



ATAR course examination, 2022

Question/Answer booklet

MATHEMATICS SPECIALIST

Section One: Calculator-free

Place one of your candidate identification labels in this box.
Ensure the label is straight and within the lines of this box.

WA student number: In figures

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In words

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional
answer booklets used
(if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.



Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	48	35
Section Two: Calculator-assumed	12	12	100	86	65
Total					100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2022: Part II Examinations*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free

35% (48 Marks)

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

Question 1

(6 marks)

Consider functions $f(x) = \sqrt{4-x}$ and $g(x) = \frac{1}{x^2}$.

(a) Determine the exact value of $g(f(-5))$. (2 marks)

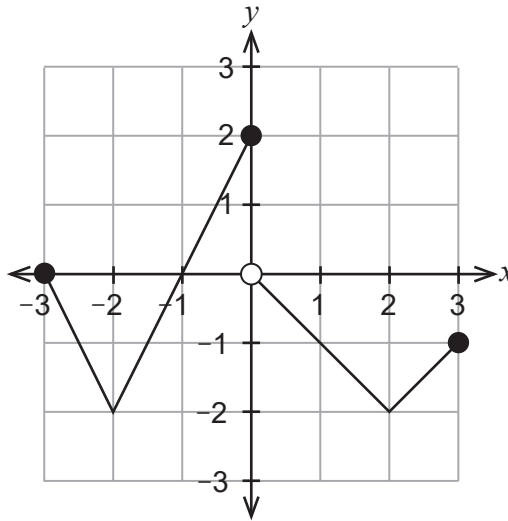
(b) Determine the domain for $f(g(x))$. (3 marks)

(c) Explain why function g is not a one-to-one function. (1 mark)

Question 2

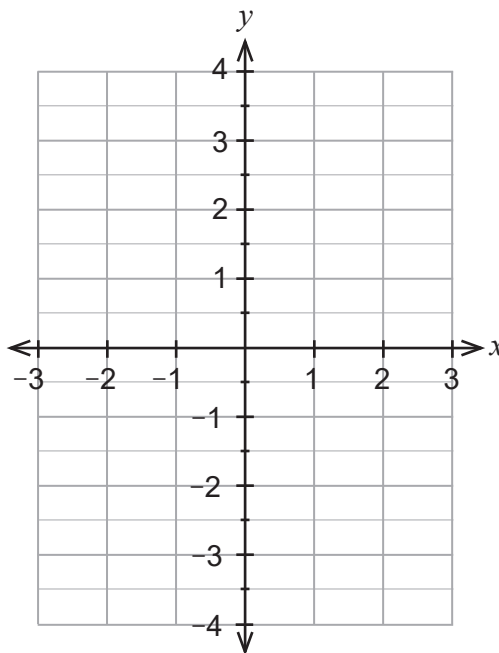
(7 marks)

The graph of $y = f(x)$ is shown below.



- (a) Solve the equation $|f(x)| = x$. (2 marks)

- (b) Sketch the graph of $y = \frac{1}{f(x)}$ on the axes below. (5 marks)



A spare grid is provided at the end of this Question/Answer booklet. If you need to use it, cross out this attempt and indicate that you have redrawn it on the spare grid.

See next page

Question 3

(5 marks)

By using one or more of the following identities:

$$\cos^2 x + \sin^2 x = 1$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\sin 2x = 2 \sin x \cos x$$

evaluate exactly $\int_0^{\frac{\pi}{2}} (\sin x + \cos x)^2 dx$.

Question 4

(8 marks)

- (a) Function $f(x) = \frac{5(x+1)}{(x-1)(x^2+3x+1)}$ can be expressed in the form $\frac{a}{x-1} + \frac{bx+c}{x^2+3x+1}$.

Determine the value of the constants a , b and c .

(3 marks)

- (b) Hence determine $\int \frac{10x + 10}{(x - 1)(x^2 + 3x + 1)} dx$. (5 marks)

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Question 5

(6 marks)

Consider the Cartesian equations for three planes:

$$\begin{aligned}2x + 2y + z &= 9 \\ -2x + 2y - 5z &= -13 \\ y - z &= -1\end{aligned}$$

(a) Show that none of these planes is parallel to another. (2 marks)

(b) Solve the above equations simultaneously. (3 marks)

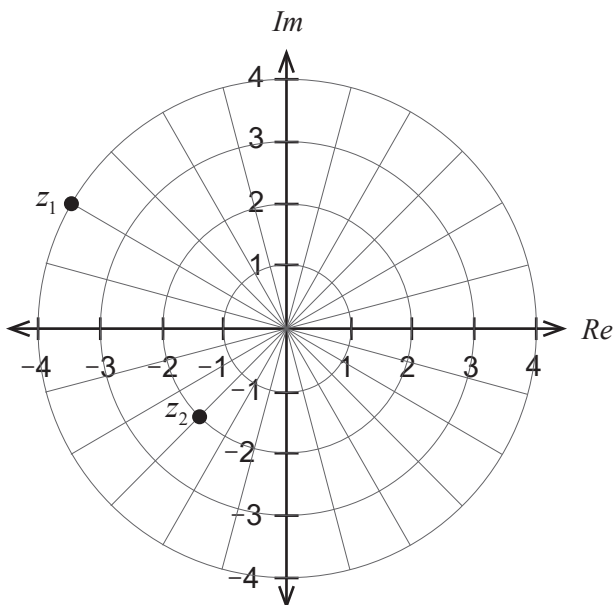
- (c) State the geometric interpretation of the solution obtained in part (b). (1 mark)

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Question 6

(8 marks)

Two complex numbers $z_1 = 4cis\left(\frac{5\pi}{6}\right)$ and z_2 are shown in the Argand plane below.



