Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS METHODS

ATAR course

Year 12 syllabus for teaching from 2027

Acknowledgement of Country

Kaya. The School Curriculum and Standards Authority (the Authority) acknowledges that our offices are on Whadjuk Noongar boodjar and that we deliver our services on the country of many traditional custodians and language groups throughout Western Australia. The Authority acknowledges the traditional custodians throughout Western Australia and their continuing connection to land, waters and community. We offer our respect to Elders past and present.

Important information

As part of the Western Australian Certificate of Education (WACE) Refreshment, the School Curriculum and Standards Authority (the Authority) has revised the course rationale and aims, and updated the General Capabilities to create clearer connections with the syllabus content.

This syllabus is effective from 1 January 2027.

Users of this syllabus are responsible for checking its currency.

Syllabuses are formally reviewed by the Authority on a cyclical basis, typically every five years.

Contents

Overview of mathematics courses

There are six mathematics courses. Each course is organised into four units. Unit 1 and Unit 2 are taken in Year 11 and Unit 3 and Unit 4 in Year 12. The ATAR course examination for each of the three ATAR courses is based on Unit 3 and Unit 4 only.

The courses are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students.

Mathematics Preliminary is a course which focuses on the practical application of knowledge, skills and understandings to a range of environments that will be accessed by students with special education needs. Grades are not assigned for these units. Student achievement is recorded as 'completed' or 'not completed'. This course provides the opportunity for students to prepare for post-school options of employment and further training.

Mathematics Foundation is a course which focuses on building the capacity, confidence and disposition to use mathematics to meet the numeracy standard for the WACE. It provides students with the knowledge, skills and understanding to solve problems across a range of contexts, including personal, community and workplace/employment. This course provides the opportunity for students to prepare for post-school options of employment and further training.

Mathematics Essential is a General course which focuses on using mathematics effectively, efficiently and critically to make informed decisions. It provides students with the mathematical knowledge, skills and understanding to solve problems in real contexts for a range of workplace, personal, further learning and community settings. This course provides the opportunity for students to prepare for post-school options of employment and further training.

Mathematics Applications is an ATAR course which focuses on the use of mathematics to solve problems in contexts that involve financial modelling, geometric and trigonometric analysis, graphical and network analysis, and growth and decay in sequences. It also provides opportunities for students to develop systematic strategies based on the statistical investigation process for answering questions that involve analysing univariate and bivariate data, including time series data.

Mathematics Methods is an ATAR course which focuses on the use of calculus and statistical analysis. The study of calculus provides a basis for understanding rates of change in the physical world, and includes the use of functions, their derivatives and integrals, in modelling physical processes. The study of statistics develops students' ability to describe and analyse phenomena that involve uncertainty and variation.

Mathematics Specialist is an ATAR course which provides opportunities, beyond those presented in the Mathematics Methods ATAR course, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. The Mathematics Specialist ATAR course contains topics in functions and calculus that build on and deepen the ideas presented in the Mathematics Methods ATAR course, as well as demonstrate their application in many areas. This course also extends understanding and knowledge of statistics and introduces the topics of vectors, complex numbers and matrices. The Mathematics Specialist ATAR course is the only ATAR mathematics course that should not be taken as a stand-alone course.

Rationale

The Mathematics Methods ATAR course is the study of calculus and statistical analysis in applied and theoretical contexts. It focuses on developing students' ability to use mathematical concepts and techniques to solve problems, analyse data and model real-world phenomena.

The course focuses on exploring differentiation and integration techniques to solve practical problems, including those involving logarithmic functions. The course furthers the application of statistics through probability distributions in applied and theoretical contexts and statistical inference.

By studying this course, students develop a strong foundation in mathematical reasoning, critical thinking and analytical problem-solving. They gain fluency in manipulating algebraic expressions, interpreting mathematical models and using statistical methods and probability techniques to draw meaningful conclusions from data. The course fosters students' ability to communicate mathematical arguments clearly and logically.

The mathematical skills and concepts covered in the course have broad applications in daily life and various professional fields. Students apply their knowledge of calculus to analyse and optimise real-world situations, such as understanding growth rates, motion and measurement calculations. Their understanding of probability and statistical inference allows students to make informed decisions by estimating outcomes and assessing risks in various contexts.

The Mathematics Methods ATAR course provides a strong foundation for students intending to pursue tertiary studies in engineering, computer science, physics, economics, health sciences and data analytics. Careers in areas such as actuarial science, medicine, finance and research heavily depend on mathematical reasoning and statistical analysis, making the study of Mathematics Methods highly beneficial.

Aims

The Mathematics Methods ATAR course aims to develop students':

- understanding of concepts and techniques drawn from algebra, functions, calculus, probability and statistics
- ability to apply mathematical knowledge to solve applied and theoretical problems
- proficiency in reasoning and interpretation within mathematical and statistical contexts
- capacity to communicate findings clearly and systematically, using precise and appropriate mathematical and statistical language
- confidence to select and use digital tools appropriately and efficiently to support problem-solving and data analysis.

Organisation

This course is organised into a Year 11 syllabus and a Year 12 syllabus. The cognitive complexity of the syllabus content increases from Year 11 to Year 12.

Structure of the syllabus

The Year 12 syllabus is divided into two units which are delivered as a pair. The notional time for the pair of units is 110 class contact hours.

Organisation of content

Unit 3

Contains the three topics:

- Further differentiation and applications
- Integrals
- Discrete random variables.

