Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS APPLICATIONS

Calculator-free

ATAR course examination 2018

Marking Key

Marking keys are an explicit statement about what the examining panel expect of candidates when they respond to particular examination items. They help ensure a consistent interpretation of the criteria that guide the awarding of marks.

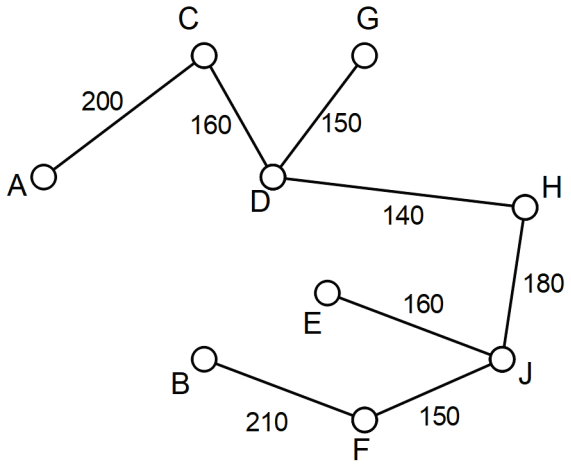
Copyright © School Curriculum and Standards Authority 2018

2018/62744

Section One: Calculator-free **35% (55 Marks)**

Question 1 **(4 marks)**

(a) Highlight, on the diagram below, the bush tracks where the pipes should be installed. (2 marks)

Solution

Specific behaviours
✓ correctly shows at least 4 arcs ✓ correctly draws the minimal spanning tree containing A

(b) Calculate the minimum length of piping required. (2 marks)

Solution
Min distance = 200 + 160 + 150 + 140 + 180 + 160 + 150 + 210 = 1350 metres
Specific behaviours
✓ calculates the correct distance ✓ correctly uses metres

Question 2 **(8 marks)**

(a) Complete the adjacency matrix for the digraph. (2 marks)

Solution
$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \end{array}$
Specific behaviours
✓ correctly gives the FC pathway ✓ completes all elements correctly in the bottom row

(b) Mai gets on a bus at E. She travels directly to B, changes bus, and continues travelling BFCDEA. Explain why her route is a trail. (2 marks)

Solution
It is a walk in which no edge is repeated.
Specific behaviours
✓ states that it is a walk ✓ states that no edge is repeated

(c) Vinh has just visited Attraction C and wishes to visit Attraction B next. Determine the route he should take to arrive at B in the shortest travelling time. State the time taken. (2 marks)

Solution
CDEAB 23 minutes
Specific behaviours
✓ correctly states the shortest route to be taken ✓ correctly states the minimum travelling time

(d) Toshi is at Attraction D. He wants to complete a Hamiltonian path. State the route he should take. (2 marks)

Solution
DGFCEAB
Specific behaviours
✓ states a path containing all vertices ✓ states the correct Hamiltonian path (finishing at a vertex different to the starting vertex)

Question 3 **(9 marks)**

(a) Describe the association between the number of people/household, P, and the daily rubbish production/household, R, in terms of strength and form. (2 marks)

Solution
Strong and non-linear
Specific behaviours
✓ states correct strength ✓ correctly identifies a non-linear form

(b) It was found that approximately 49% of the variation in daily water usage, W, could be explained by the variation in the number of people/household, P. Determine the correlation coefficient (r_{PW}). (1 mark)

Solution
$r_{PW} = \sqrt{0.49}$ $r_{PW} = 0.7$
Specific behaviours
✓ states correct value for r_{PW}

(c) The equation of the least-squares line for the graph showing daily water usage/household is $W = 0.83P + 4.7$.

