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WACE
Western Australian Certificate of Education

MATHEMATICS: SPECIALIST

UNITS 3C AND 3D

FORMULA SHEET
2014

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Vectors

Magnitude: $|(a_1, a_2, a_3)| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

Dot product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta = a_1b_1 + a_2b_2 + a_3b_3$

Triangle inequality: $|\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$

Vector equation of a line in space:
- one point and the slope: $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$
- two points A and B: $\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$

Cartesian equations of a line in space: $\dfrac{x - a_1}{b_1} = \dfrac{y - a_2}{b_2} = \dfrac{z - a_3}{b_3}$

Parametric form of vector equation of a line in space:

$$x = a_1 + \lambda b_1 \text{......}(1)$$
$$y = a_2 + \lambda b_2 \text{......}(2)$$
$$z = a_3 + \lambda b_3 \text{......}(3)$$

Vector equation of a plane in space: $\mathbf{r} \cdot \mathbf{n} = c$ or $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$

Trigonometry

In any triangle ABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$A = \frac{1}{2}ab\sin C$$

In a circle of radius r, for an arc subtending angle θ (radians) at the centre:

Length of arc $= r\theta$

Area of segment $= \frac{1}{2}r^2(\theta - \sin\theta)$ Area of sector $= \frac{1}{2}r^2\theta$

Identities:

$\cos^2\theta + \sin^2\theta = 1$

$\cos(\theta \pm \varphi) = \cos\theta\cos\varphi \mp \sin\theta\sin\varphi$

$\sin(\theta \pm \varphi) = \sin\theta\cos\varphi \pm \cos\theta\sin\varphi$

$\tan(\theta \pm \varphi) = \dfrac{\tan\theta \pm \tan\varphi}{1 \mp \tan\theta\tan\varphi}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

$\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$ $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{x} = 0$

Simple Harmonic Motion: If $\dfrac{d^2x}{dt^2} = -k^2x$ then $x = A\sin(kt + \alpha)$ or $x = A\cos(kt + \beta)$ and $v^2 = k^2(A^2 - x^2)$, where A is the amplitude of the motion, α and β are phase angles, v is the velocity and x is the displacement.

Functions

Differentiation:

If $f(x) = y$ then $f'(x) = \dfrac{dy}{dx}$

If $f(x) = x^n$ then $f'(x) = nx^{n-1}$

If $f(x) = e^x$ then $f'(x) = e^x$

If $f(x) = \ln x$ then $f'(x) = \dfrac{1}{x}$

If $f(x) = \sin x$ then $f'(x) = \cos x$

If $f(x) = \cos x$ then $f'(x) = -\sin x$

If $f(x) = \tan x$ then $f'(x) = \sec^2 x = \dfrac{1}{\cos^2 x}$

Product rule:

If $y = f(x)\, g(x)$

then $y' = f'(x)\, g(x) + f(x)\, g'(x)$

or

If $y = uv$

then $\dfrac{dy}{dx} = \dfrac{du}{dx} v + u \dfrac{dv}{dx}$

Quotient rule:

If $y = \dfrac{f(x)}{g(x)}$

then $y' = \dfrac{f'(x)\, g(x) - f(x)\, g'(x)}{(g(x))^2}$

or

If $y = \dfrac{u}{v}$

then $\dfrac{dy}{dx} = \dfrac{\dfrac{du}{dx} v - u \dfrac{dv}{dx}}{v^2}$

Incremental formula: $\delta y \simeq \dfrac{dy}{dx} \delta x$ or $f(x+h) - f(x) \simeq f'(x)h$

Chain rule:

If $y = f(g(x))$

then $y' = f'(g(x))\, g'(x)$

or

If $y = f(u)$ and $u = g(x)$

then $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

Integration:

Powers: $\int x^n dx = \dfrac{x^{n+1}}{n+1} + c,\ n \neq -1$

Exponentials: $\int e^x dx = e^x + c$

Logarithms: $\int \dfrac{1}{x}\, dx = \ln|x| + c$

Trigonometric:

$\int \sin x\, dx = -\cos x + c$

$\int \cos x\, dx = \sin x + c$

$\int \dfrac{1}{\cos^2 x}\, dx = \tan x + c$

Fundamental Theorem of Calculus:

$\dfrac{d}{dx}\int_a^x f(t)\, dt = f(x)$ and $\int_a^b f'(x)\, dx = f(b) - f(a)$

Functions

Quadratic function:

If $y = ax^2 + bx + c$ and $y = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for $x \in C$

Piecewise-defined functions:

Absolute value function: $|x| = \begin{cases} x, & \text{for } x \geq 0 \\ -x, & \text{for } x < 0 \end{cases}$