The study of calculus continues by introducing the derivatives of exponential and trigonometric functions and their applications, as well as some basic differentiation techniques and the concept of a second derivative, its meaning and applications. The aim is to demonstrate to students the beauty and power of calculus and the breadth of its applications. The unit includes integration, both as a process that reverses differentiation and as a way of calculating areas. The fundamental theorem of calculus as a link between differentiation and integration is emphasised. Discrete random variables are introduced, together with their uses in modelling random processes involving chance and variation. The purpose here is to develop a framework for statistical inference.

Unit 4

Contains the three topics:

- The logarithmic function
- Continuous random variables and the normal distribution
- Interval estimates for proportions.

The logarithmic function and its derivative are studied. Continuous random variables are introduced and their applications examined. Probabilities associated with continuous distributions are calculated using definite integrals. In this unit, students are introduced to one of the most important parts of statistics, namely, statistical inference, where the goal is to estimate an unknown parameter associated with a population using a sample of that population. In this unit, inference is restricted to estimating proportions in two-outcome populations. Students will already be familiar with many examples of these types of populations.

Each unit includes:

- a unit description – a short description of the focus of the unit
- learning outcomes – a set of statements describing the learning expected as a result of studying the unit
- unit content – the content to be taught and learned.

Role of technology

It is assumed that students will be taught this course with an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of the course. However, students also need to continue to develop skills that do not depend on technology. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills in this course.

Representation of the General Capabilities

The General Capabilities encompass the knowledge, skills, behaviours and dispositions that will support students to live and work successfully now and into the future. They are not assessed unless identified within the specified unit content. Teachers should find opportunities to incorporate the following General Capabilities into the teaching and learning program for the Mathematics Methods ATAR course.

Critical and creative thinking

Students identify and clarify information from a range of sources, including visual information and digital sources. They identify the relevant aspects of a concept or problem, understanding that a single mathematical process can be used in seemingly different situations and that approaches may change depending on the context or nature of the problem. Students draw conclusions and make choices when completing tasks by connecting evidence from within and across the mathematical content to provide reasons and evaluate arguments for choices made.

Digital literacy

Students use appropriate digital tools to support the development of mathematical understanding and to apply mathematical knowledge to a range of problems. They use technologies aligned with areas of work they may be involved with such as statistical and graphical analysis, generation of algorithms, algebraic manipulation and complex calculations. Students use digital tools to make connections between mathematical theory, practice and application.

Literacy

Students develop literacy skills and strategies that enable them to express, interpret and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their ability to read, write, visualise and talk about complex situations involving a range of mathematical ideas. They can apply and further develop their literacy skills and strategies by shifting between verbal, graphical, numerical and symbolic forms of representing problems to formulate, understand and solve problems, and communicate results. Students learn to communicate their findings in different ways, using multiple systems of representation to illustrate the relationships they have observed or constructed.

Numeracy

Students develop their numeracy skills at a more sophisticated level, making decisions about the relevant mathematics to use, following through with calculations, selecting appropriate methods and being confident of their results. This course contains topics that will equip students for the increasing demands of the information age, developing the skills of critical evaluation of numerical information. They will enhance their numerical operation skills by solving practical problems using the calculus of trigonometric, exponential and logarithmic functions, and by working with discrete and continuous random variables and associated data distributions.

Addressing the other General Capabilities

Although the following General Capabilities have not been identified as a focus in the Mathematics Methods ATAR Year 12 syllabus, teachers may find opportunities to incorporate them into the teaching and learning program.

- Ethical understanding
- Intercultural understanding
- Personal and social capability

Such opportunities may occur through the application of different contexts, pedagogical practices and/or assessment strategies that relate to the syllabus as part of the teaching and learning program.

Summary representation of the General Capabilities in the Mathematics Methods ATAR course

The unit content and assessment types for this course provide students with the opportunity to develop the General Capabilities summarised in the table below.

Year	**Course**	**Course type**	**General Capabilities**						
			CCT	**DL**	**EU**	**IU**	**L**	**N**	**PSC**
Year 11	Mathematics Methods (AEMAM)	ATAR	✓	✓			✓	✓	
Year 12	Mathematics Methods (ATMAM)	ATAR	✓	✓			✓	✓	

Key
CCT: Critical and creative thinking, DL: Digital literacy, EU: Ethical understanding, IU: Intercultural understanding, L: Literacy, N: Numeracy, PSC: Personal and social capability

Representation of the Cross-curriculum Priorities

The Cross-curriculum Priorities address contemporary issues which students face in a globalised world. Teachers may find opportunities to incorporate them into the teaching and learning program for the Mathematics Methods ATAR course. The Cross-curriculum Priorities are not assessed unless they are identified within the specified unit content.

Aboriginal and Torres Strait Islander histories and cultures

Mathematics courses value the histories, cultures, traditions and languages of Aboriginal and Torres Strait Islander Peoples' past and ongoing contributions to contemporary Australian society and culture. Through the study of mathematics within relevant contexts, opportunities will allow for the development of students' understanding and appreciation of the diversity of Aboriginal and Torres Strait Islander Peoples' histories and cultures.

Asia and Australia's engagement with Asia

There are strong social, cultural and economic reasons for Australian students to engage with the countries of Asia and with the past and ongoing contributions made by the peoples of Asia in Australia. It is through the study of mathematics in an Asian context that students engage with Australia's place in the region. By analysing relevant data, students have opportunities to further develop an understanding of the diverse nature of Asia's environments and traditional and contemporary cultures.