(i) Interpret the slope of this line. (2 marks)

Solution
For each additional person in the household, the daily water usage increases by approximately 83 Litres.
Specific behaviours
✓ correctly identifies an increase ✓ of 83 Litres/person

(ii) Predict the daily water usage for a household with 10 people. Comment on the likelihood of this being a valid prediction and justify your answer. (3 marks)

Solution
$W = 0.83(10) + 4.7$ $W = 8.3 + 4.7$ $W = 13$ Therefore, the daily water usage is 1300 Litres. This is extrapolation, so the prediction is not very likely.
Specific behaviours
✓ correctly predicts water usage for 10 people ✓ correctly identifies extrapolation ✓ states the prediction is not likely

(d) The council argued that increasing the number of people/household causes the daily water usage to increase. Provide a non-causal explanation for the association between these two variables. (1 mark)

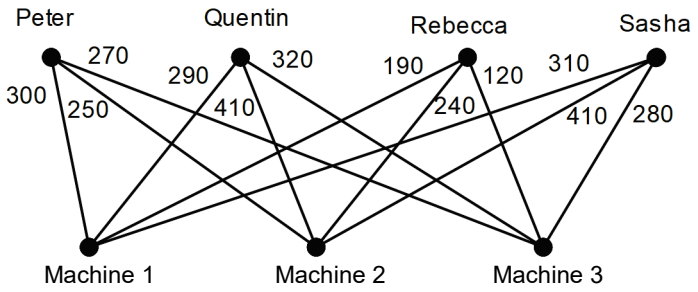
Solution
For example: • It is simply a coincidence. • There may be another variable/factor affecting the amount of water being used, e.g. the size of the garden.
Specific behaviours
✓ correctly identifies a non-causal explanation

Question 4 **(8 marks)**

The table below shows the number of metres of shade cloth that can be produced in a day by each machine operator.

		Machines		
		1	2	3
Workers	Peter	300	250	270
	Quentin	290	410	320
	Rebecca	190	240	120
	Sasha	310	410	280

(a) Draw the weighted bipartite graph below, showing the possible allocations for each of the workers. (2 marks)

Solution

Specific behaviours
✓ correctly shows allocated pairings ✓ correctly labels graph with lengths of shade cloth

(b) Why has she added the column of zeros? (1 mark)

Solution
The number of columns does not equal the number of rows, i.e. not a square.
Specific behaviours
✓ because the given matrix is not square

(c) Continue the steps of the Hungarian algorithm, showing the optimum allocation of workers to machines in the table below. State the maximum total length of shade cloth that can be produced in the day. (5 marks)

Solution

$$\begin{bmatrix} 110 & 160 & 140 & 410 \\ 120 & 0 & 90 & 410 \\ 220 & 170 & 290 & 410 \\ 100 & 0 & 130 & 410 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 160 & 50 & 0 \\ \cancel{20} & \cancel{0} & \cancel{0} & \cancel{0} \\ 120 & 170 & 200 & 0 \\ \cancel{0} & \cancel{0} & \cancel{40} & \cancel{0} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 150 & 40 & 0 \\ \cancel{20} & \cancel{0} & \cancel{0} & \cancel{10} \\ 110 & 160 & 190 & 0 \\ \cancel{0} & \cancel{0} & \cancel{40} & \cancel{10} \end{bmatrix}$$

Worker	Peter	Quentin	Rebecca	Sasha
Machine	1	3	none	2

Maximum total length of shade cloth = 300 + 320 + 410 = 1030 metres

Specific behaviours

✓ subtracts all numbers from the largest number, 410
✓ subtracts the smallest number from each element in each column
✓ subtracts lowest number uncovered from each uncovered number and adds lowest number uncovered to the intersection points
✓ states correct allocations
✓ states correct maximum length of shade cloth

Question 5 **(9 marks)**

(a) Consider the adjacency matrix for Town 1.

(i) Explain why the network represented by this matrix is a directed graph. (1 mark)

Solution
The adjacency matrix is not symmetrical about the leading diagonal. **or** There are two ways from B to C but only one way from C to B (or two ways from D to C but only one way from C to D).
Specific behaviours
✓ states a valid reason

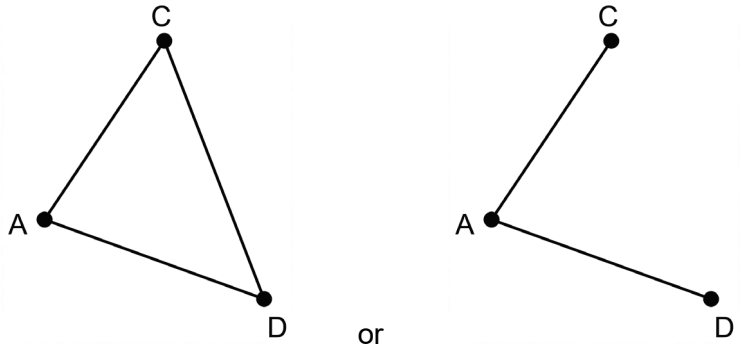
(ii) Give **two** reasons why the network represented by this matrix is not a simple graph. (2 marks)

