebounds (y)
1	198	10
2	174	2
3	200	12
4	182	2
5	184	3
6	209	12
7	196	19
8	185	8
9	202	14
10	179	1

(a) Identify the explanatory variable. (1 mark)

(b) Complete the scatter graph below by plotting the missing data from the table.

(2 marks)
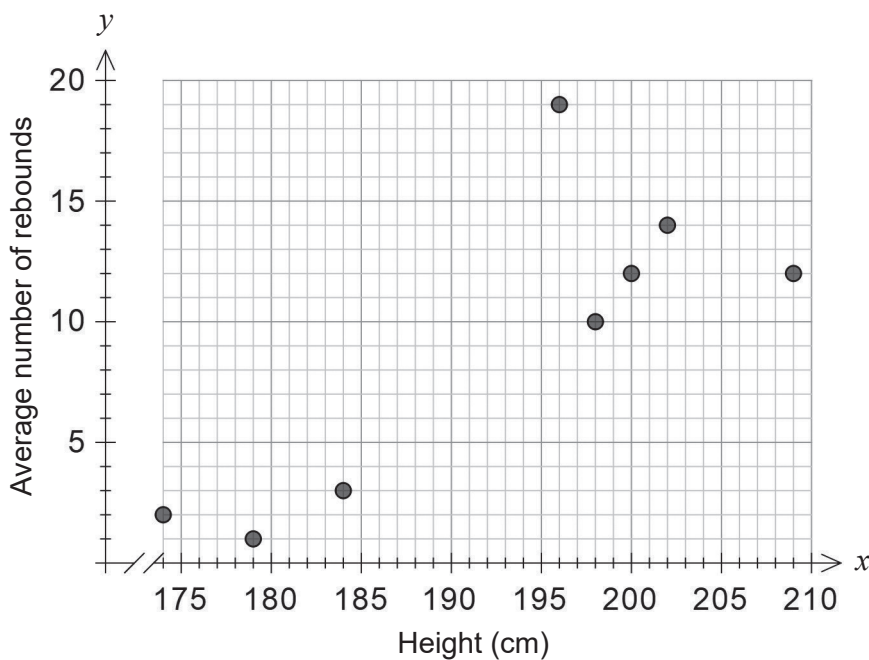
The data has a correlation coefficient of 0.814, and the equation of the least-squares line is $y = 0.43x - 74.23$.

(c) Draw the least-squares line on the graph above. (2 marks)

See next page

(d)   Describe the association between players' heights and average number of rebounds in terms of direction and strength.   (2 marks)

(e)   Determine the coefficient of determination and state its meaning in the context of the question.   (2 marks)

Question 8 **(16 marks)**

The table below shows the average price per kilogram of watermelon in a supermarket for each of the four seasons of the year over a four-year period.

Year	Quarter	Time (t)	Average price ($)	4-point centred moving average	Quarterly mean	Average price as a percentage of the quarterly mean	Deseasonalised data
2019	Summer	1	1.06	-	1.85	57.30	1.80
	Autumn	2	1.36	-		73.51	1.79
	Winter	3	2.94	1.87		158.92	1.89
	Spring	4	2.03	1.92		109.73	1.86
2020	Summer	5	1.24	1.97	P	59.05	2.11
	Autumn	6	1.58	2.04		75.24	2.07
	Winter	7	3.12	2.12		148.57	2.01
	Spring	8	2.44	2.20		116.19	2.23
2021	Summer	9	1.47	2.29	2.39	61.51	2.50
	Autumn	10	1.93	2.36		80.75	2.53
	Winter	11	3.48	2.41		145.61	2.23
	Spring	12	2.69	2.44		112.55	2.46
2022	Summer	13	**Q**	**R**	2.75	57.09	2.69
	Autumn	14	2.07	2.75		**S**	2.72
	Winter	15	4.65	-		169.09	2.99
	Spring	16	2.71	-		98.55	2.48

(a) Determine the values **P**, **Q**, **R** and **S** in the table above. (4 marks)

(b) Complete the following table. (1 mark)

Season	Summer	Autumn	Winter	Spring
Seasonal index	58.83%	76.19%		109.26%

(c) Interpret the seasonal index for Summer in the context of this question. (1 mark)

(d)   Define 'deseasonalised data'.   (1 mark)

(e)   Using information from the tables on page 6, show how the deseasonalised data value
         for Autumn 2021 was calculated.   (1 mark)