Sustainability

Each of the mathematics courses provides the opportunity for the development of informed and reasoned points of view, discussion of issues, research and problem solving. Teachers are therefore encouraged to select contexts for discussion that are connected with sustainability. Through the analysis of data, students have the opportunity to research and discuss sustainability and learn the importance of respecting and valuing a wide range of world perspectives.

Unit 3

Unit description

The study of calculus continues with the derivatives of exponential and trigonometric functions and their applications, together with some differentiation techniques and applications to optimisation problems and graph sketching. It concludes with integration, both as a process that reverses differentiation and as a way of calculating areas. The fundamental theorem of calculus as a link between differentiation and integration is emphasised. In statistics, discrete random variables are introduced, together with their uses in modelling random processes involving chance and variation. This supports the development of a framework for statistical inference.

Access to technology to support the computational aspects of these topics is assumed.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques in calculus, probability and statistics
- solve problems in calculus, probability and statistics
- apply reasoning skills in calculus, probability and statistics
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems.
- communicate their arguments and strategies when solving problems.

Unit content

An understanding of the Year 11 content is assumed knowledge for students in Year 12. It is recommended that students studying Unit 3 and Unit 4 have completed Unit 1 and Unit 2.

This unit includes the knowledge, understandings and skills described below. This is the examinable content.

Topic 3.1: Further differentiation and applications (20 hours)

Exponential functions

3.1.1 estimate the limit of $\frac{a^h - 1}{h}$ as $h \to 0$, using technology, for various values of $a > 0$

3.1.2 identify that e is the unique number a for which the above limit is 1

3.1.3 establish and use the formula $\frac{d}{dx}(e^x) = e^x$

3.1.4 use exponential functions of the form Ae^{kx} and their derivatives to solve practical problems

Trigonometric functions

3.1.5 establish the formulas $\frac{d}{dx}(\sin x) = \cos x$ and $\frac{d}{dx}(\cos x) = -\sin x$ by graphical treatment, numerical estimations of the limits, and informal proofs based on geometric constructions

3.1.6 use trigonometric functions and their derivatives to solve practical problems

Differentiation rules

3.1.7 examine and use the product and quotient rules

3.1.8 examine the notion of composition of functions and use the chain rule for determining the derivatives of composite functions

3.1.9 apply the product, quotient and chain rule to differentiate functions such as xe^x, $\tan x$, $\frac{1}{x^n}$, $x \sin x$, $e^{-x} \sin x$ and $f(ax - b)$

The second derivative and applications of differentiation

3.1.10 use the increments formula: $\delta y \approx \frac{dy}{dx} \times \delta x$ to estimate the change in the dependent variable y resulting from changes in the independent variable x

3.1.11 apply the concept of the second derivative as the rate of change of the first derivative function

3.1.12 identify acceleration as the second derivative of position with respect to time

3.1.13 examine the concepts of concavity and points of inflection and their relationship with the second derivative

3.1.14 apply the second derivative test for determining local maxima and minima

3.1.15 sketch the graph of a function using first and second derivatives to locate stationary points and points of inflection

3.1.16 solve optimisation problems from a wide variety of fields using first and second derivatives

Topic 3.2: Integrals (20 hours)

Anti-differentiation

3.2.1 identify anti-differentiation as the reverse of differentiation

3.2.2 use the notation $\int f(x)dx$ for anti-derivatives or indefinite integrals

3.2.3 establish and use the formula $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$ for $n \neq -1$

3.2.4 establish and use the formula $\int e^x dx = e^x + c$

3.2.5 establish and use the formulas $\int \sin x \, dx = -\cos x + c$ and $\int \cos x \, dx = \sin x + c$

3.2.6 identify and use linearity of anti-differentiation

3.2.7 determine indefinite integrals of the form $\int f(ax - b)dx$

3.2.8 identify families of curves with the same derivative function

3.2.9 determine $f(x)$, given $f'(x)$ and an initial condition $f(a) = b$

Definite integrals

3.2.10 examine the area problem and use sums of the form $\sum_i f(x_i)\ \delta x_i$ to estimate the area under the curve $y = f(x)$

3.2.11 identify the definite integral $\int_a^b f(x)dx$ as a limit of sums of the form $\sum_i f(x_i)\ \delta x_i$

3.2.12 interpret the definite integral $\int_a^b f(x)dx$ as area under the curve $y = f(x)$ if $f(x) > 0$

3.2.13 interpret $\int_a^b f(x)dx$ as a sum of signed areas

3.2.14 apply the additivity and linearity of definite integrals

Fundamental theorem

3.2.15 examine the concept of the signed area function $F(x) = \int_a^x f(t)dt$

3.2.16 apply the theorem: $F'(x) = \frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$, and illustrate its proof geometrically

3.2.17 develop the formula $\int_a^b f'(x)dx = f(b) - f(a)$ and use it to calculate definite integrals

Applications of integration

3.2.18 calculate total change by integrating instantaneous or marginal rate of change

3.2.19 calculate the area under a curve

3.2.20 calculate the area between curves determined by functions of the form $y = f(x)$

3.2.21 determine displacement given velocity in linear motion problems

3.2.22 determine positions given linear acceleration and initial values of position and velocity.