Solution
A simple graph contains no loops or multiple edges. There is a loop joining C to C. There are two edges between A and C and D and C.
Specific behaviours
✓ correctly identifies a reason ✓ correctly identifies a second reason

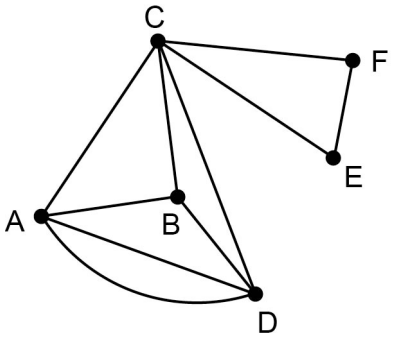
(b) (i) Explain the significance of the element in Row 3, Column 4. (2 marks)

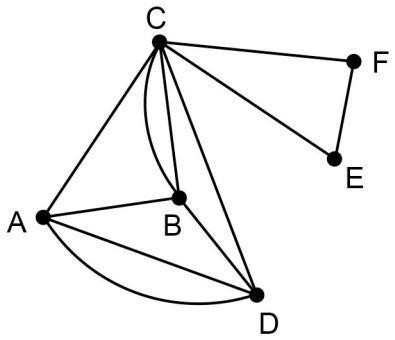
Solution
The element 3 indicates that there are three 2-stage routes from C to D.
Specific behaviours
✓ identifies the element value correctly ✓ states the relevance of the element correctly

(ii) Draw a connected subgraph containing only vertices A, C and D. (2 marks)

Solution
For example: 
Specific behaviours
✓ draws graph containing only vertices A, C and D ✓ draws graph using only existing edges between A, C and D

(c) (i) Draw an edge on the graph below that allows this to occur. (1 mark)



Solution

Specific behaviours
✓ correctly draws an edge between C and B

(ii) Explain why an Eulerian trail is now possible. (1 mark)

Solution
An Eulerian trail is only possible with no odd vertices.
Specific behaviours
✓ correctly identifies why a Eulerian trail is possible.

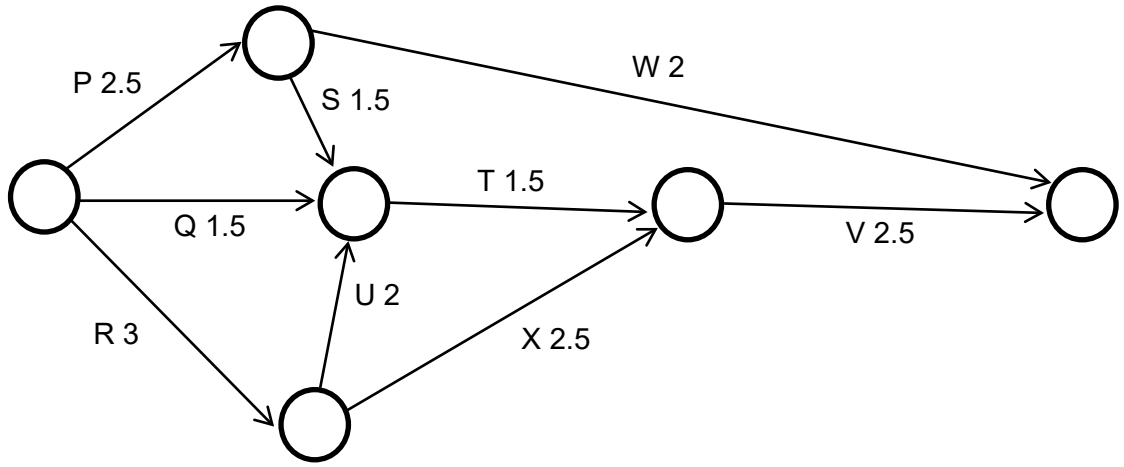
Question 6 **(11 marks)**

Yana has booked three gardeners to landscape her garden. The table below shows the required activities, together with the times taken (in hours) and the immediate predecessors for each activity.