(f)   Determine the equation of the least-squares line using the deseasonalised data. (1 mark)

(g)   The graph below is the time series plot for these data. On this graph draw the
         least-squares line determined in part (f).   (2 marks)

Average price ($)
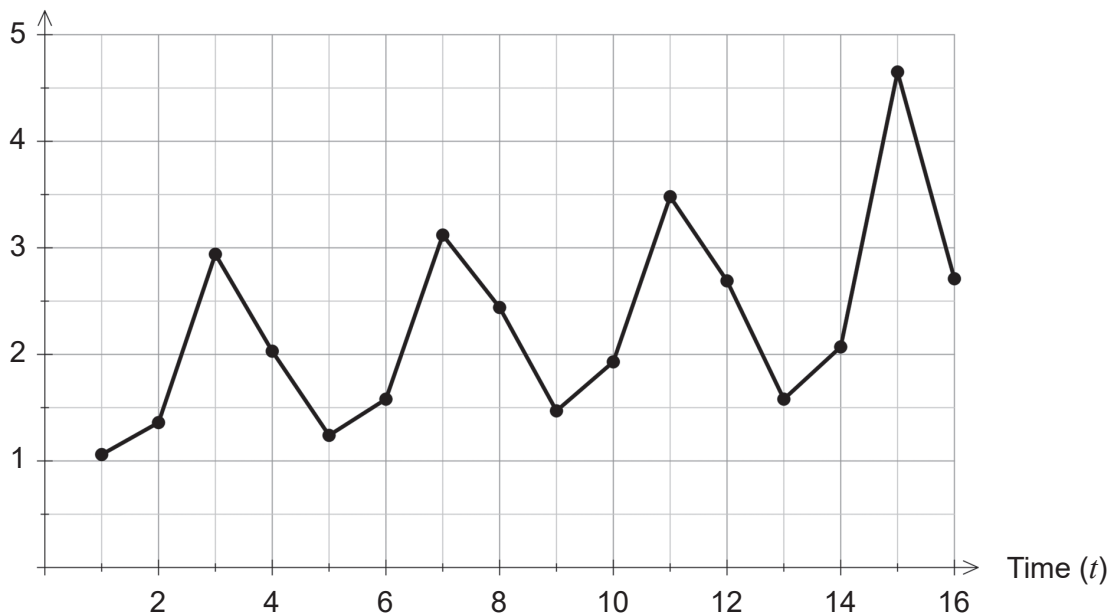
(h)   Predict the average price per kilogram of watermelon from the supermarket in
         Spring 2024.   (3 marks)

(i)   Discuss the reliability of the prediction in part (h).   (2 marks)

See next page

Question 9 **(16 marks)**

Sonia secures a bank loan to buy a professional gaming computer. The loan has reducible interest. Information about the loan is shown below.

Loan issued: Start of October 2023.
Starting balance: $9200.
Interest: Compounded monthly.
Repayments: $290 per month.

After the first monthly payment at the end of October 2023, Sonia's balance is $8992.80.

(a) Use the information above to show that the annual interest rate is 10.8%. (2 marks)

(b) Determine a recursive rule to model the balance of the loan at the end of each month. (2 marks)

(c) Determine

(i) the balance of the loan at the end of November 2023. (1 mark)

(ii) the total amount of interest incurred in the first three months. (2 marks)

(iii) the balance of the loan at the end of May 2024. (1 mark)

(d)   Determine how many months it takes to repay the loan.   (1 mark)

(e)   Determine the final repayment and the total amount repaid.   (2 marks)

(f)   Calculate the total interest paid on the loan.   (1 mark)

(g)   Sonia is paid every fortnight in her employment. Instead of monthly repayments of $290, she is now considering making fortnightly repayments of $145, with the interest calculated fortnightly. Use mathematical evidence to show what difference this would make and advise Sonia what her savings might be.   (4 marks)

Question 10 (18 marks)

Data concerning rental properties have been collected from 10 suburbs of a city. The data is for median property value (p) ($'000), median weekly rent ($\$w$) and percentage vacancy rate ($v\%$) within each suburb.

The data in the table below show the median property value and the median weekly rent for the 10 suburbs.