Topic 3.3: Discrete random variables (15 hours)

General discrete random variables

3.3.1 develop the concepts of a discrete random variable and its associated probability function, and their use in modelling data

3.3.2 use relative frequencies obtained from data to obtain point estimates of probabilities associated with a discrete random variable

3.3.3 identify uniform discrete random variables and use them to model random phenomena with equally likely outcomes

3.3.4 examine simple examples of non-uniform discrete random variables

3.3.5 define the mean or expected value of a discrete random variable and interpret it as a measure of location, and evaluate it in simple cases

3.3.6 define the variance and standard deviation of a discrete random variable and interpret them as measures of spread, and evaluate them using technology

3.3.7 examine the effects of linear changes of scale and origin on the mean and the standard deviation

3.3.8 use discrete random variables and associated probabilities to solve practical problems

Bernoulli distributions

3.3.9 use a Bernoulli random variable as a model for two-outcome situations

3.3.10 identify contexts suitable for modelling by Bernoulli random variables

3.3.11 determine the mean p and variance $p(1-p)$of the Bernoulli distribution with parameter p

3.3.12 use Bernoulli random variables and associated probabilities to model data and solve practical problems

Binomial distributions

3.3.13 examine the concept of Bernoulli trials and the concept of a binomial random variable as the number of 'successes' in n independent Bernoulli trials, with the same probability of success p in each trial

3.3.14 identify contexts suitable for modelling by binomial random variables

3.3.15 determine and use the probabilities $P(X=x)=\binom{n}{x}p^x(1-p)^{n-x}$ associated with the binomial distribution with parameters n and p; note the mean np and variance $np(1-p)$ of a binomial distribution

3.3.16 use binomial distributions and associated probabilities to solve practical problems

Unit 4

Unit description

The calculus in this unit deals with derivatives of logarithmic functions. In probability and statistics, continuous random variables and their applications are introduced and the normal distribution is used in a variety of contexts. The study of statistical inference in this unit is the culmination of earlier work on probability and random variables. Statistical inference is one of the most important parts of statistics, in which the goal is to estimate an unknown parameter associated with a population using a sample of data drawn from that population. In the Mathematics Methods ATAR course, statistical inference is restricted to estimating proportions in two-outcome populations.

Access to technology to support the computational aspects of these topics is assumed.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques in calculus, probability and statistics
- solve problems in calculus, probability and statistics
- apply reasoning skills in calculus, probability and statistics
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems.
- communicate their arguments and strategies when solving problems.

Unit content

This unit builds on the content covered in Unit 3.

This unit includes the knowledge, understandings and skills described below. This is the examinable content.

Topic 4.1: The logarithmic function (18 hours)

Logarithmic functions

4.1.1 define logarithms as indices: $a^x = b$ is equivalent to $x = \log_a b$ i.e. $a^{\log_a b} = b$

4.1.2 establish and use the algebraic properties of logarithms

4.1.3 examine the inverse relationship between logarithms and exponentials: $y = a^x$ is equivalent to $x = \log_a y$

4.1.4 interpret and use logarithmic scales

4.1.5 solve equations involving indices using logarithms

4.1.6 identify the qualitative features of the graph of $y = \log_a x$ $(a > 1)$, including asymptotes, and of its translations $y = \log_a x + b$ and $y = \log_a (x - c)$

4.1.7 solve simple equations involving logarithmic functions algebraically and graphically

4.1.8 identify contexts suitable for modelling by logarithmic functions and use them to solve practical problems

Calculus of the natural logarithmic function

4.1.9 define the natural logarithm $\ln x = \log_e x$

4.1.10 examine and use the inverse relationship of the functions $y = e^x$ and $y = \ln x$

4.1.11 establish and use the formula $\frac{d}{dx}(\ln x) = \frac{1}{x}$

4.1.12 establish and use the formula $\int \frac{1}{x} dx = \ln\ x\ + c$, for $x > 0$

4.1.13 determine derivatives of the form $\frac{d}{dx}(\ln f(x))$ and integrals of the form $\int \frac{f'(x)}{f(x)} dx$ for $f\ (x) > 0$

4.1.14 use logarithmic functions and their derivatives to solve practical problems

Topic 4.2: Continuous random variables and the normal distribution (15 hours)

General continuous random variables

4.2.1 use relative frequencies and histograms obtained from data to estimate probabilities associated with a continuous random variable

4.2.2 examine the concepts of a probability density function, cumulative distribution function, and probabilities associated with a continuous random variable given by integrals; examine simple types of continuous random variables and use them in appropriate contexts

4.2.3 define the expected value, variance and standard deviation of a continuous random variable and evaluate them in simple cases, using technology where required

4.2.4 examine the effects of linear changes of scale and origin on the mean and the standard deviation

Normal distributions

4.2.5 identify contexts, such as naturally occurring variation, that are suitable for modelling by normal random variables

4.2.6 identify features of the graph of the probability density function of the normal distribution with mean μ and standard deviation σ and the use of the standard normal distribution

4.2.7 calculate probabilities and quantiles associated with a given normal distribution using technology, and use these to solve practical problems

Topic 4.3: Interval estimates for proportions (22 hours)

Random sampling

4.3.1 examine the concept of a random sample

4.3.2 discuss sources of bias in samples, and procedures to ensure randomness

4.3.3 use graphical displays of simulated data to investigate the variability of random samples from various types of distributions, including uniform, normal and Bernoulli

Sample proportions

4.3.4 examine the concept of the sample proportion $\hat{p}$ as a random variable whose value varies between samples, and the formulas for the mean p and standard deviation $\sqrt{\frac{p(1-p)}{n}}$ of the sample proportion $\hat{p}$