Activity	P	Q	R	S	T	U	V	W	X
Time (hours)	2.5	1.5	3	1.5	1.5	2	2.5	2	2.5
Immediate Predecessors	–	–	–	P	S, Q, U	R	T, X	P	R

(a) Complete the network diagram below, showing all tasks and durations. (3 marks)

Solution



Specific behaviours

✓ starts diagram with S and W following P, as well as U and X following R
✓ completes network diagram with correct edges
✓ network is labelled with tasks, durations and arrows

(b) Determine the critical path and the minimum completion time for the project. (2 marks)

Solution

The critical path is RUTV.
The minimum completion time is 9 hours.

Specific behaviours

✓ states the correct critical path
✓ correctly states the minimum completion time

(c) Calculate the float time for

(i) Activity W. (1 mark)

Solution
4.5 hours
Specific behaviours
✓ correctly states float time for W

(ii) Activity U. (1 mark)

Solution
0 hours
Specific behaviours
✓ correctly states there is no float time for U

(d) (i) Determine the latest starting time for Activity P. (1 mark)

Solution
7.30 am
Specific behaviours
✓ correctly determines latest starting time

(ii) Determine the earliest starting time for Activity V. (1 mark)

Solution
Adding a break of half an hour means the project takes 9.5 hours (finishing at 4 pm). Therefore, V must start no later than 1.30 pm.
Specific behaviours
✓ correctly determines earliest starting time

(e) One of the gardeners becomes ill and is unable to work on Yana's landscaping job. How, if at all, will this affect the minimum completion time for this project (excluding the gardeners' break)? Explain your answer. (2 marks)

Solution
Minimum completion time is now 10 hours (a delay of an hour). One gardener can do all tasks on the path (R, U and V). A second gardener will need to do P, S and Q. As these take a total of 5.5 hours, T will have to be delayed by half an hour. The second gardener can then do T and W, in that order, which would also add an extra half an hour.
Specific behaviours
✓ correctly determines the increased minimum completion time ✓ correctly justifies the increased time

Question 7 **(6 marks)**

(a) Write the recursive rule for this sequence. (3 marks)

Solution
$\frac{24}{36}=\frac{2}{3}\therefore T_{n+1}=\frac{2}{3}T_n, T_0=54$
Specific behaviours
✓ correctly states common ratio ✓ states a decreasing geometric recursive rule ✓ correctly states the value of the initial term

(b) Write the rule for the n^{th} term of this sequence. (1 mark)

Solution
$T_n=36\left(\frac{2}{3}\right)^{n-1}$ or $T_n=54\left(\frac{2}{3}\right)^{n}$
Specific behaviours
✓ states correct rule

(c) Show that the height reached by the golf ball above the ground after the fifth bounce is $\frac{64}{9}$ cm. (2 marks)

Solution
$T_5=36\left(\frac{2}{3}\right)^4=36\times\frac{16}{81}=\frac{64}{9}$ **or** $T_5=54\left(\frac{2}{3}\right)^5=54\times\frac{32}{243}=\frac{64}{9}$
Specific behaviours
✓ correctly evaluates $\left(\frac{2}{3}\right)^4$ **or** $\left(\frac{2}{3}\right)^5$ ✓ correctly simplifies

This document – apart from any third party copyright material contained in it – may be freely copied, or communicated on an intranet, for non-commercial purposes in educational institutions, provided that it is not changed and that the School Curriculum and Standards Authority is acknowledged as the copyright owner, and that the Authority's moral rights are not infringed.

Copying or communication for any other purpose can be done only within the terms of the *Copyright Act 1968* or with prior written permission of the School Curriculum and Standards Authority. Copying or communication of any third party copyright material can be done only within the terms of the *Copyright Act 1968* or with permission of the copyright owners.

Any content in this document that has been derived from the Australian Curriculum may be used under the terms of the Creative Commons Attribution 4.0 International (CC BY) licence.

Published by the School Curriculum and Standards Authority of Western Australia
303 Sevenoaks Street
CANNINGTON WA 6107