Median property value (p) ($'000)	395	470	550	725	580	780	700	740	690	585
Median weekly rent (w) ($)	445	460	590	630	530	850	680	690	640	575

(a) Calculate the correlation coefficient and the equation of the least-squares line for these data. (3 marks)

(b) In the context of this question, interpret the gradient of the least-squares line determined in part (a). (2 marks)

(c) The graph below shows the residual plot for the first nine suburbs as given in the table. Determine the residual for the 10th suburb and plot this value on the graph. (2 marks)
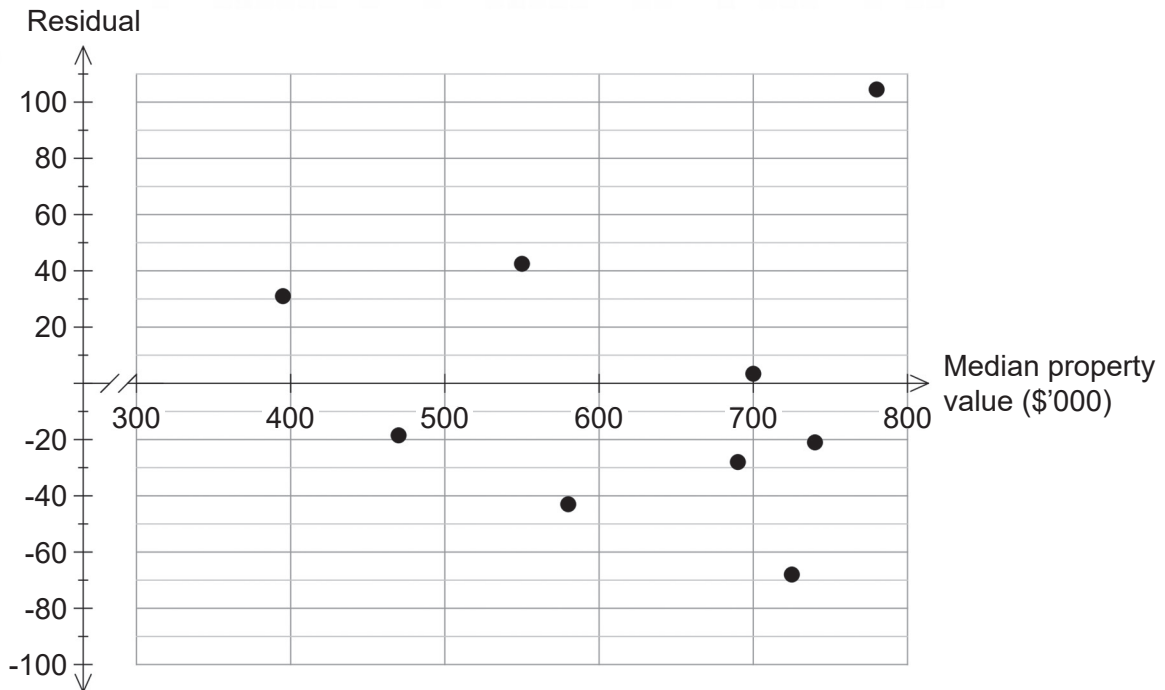
See next page

(d) State a conclusion that can be drawn from the residual plot. (1 mark)

(e) The predicted weekly rent of a property was calculated to be $612. What property value was this based on? (2 marks)

(f) If the data point (780, 850) was removed from all calculations, would the gradient of the least-squares line determined in part (a) increase, decrease or stay the same? (1 mark)

Bivariate data analysis between percentage vacancy rate and median weekly rent produced the following: $r_{vw}^2 = 0.85$ and $w = -82.64v + 940.64$.

(g) Explain why $r_{vw} = -0.92$. (2 marks)

(h) A property has a vacancy rate of 4.1% and a median property value of $605 000. Predict the median weekly rent using the most reliable predictor. Justify which predictor is used. (2 marks)

(i) Calculate the expected change in the weekly rent if the percentage vacancy rate increases by 0.4%. (1 mark)

(j) Comment on the statement 'it is clear both property price and vacancy rate will cause changes to the median weekly rent'. Justify your answer. (2 marks)

Question 11 **(10 marks)**

A ferry service connects a group of islands in the Pacific Ocean.