4.3.5 examine the approximate normality of the distribution of $\hat{p}$ for large samples

4.3.6 simulate repeated random sampling, for a variety of values of p and a range of sample sizes, to illustrate the distribution of $\hat{p}$ and the approximate standard normality of $\frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}}$ where the closeness of the approximation depends on both n and p

Confidence intervals for proportions

4.3.7 examine the concept of an interval estimate for a parameter associated with a random variable

4.3.8 use the approximate confidence interval $\left(\hat{p}-z\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{n}\right)}, \hat{p}+z\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{n}\right)}\right)$ as an interval estimate for p, where z is the appropriate quantile for the standard normal distribution

4.3.9 define the approximate margin of error $E = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ and understand the trade-off between margin of error and level of confidence

4.3.10 use simulation to illustrate variations in confidence intervals between samples and to show that most, but not all, confidence intervals contain p

School-based assessment

The *Western Australian Certificate of Education (WACE) Manual* contains essential information on principles, policies and procedures for school-based assessment that needs to be read in conjunction with this syllabus.

Teachers design school-based assessment tasks to meet the needs of students. The table below provides details of the assessment types for the Mathematics Methods ATAR Year 12 syllabus and the weighting for each assessment type.

Assessment table – Year 12

Type of assessment	Weighting
Response Students apply mathematical knowledge and understanding of concepts, techniques and relationships to solve a mix of routine and non-routine questions, demonstrating their interpretation of concepts and results in applied and theoretical contexts. Response tasks can include: tests, assignments and multimedia representations.	40%
Investigation Students use the mathematical thinking process to plan, research, conduct and communicate the findings of an investigation. They can investigate problems to identify the underlying mathematics, or select, adapt and apply models and procedures to solve problems. This assessment type provides for the assessment of the mathematical thinking process using course-related knowledge and modelling skills. Evidence can include: observation and interview, written work or multimedia presentations.	20%
Examination Students apply mathematical understanding and skills to analyse, interpret and respond to questions and situations. Examinations provide for the assessment of conceptual understandings, knowledge of mathematical facts and terminology, problem-solving skills, and the use of algorithms. Examination questions can range from those of a routine nature, assessing lower level concepts, through to those that require responses at the highest level of conceptual thinking. Typically conducted at the end of each semester and/or unit and reflecting the examination design brief for this syllabus.	40%

Teachers are required to use the assessment table to develop an assessment outline for the pair of units.

The assessment outline must:

- include a set of assessment tasks
- include a general description of each task
- indicate the unit content to be assessed
- indicate a weighting for each task and each assessment type
- include the approximate timing of each task (for example, the week the task is conducted, or the issue and submission dates for an extended task).

In the assessment outline for the pair of units, each assessment type must be included at least once over the year/pair of units.

The set of assessment tasks must provide a representative sampling of the content for Unit 3 and Unit 4.

Assessment tasks not administered under test/controlled conditions require appropriate validation/authentication processes. This may include observation, annotated notes, checklists, interview, presentations or in-class tasks assessing related content and processes.

Grading

Schools report student achievement in terms of the following grades:

Grade	Interpretation
A	Excellent achievement
B	High achievement
C	Satisfactory achievement
D	Limited achievement
E	Very low achievement

The teacher prepares a ranked list and assigns the student a grade for the pair of units. The grade is based on the student's overall performance as judged by reference to a set of pre-determined standards. These standards are defined by grade descriptions and annotated work samples. The grade descriptions for the Mathematics Methods ATAR Year 12 syllabus are provided in Appendix 1. They can also be accessed, together with annotated work samples, through the Guide to Grades link on the course page of the Authority website at www.scsa.wa.edu.au.

To be assigned a grade, a student must have had the opportunity to complete the education program, including the assessment program (unless the school accepts that there are exceptional and justifiable circumstances).

Refer to the *WACE Manual* for further information about the use of a ranked list in the process of assigning grades.

ATAR course examination

All students enrolled in the Mathematics Methods ATAR Year 12 course are required to sit the ATAR course examination. The examination is based on a representative sampling of the content for Unit 3 and Unit 4. Details of the ATAR course examination are prescribed in the examination design brief on the following page.

Refer to the *WACE Manual* for further information.

Examination design brief – Year 12

This examination consists of two sections.

Section One: Calculator-free

Time allowed
Reading time before commencing work: five minutes
Working time for section: fifty minutes

Permissible items
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction, fluid/tape, eraser, ruler, highlighters
Special items: nil

Additional information
Changeover period during which the candidate is not permitted to work: up to 15 minutes

Section Two: Calculator-assumed

Time allowed
Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Permissible items
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Provided by the supervisor
A formula sheet

Additional information
It is assumed that candidates sitting this examination have a calculator with CAS capabilities for Section Two.

The candidate is required to demonstrate knowledge of mathematical facts, conceptual understandings, use of algorithms, use and knowledge of notation and terminology, and problem-solving skills.

Questions can require the candidate to solve problems, investigate mathematical patterns, apply reasoning skills, interpret results and communicate their arguments and strategies to justify mathematical relationships. Examination questions can range from those of a routine nature, assessing lower level concepts, through to those that require responses at the highest level of conceptual thinking.

Instructions to candidates indicate that, for any question or part question worth more than two marks, valid working or justification is required to receive full marks.