Sign function: $\text{sgn}\,(x) = \begin{cases} 1, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \\ -1, & \text{for } x < 0 \end{cases}$

Greatest integer function: $\text{int}\,(x)$ = greatest integer $\leq x$ for all x

Matrices

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $|A| = \det A = ad - bc$

$A^{-1} = \dfrac{1}{(ad - bc)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Dilation = $\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$

Shear = $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix}$

Rotation = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Reflection = $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

Complex numbers

For $z = a + ib$, where $i^2 = -1$

Argument: $\arg z = \theta$, where $\tan \theta = \frac{b}{a}$ and $-\pi < \theta \leq \pi$

Modulus: $\text{mod } z = |z| = |a + ib| = \sqrt{a^2 + b^2} = r$

Product: $|z_1 z_2| = |z_1|\,|z_2|$ $\quad$ $\arg(z_1 z_2) = \arg z_1 + \arg z_2$

Quotient: $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ $\quad$ $\arg \frac{z_1}{z_2} = \arg z_1 - \arg z_2$

Polar form:

For $z = r \text{ cis}\theta$, where $r = |z|$ and $\theta = \arg z$:

$\text{cis}(\theta + \varphi) = \text{cis } \theta \text{ cis } \varphi$ $\quad$ $\text{cis } \theta = \cos \theta + i \sin \theta$

$\text{cis}(-\theta) = \frac{1}{\text{cis } \theta}$ $\quad$ $\text{cis}(0) = 1$

$z_1 z_2 = r_1 r_2 \text{cis}(\theta + \varphi)$ $\quad$ $\frac{z_1}{z_2} = \frac{r_1}{r_2} \text{cis}(\theta - \varphi)$

Exponential form:

$z = re^{i\theta}$, where $r = |z|$ and $\theta = \arg z$

For complex conjugates:

$z = a + bi$ $\quad$ $\bar{z} = a - bi$

$z = r \text{ cis } \theta$ $\quad$ $\bar{z} = r \text{ cis } (-\theta)$

$z = re^{i\theta}$ $\quad$ $\bar{z} = re^{-i\theta}$

$z\bar{z} = |z|^2$ $\quad$ $z^{-1} = \frac{1}{z} = \frac{\bar{z}}{|z|^2}$

$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$ $\quad$ $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$

Exponentials and logarithms

For a, $b > 0$ and m, n real:

$$a^m a^n = a^{m+n} \qquad \frac{a^m}{a^n} = a^{m-n}$$

$$a^0 = 1 \qquad a^{-n} = \frac{1}{a^n}$$

$$(a^m)^n = a^{mn} \qquad (ab)^m = a^m b^m$$

For m an integer and n a positive integer:

$$a^{\frac{1}{n}} = \sqrt[n]{a} \qquad a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

For a, b, y, m and n positive real and k real:

$$1 = a^0 \Leftrightarrow \log_a 1 = 0 \qquad y = a^x \Leftrightarrow \log_a y = x$$

$$\log_a mn = \log_a m + \log_a n \qquad a = a^1 \Leftrightarrow \log_a a = 1$$

$$\log_a m = \frac{\log_b m}{\log_b a} \text{ (change of base)} \qquad \log_a(m^k) = k \log_a m$$

If $\dfrac{dP}{dt} = kP$, then $P = P_0 e^{kt}$

Mathematical reasoning

De Moivre's theorem:

$$(\text{cis } \theta)^n = (\cos\theta + i\sin\theta)^n$$

$$(\text{cis } \theta)^n = \cos n\theta + i\sin n\theta$$

$$z^n = |z|^n \text{cis } (n\theta)$$

$$z^{\frac{1}{q}} = |z|^{\frac{1}{q}}\left[\cos\left(\frac{\theta + 2\pi k}{q}\right) + i\sin\left(\frac{\theta + 2\pi k}{q}\right)\right] \text{ for } k \text{ an integer.}$$

Measurement

Circle: $C = 2\pi r = \pi D$, where C is the circumference, r is the radius and D is the diameter
$A = \pi r^2$, where A is the area

Triangle: $A = \frac{1}{2}bh$, where b is the base and h is the perpendicular height

Parallelogram: $A = bh$

Trapezium: $A = \frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides

Prism: $V = Ah$, where V is the volume and A is the area of the base

Pyramid: $V = \frac{1}{3} Ah$

Cylinder: $S = 2\pi rh + 2\pi r^2$, where S is the total surface area
$V = \pi r^2 h$

Cone: $S = \pi rs + \pi r^2$, where s is the slant height
$V = \frac{1}{3}\pi r^2 h$

Sphere: $S = 4\pi r^2$
$V = \frac{4}{3}\pi r^3$

Note: Any additional formulas identified by the examination panel as necessary will be included in the body of the particular question.