(a) The network below is for the hourly departures from A and shows the number of passengers that can be carried between each island, A, B, C, D, E, F and G. Determine the maximum number of passengers that can be carried from A to F each hour. Show systematic workings. (4 marks)
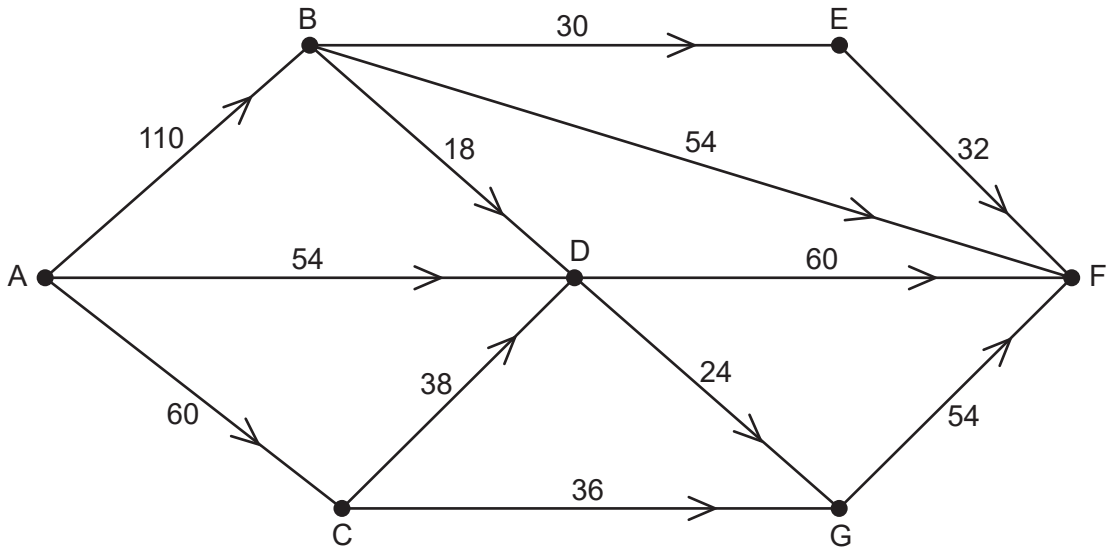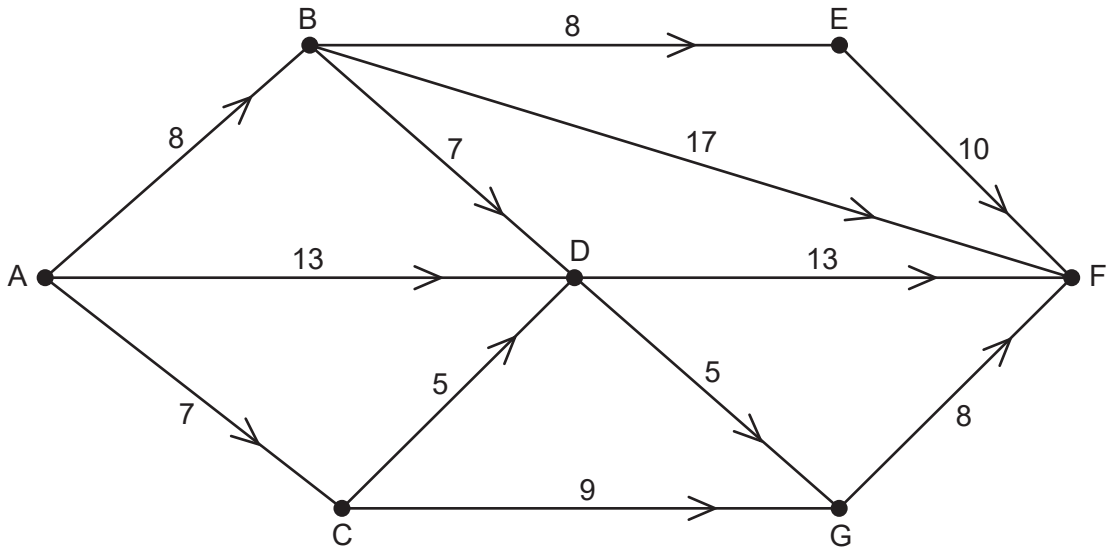

(b) On the network below, draw the minimum cut that corresponds to the maximum flow determined in part(a).