Section	Supporting information
Section One: Calculator-free 35% of the total examination 5–10 questions Working time: 50 minutes	Questions examine content and procedures that can reasonably be expected to be completed without the use of a calculator, i.e. without undue emphasis on algebraic manipulations or time-consuming calculations. The candidate is required to provide answers that include calculations, tables, graphs, interpretation of data, descriptions and/or conclusions. Stimulus material can include: diagrams, tables, graphs, drawings, print text and/or data gathered from the media.
Section Two: Calculator-assumed 65% of the total examination 8–13 questions Working time: 100 minutes	Questions examine content and procedures for which the use of a calculator is assumed. The candidate can be required to provide answers that include calculations, tables, graphs, interpretation of data, descriptions and conclusions. Stimulus material can include: diagrams, tables, graphs, drawings, print text and/or data gathered from the media. The candidate can be required to solve problems, investigate mathematical patterns, apply reasoning skills, interpret results and communicate their arguments and strategies to justify mathematical relationships. The candidate can be required to solve problems from unfamiliar situations, choosing and using mathematical models with adaptations where necessary, comparing their solutions with the situations concerned, and then presenting their findings in context.

Appendix 1 – Grade descriptions Year 12

A	**Identifies and organises relevant information** Identifies and organises relevant information from previous parts of a problem and brings them together to solve subsequent problems. Defines variables and equations from text and diagrams. Organises data in a concise, clear format and appropriately presents it in tabular, diagrammatic and/or graphical form. Identifies the underlying assumptions related to the relevant mathematics of an investigation.
	Chooses effective models and methods and carries through the methods correctly Selects an appropriate strategy and applies mathematical knowledge to solve non-routine problems. Generalises and extends models from previous parts of the question. Translates between representations in unpractised ways. Selects appropriate calculator techniques to solve multi-step problems in unfamiliar contexts. Selects and appropriately uses numerical, graphical, symbolic and statistical methods to develop mathematical ideas. Produces results, carries out analysis and generalises in situations requiring investigative techniques.
	Follows mathematical conventions and attends to accuracy Follows mathematical conventions and attends to accuracy in non-routine situations. Completes concise and accurate solutions to mathematical problems set in applied and theoretical contexts. Selects, extends and applies mathematical and/or statistical procedures to investigate a problem.
	Links mathematical results to data and contexts to reach reasonable conclusions Recognises implied conditions in extended responses and defines and explains the limitations of models. Interprets the result and draws the correct conclusion about the effect of changing conditions. Considers the strengths and limitations of an investigation and refines the results to make sensible conclusions.
	Communicates mathematical reasoning, results and conclusions Sets out the steps of the solution in a clear and logical sequence, including suitable justification and explanation of methods and processes used. Adds a detailed diagram to illustrate and use in the solution of a problem. Presents work with the final answer clearly identified, using the correct units and relating to the context of the question. Communicates investigation findings with a comprehensive interpretation of mathematical results in the context of the investigation.

B	**Identifies and organises relevant information** Identifies and organises relevant information for problems involving a few steps or processes. Draws a diagram and labels it with appropriate variables. Organises data clearly and appropriately presents it in tabular, diagrammatic and/or graphical form. Identifies suitable variables and constant parameters related to various aspects of an investigation.
	Chooses effective models and methods and carries through the methods correctly Selects an appropriate strategy and applies mathematical knowledge to solve simple non-routine problems. Translates between representations in practised ways. Selects appropriate calculator techniques to solve multi-step problems. Selects and appropriately uses numerical, graphical, symbolic and statistical methods to develop mathematical ideas. Attempts to analyse and calculate specific cases of generalisation in situations requiring investigative techniques.
	Follows mathematical conventions and attends to accuracy Interprets and uses mathematical terminology, symbols and conventions in routine situations. Rounds, unprompted, to suit context or correctly to specified accuracy. Completes mostly accurate solutions to mathematical problems set in applied and theoretical contexts. Selects and applies mathematical and/or statistical procedures previously learnt to investigate a problem.
	Links mathematical results to data and contexts to reach reasonable conclusions Identifies specified conditions, recognises and rejects inappropriate solutions. Links the effect of changing conditions to the original solution. Uses examples in mathematical analysis of an investigation and draws valid conclusions related to a given context.
	Communicates mathematical reasoning, results and conclusions Carries through calculations and simplifications in a clear sequence, showing a logical line of reasoning. Defines variables associated with a given diagram and uses them in the working of a problem. Presents work with the final answer clearly identified and using the correct units. Communicates investigation findings in a systematic and concise way using mathematical language and relating the solution to the original problem or statement.

C	**Identifies and organises relevant information** Identifies and extracts key information needed to solve a familiar problem. Identifies variables in a given diagram. Organises some data and presents it in tabular, diagrammatic and/or graphical form. Identifies the key mathematical content related to various aspects of an investigation in a given context.
	Chooses effective models and methods and carries through the methods correctly Selects from a range of strategies and formulae and applies mathematical knowledge in practised ways to solve routine problems. Recognises and uses information in different representations. Uses familiar calculator applications to solve routine problems. Selects appropriate numerical, graphical, symbolic and statistical methods to carry through a single thread of reasoning in situations requiring investigative techniques.
	Follows mathematical conventions and attends to accuracy Applies mathematical definitions, rules and procedures in practised situations. Applies basic conventions for diagrams and graphs. Rounds appropriately in a given context and to specified accuracy in short responses. Generates some accurate and generally complete solutions to mathematical problems set in applied and theoretical contexts. Selects and applies, with direction, mathematical and/or statistical procedures previously learnt to investigate a problem.
	Links mathematical results to data and contexts to reach reasonable conclusions Identifies specified conditions and recognises inappropriate solutions in routine problems. Recognises that changing conditions will affect the outcome. Makes inferences from analysis and uses these to draw conclusions related to a given context for investigation.
	Communicates mathematical reasoning, results and conclusions Shows adequate working and supports answers with simple or routine statements. Relates the working to a labelled diagram that has been given as part of the question. Presents work with the final answer, but not always clearly identified. Communicates investigation findings in a systematic way using some mathematical expression and everyday language.