- (a) Determine the exact polar form for z_2 . (2 marks)

- (b) Plot the complex number $w = z_1 \times (z_2)^{-1}$ on the Argand diagram above. (3 marks)

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- (c) If $z_1 = 4\text{cis}\left(\frac{5\pi}{6}\right)$ is a solution of the equation $z^n = r$ where r is a positive real number and n is a positive integer, determine the smallest possible value for r in the form 2^p .

Justify your answer.

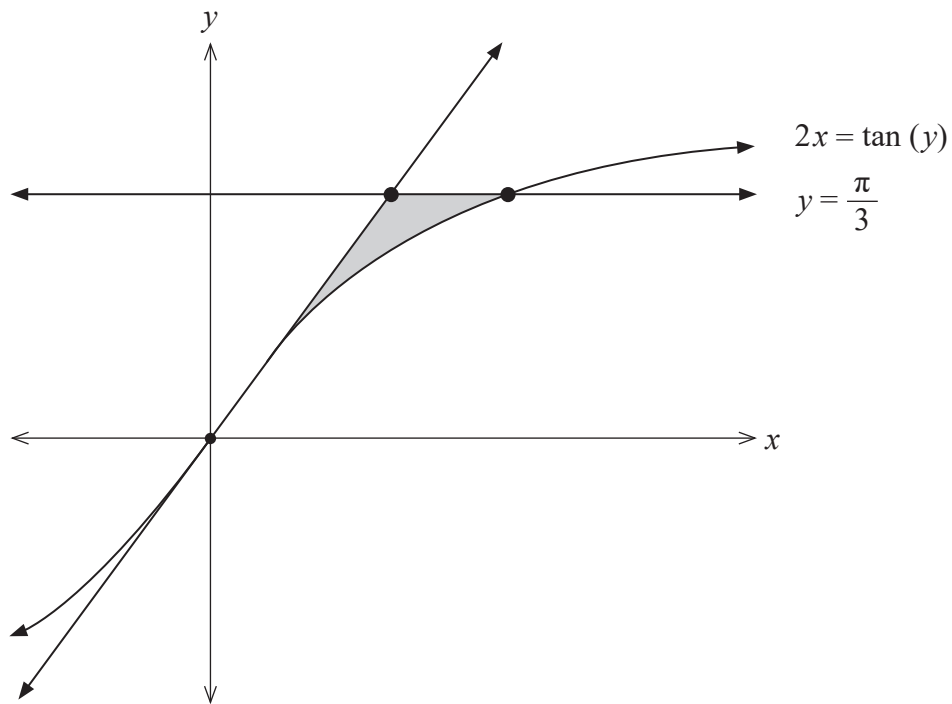
(3 marks)

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Question 7

(8 marks)

The graph of $2x = \tan(y)$ is shown along with the tangent at $x = 0$. The horizontal line $y = \frac{\pi}{3}$ is also shown.



- (a) Using implicit differentiation, determine the equation of the tangent drawn at $x = 0$.
(3 marks)

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The shaded region is bounded by the curve $2x = \tan(y)$, the tangent drawn and $y = \frac{\pi}{3}$.

(b) Write the expression for the area of the shaded region. (2 marks)

(c) Evaluate this area exactly. (3 marks)

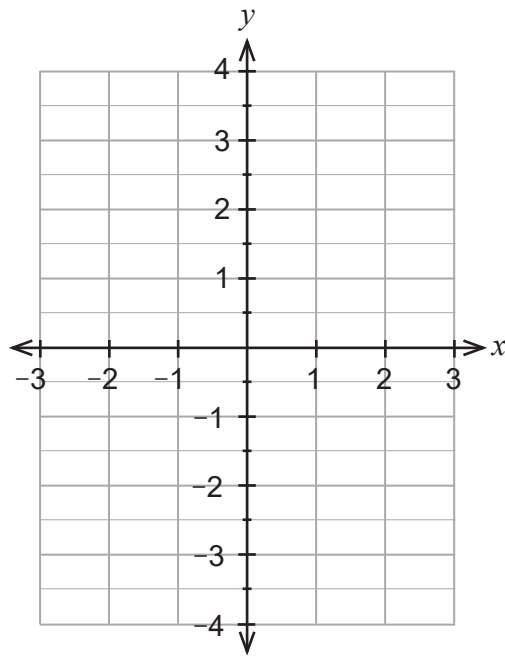
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Supplementary page

Question number: _____

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Spare grid for Question 2(b)



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