(1 mark)
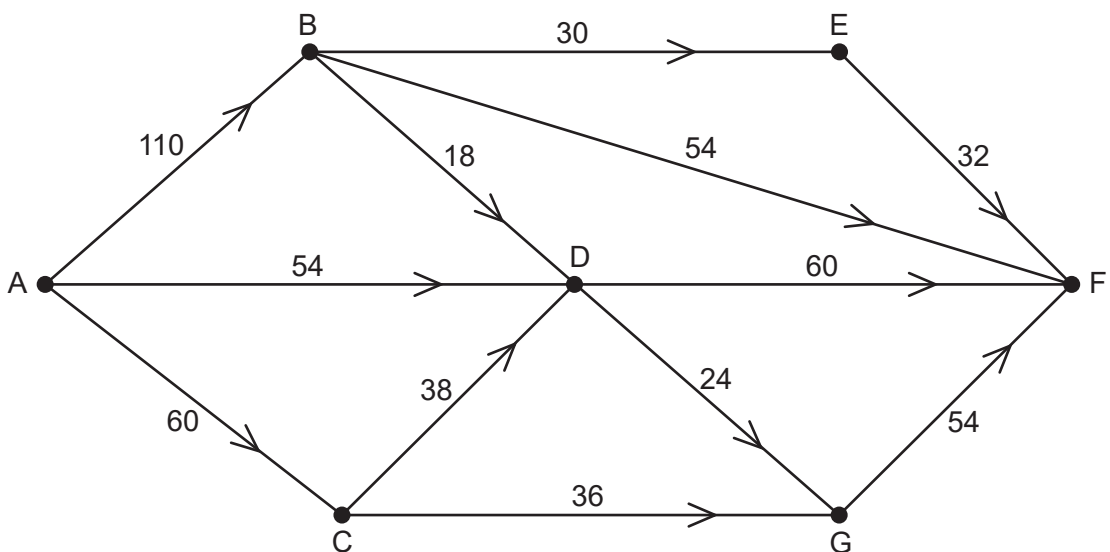
(c)   The network below shows the distance in kilometres between the islands. Ferry passengers are charged a $10 booking fee plus $1.50 for each kilometre they plan to travel. Calculate the minimum cost of travelling from A to F.   (3 marks)

(d)   During the off-season when there are fewer passengers, the ferry company will only maintain the services that form the minimum spanning tree. On the network below, highlight clearly the services that will be maintained.   (2 marks)
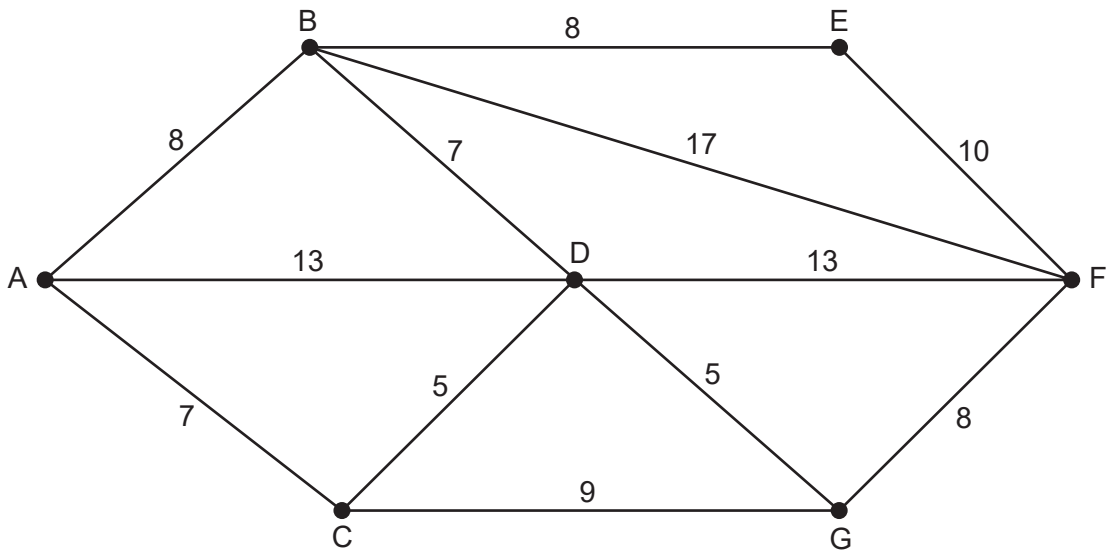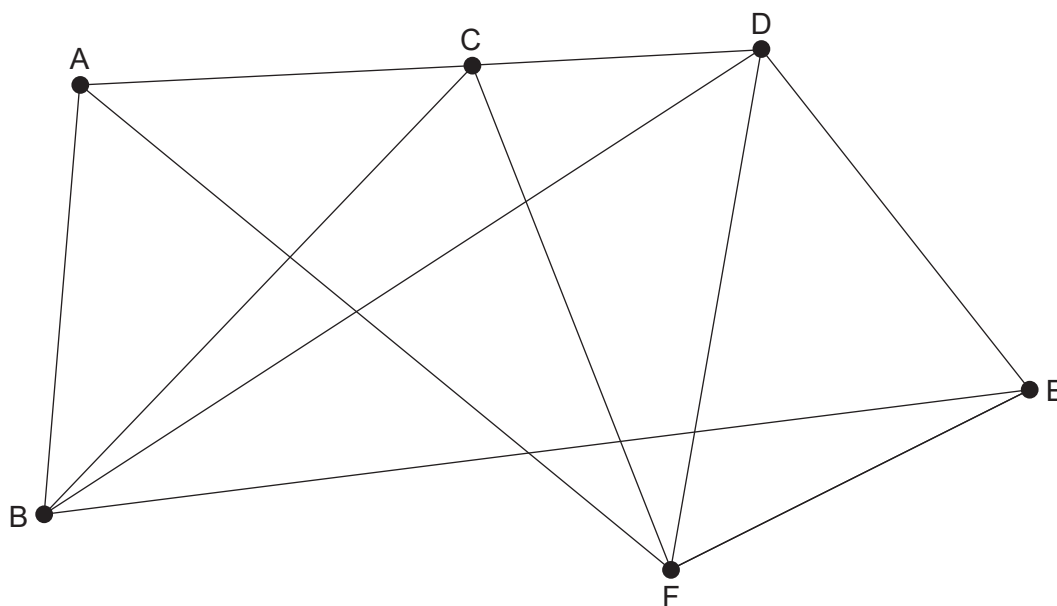
Question 12 **(7 marks)**