D	**Identifies and organises relevant information** Uses given information to solve some simple routine problems. Identifies variables in a simple diagram. Displays data using an inappropriate presentation format. Identifies some mathematical content of an investigation.
	Chooses effective models and methods and carries through the methods correctly Follows an appropriate strategy to solve simple routine and familiar problems that require short responses. Deals with information in familiar representations only. Uses a calculator for straightforward problems. Makes some attempt to select appropriate numerical, graphical, symbolic and statistical methods in situations requiring investigative techniques.
	Follows mathematical conventions and attends to accuracy Applies limited mathematical conventions to practised problems. Rounds inconsistently or inappropriately. Generates partly accurate and generally incomplete solutions to mathematical problems set in applied and theoretical contexts. Attempts to apply, with direction, mathematical and/or statistical procedures previously learnt to investigate a problem.
	Links mathematical results to data and contexts to reach reasonable conclusions Is unable to recognise specified or changing conditions in routine problems. Draws some conclusions from the results of an investigation.
	Communicates mathematical reasoning, results and conclusions Shows some working in an attempt to answer simple questions. Sets out calculations in a manner that is difficult to check for accuracy. Presents working with no clear indication of the final answer evident. Offers simple conclusions that are not supported by data or calculations.

E	Does not meet the requirements of a D grade and/or has completed insufficient assessment tasks to be assigned a higher grade.

Appendix 2 – Glossary

This glossary is provided to enable a common understanding of the key terms in this syllabus.

Unit 3

Further differentiation and applications

Composition of functions
If $y = g(x)$ and $z = f(y)$ for functions f and g, then z is a composite function of x. We write $z = f \circ g(x) = f(g(x))$. For example, $z = \sqrt{x^2 + 3}$ expresses z as a composite of the functions $f(y) = \sqrt{y}$ and $g(x) = x^2 + 3$.

Chain rule
The chain rule relates the derivative of the composite of two functions to the functions and their derivatives.

If $h(x) = f \circ g(x)$ then $(f \circ g)'(x) = f'\big(g(x)\big)g'(x)$,

and in Leibniz notation: $\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$

Euler's number
Euler's number e is an irrational number whose decimal expansion begins

$e = 2.7182818284590452353602874713527$

It is the base of the natural logarithms, and can be defined in various ways, including:

$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$ and $e = \lim_{n \to \infty} (1 + \frac{1}{n})^n$

Point of inflection
A point P on the graph of $y = f(x)$ is a point of inflection if the concavity changes at P, i.e. points near P on one side of P lie above the tangent at P and points near P on the other side of P lie below the tangent at P.

Product rule
The product rule relates the derivative of the product of two functions to the functions and their derivatives.

If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$,

and in Leibniz notation: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + \frac{du}{dx}v$

Quotient rule
The quotient rule relates the derivative of the quotient of two functions to the functions and their derivatives

If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

and in Leibniz notation: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Second derivative test
According to the second derivative test, if $f'(x) = 0$, then $f(x)$ is a local maximum of f if $f''(x) < 0$ and $f(x)$ is a local minimum if $f''(x) > 0$.

Integrals

Additivity property of definite integrals
The additivity property of definite integrals refers to 'addition of intervals of integration':

$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$ for any numbers a, b and c and any function $f(x)$.

Anti-differentiation
An anti-derivative, primitive or indefinite integral of a function $f(x)$ is a function $F(x)$ whose derivative is $f(x)$, i.e. $F'(x) = f(x)$.

The process of solving for anti-derivatives is called anti-differentiation.

Anti-derivatives are not unique. If $F(x)$ is an anti-derivative of $f(x)$, then so too is the function $F(x) + c$ where c is any number. We write $\int f(x)\,dx = F(x) + c$ to denote the set of all anti-derivatives of $f(x)$. The number c is called the constant of integration. For example, since $\frac{d}{dx}(x^3) = 3x^2$, we can write $\int 3x^2\,dx = x^3 + c$.

The fundamental theorem of calculus
The fundamental theorem of calculus relates differentiation and definite integrals.

It has two forms: $\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$ and $\int_a^b f'(x)dx = f(b) - f(a)$.

The linearity property of anti-differentiation
The linearity property of anti-differentiation is summarised by the equations:

$\int kf(x)dx = k\int f(x)dx$ for any constant k and

$\int\big(f_1(x) + f_2(x)\big)dx = \int f_1(x)dx + \int f_2(x)\,dx$ for any two functions $f_1(x)$ and $f_2(x)$

Similar equations describe the linearity property of definite integrals:

$\int_a^b kf(x)dx = k\int_a^b f(x)dx$ for any constant k and

$\int_a^b\big(f_1(x) + f_2(x)\big)dx = \int_a^b f_1(x)dx + \int_a^b f_2(x)dx$ for any two functions $f_1(x)$ and $f_2(x)$.

Discrete random variables

Bernoulli random variable
A Bernoulli random variable has two possible values, namely 0 and 1. The parameter associated with such a random variable is the probability p of obtaining a 1.

Bernoulli trial
A Bernoulli trial is a chance experiment with possible outcomes, typically labelled 'success' and 'failure'.

Effect of linear change
The effects of linear changes of scale and origin on the mean and variance of a random variable are summarised as follows:

If X is a random variable and $Y = aX + b$, where a and b are constants, then $E(Y) = aE(X) + b$ and $Var(Y) = a^2Var(X)$.

Expected value
The expected value $E(X)$ of a random variable X is a measure of the central tendency of its distribution.

If X is discrete, $E(X) = \sum_i p_i x_i$, where the x_i are the possible values of X and $p_i = P(X = x_i)$.

If X is continuous, $E(x) = \int_{-\infty}^{\infty} xp(x)dx$, where $p(x)$ is the probability density function of X.

Mean of a random variable
The mean of a random variable is another name for its expected value.

Point and interval estimates
In statistics estimation is the use of information derived from a sample to produce an estimate of an unknown probablity or population parameter. If the estimate is a single number, this number is called a point estimate.

An interval estimate is an interval derived from the sample that, in some sense, is likely to contain the parameter.

A simple example of a point estimate of the probability p of an event is the relative frequency f of the event in a large number of Bernoulli trials. An example of an interval estimate for p is a confidence interval centred on the relative frequency f.

Probability distribution
The probability distribution of a discrete random variable is the set of probabilities for each of its possible values.

Random variable
A random variable is a numerical quantity whose value depends on the outcome of a chance experiment. Typical examples are the number of people who attend an AFL grand final, the proportion of heads observed in 100 tosses of a coin, and the number of tonnes of wheat produced in Australia in a year.

A discrete random variable is one whose possible values are the counting numbers $0,1,2,3,\cdots$, or form a finite set, as in the first two examples.

A continuous random variable is one whose set of possible values are all of the real numbers in some interval.

Standard deviation of a random variable
The standard deviation of a random variable is the square root of its variance.

Uniform discrete random variable
A uniform discrete random variable is one whose possible values have equal probability of occurrence. If there are n possible values, the probability of occurrence of any one of them is $\frac{1}{n}$.

Variance of a random variable
The variance $Var(X)$ of a random variable X is a measure of the 'spread' of its distribution.

If X is discrete, $Var(X) = \sum_i p_i(x_i - \mu)^2$, where $\mu = E(X)$ is the expected value.

If X is continuous, $Var(X) = \int_{-\infty}^{\infty}(x - \mu)^2 p(x)dx$.

Unit 4

The logarithmic function

Algebraic properties of logarithms
The algebraic properties of logarithms are the rules:

- $\log_a(xy) = \log_a x + \log_a y$
- $\log_a x^n = n \log_a x$
- $\log_a \frac{1}{x} = -\log_a x$
    - $\log_a 1 = 0$, for n any integer and for positive real numbers x, y and a.

Continuous random variables and the normal distribution

Probability density function
The probability density function of a continuous random variable is a function that describes the relative likelihood that the random variable takes a particular value. Formally, if $p(x)$ is the probability density of the continuous random variable X, then the probability that X takes a value in some interval $[a, b]$ is given by $\int_a^b p(x)\, dx$.

Quantile
A quantile t_α for a continuous random variable X is defined by $P(X < t_\alpha) = \alpha$, where $0 < \alpha < 1$.

The 75th percentile of X is the quantile corresponding to $\alpha = 0.75$: $P(X < t_\alpha) = 0.75$,

Thus the 0·75 quantile (also called the 75th percentile or upper quartile) is the score that 75% of the population lies below.

Triangular continuous random variable
A triangular continuous random variable X is one whose probability density function $p(x)$ has a graph with the shape of a triangle.

Uniform continuous random variable
A uniform continuous random variable X is one whose probability density function $p(x)$ has constant value on the range of possible values of X. If the range of possible values is the interval $[a, b]$, then $p(x) = \frac{1}{b-a}$ if $a \leq x \leq b$ and $p(x) = 0$ otherwise.

Interval estimates for proportions

Central limit theorem

There are various forms of the central limit theorem, a result of fundamental importance in statistics. For the purposes of this course, it can be expressed as follows:

"If $\bar{X}$ is the mean of n independent values of random variable X which has a finite mean μ and a finite standard deviation σ, then as $n \to \infty$ the distribution of $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ approaches the standard normal distribution."

In the special case where X is a Bernoulli random variable with parameter p, $\bar{X}$ is the sample proportion $\hat{p}$, $\mu = p$ and $\sigma = \sqrt{\frac{p(1-p)}{n}}$. In this case, the central limit theorem is a statement that as $n \to \infty$ the distribution of $\frac{\hat{p}-p}{\sqrt{p(1-p)/n}}$ approaches the standard normal distribution.

Margin of error

The margin of error of a confidence interval of the form $f - E < p < f + E$ is E, the half-width of the confidence interval. It is the maximum difference between f and p if p is actually in the confidence interval.

Confidence level

A confidence level is the percentage with which a given confidence interval contains the true value of the parameter being estimated. For example a 95% confidence level associated with a confidence interval for estimating a population parameter indicates that 95% of all estimated confidence intervals based on random samples of a given sample size will contain the true value of the population parameter.