An event organiser is setting up a music festival with multiple stages. The graph below shows the stages as vertices, and the edges are the walkways that connect each stage.

(a) To avoid congestion, the organisers want to move the stages and re-position some connecting walkways so no walkways intersect. Redraw the graph to meet these conditions. (2 marks)

(b)   Use Euler's formula, stating the number of vertices, edges and faces, to verify that this
       graph is planar.   (2 marks)

(c)   State a reason why the graph is not Eulerian.   (1 mark)

One attendee wishes to visit every stage to make the most of the festival.

(d)   (i)   Write down a pathway the attendee can take to visit every stage exactly once if
              they start at Stage F.   (1 mark)

       (ii)   What is the name given to this path?   (1 mark)

Question 13 **(10 marks)**

Chau purchased a brand-new motorcycle valued at $17 000. Once Chau drove the motorcycle out of the dealer's yard, it immediately depreciated in value by 15%. By the end of the first year, it depreciated a further 5%. After the first year the motorcycle depreciated at a constant rate of 7% per year.

(a)

n (years)	1	2	3	4
Value of the motorcycle ($)	13 727.50			

    (i)   Show how the value of the motorcycle at the end of the first year was calculated.

                                                        (2 marks)

    (ii)   Complete the remainder of the table.   (2 marks)

(b)   Determine the percentage by which the motorcycle depreciated in the first year.

                                                         (2 marks)

(c)   Write a recursive rule that represents the amount the motorcycle will be worth at the end of each year, after the first year.   (2 marks)

(d)   Determine how much the motorcycle will be worth at the end of eight years.   (1 mark)

(e)   When will the motorcycle be first worth less than $5000?   (1 mark)

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Question 14 **(11 marks)**

Ravi retired at the beginning of the month, with a superannuation balance of $945 864. He has a written guarantee of a 7.5% per annum return on his superannuation with interest added at the end of each month. Ravi will receive an annuity of $3200 paid at the end of each fortnight.

Ravi has also committed to purchasing a new car for a price of $37 000. There is a 12-month waiting period for the delivery of the car and he plans to pay for the car by withdrawing the cost from his superannuation account at time of delivery, when payment is due.

(a)   Calculate the balance in the superannuation account at the end of two years.   (6 marks)

(b)   After 15 years of retirement Ravi expects to have a less active lifestyle and his living expenses will not be as high. He also understands that the balance of his superannuation needs to last for his remaining years. Ravi decides the most suitable plan for him is to place the balance of his superannuation into a perpetuity with a fortnightly payment.

Assuming the 7.5% interest rate is maintained, what fortnightly amount can Ravi expect from the perpetuity?   (5 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

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