Government of **Western Australia**
School Curriculum and Standards Authority

Western Australian Certificate of Education
ATAR course examination, 2016

Question/Answer booklet

MATHEMATICS METHODS

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

Section Two: Calculator-assumed

Student number: In figures

In words ______________________________

____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Ref: 16-050
MAM-S2

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	49	35
Section Two: Calculator-assumed	13	13	100	101	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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Section Two: Calculator-assumed 65% (101 Marks)

This section has **13** questions. Answer **all** questions. Write your answers in the spaces provided.

Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.

Suggested working time: 100 minutes.

Question 9 (7 marks)

Fermium-257 is a radioactive substance whose decay rate can be modelled by the formula $P = P_0 e^{kt}$, where P is the mass in grams and t is measured in days and P_0 = original amount and k is a constant. The time taken to decay to half of the original amount is known as half-life. The half-life of Fermium-257 is 100.5 days.

(a) Determine the value of k to three significant figures. (3 marks)

(b) How many days will it take for 100 grams of the substance to first decay below five grams? (2 marks)

(c) Determine the rate of change of the amount of Fermium on the day found in part (b). (2 marks)

Question 10 **(12 marks)**

A survey in Western Australia was conducted on the popularity of a calculator known as Type A. Out of 1450 Year 12 students, the survey found that 986 students used the Type A calculator.

Determine the following.

(a) A 90% confidence interval, to three decimal places, for the proportion of Western Australian Year 12 students who use the Type A calculator. What assumption was made in calculating this interval? (3 marks)

(b) The margin of error in this confidence interval. (2 marks)

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Another three surveys of Year 12 students were conducted on the use of Type A calculators across Australia.

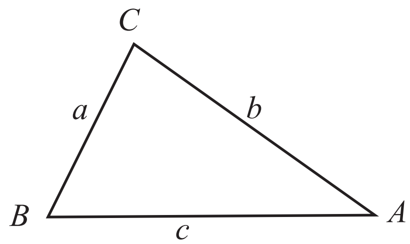
Survey 2	**Survey 3**	**Survey 4**
Type A usage 1772 out of 3221 Year 12 students	Type A usage 1021 out of 1566 Year 12 students	Type A usage 2203 out of 3221 Year 12 students

(c) Determine which of these surveys were more likely to have been taken outside of Western Australia. Justify your answer(s). (3 marks)

(d) Using the sample proportion of the survey at the start of the question, determine a sample size that will halve the margin of error for the proportion of Western Australian Year 12 students who use the Type A calculator, with a confidence of 90%. (4 marks)

Question 11 **(3 marks)**

The area of a triangle can be found by the formula: $Area = \dfrac{ab\sin C}{2}$.



Using the incremental formula, determine the approximate change in area of an equilateral triangle, with each side of 10 cm, when each side increases by 0.1 cm.

Question 12 **(3 marks)**

The Richter magnitude, M, of an earthquake is determined from the logarithm of the amplitude, A, of waves recorded by seismographs.

$M = \log_{10}\dfrac{A}{A_o}$, where A_o is a reference value.

An earthquake in a town in New Zealand in November 2015 was estimated at 5.5 on the Richter scale, while the earthquake just north of Hayman Island measured 3.4 on the same scale. How many times larger was the amplitude of the waves in New Zealand compared to those at Hayman Island?

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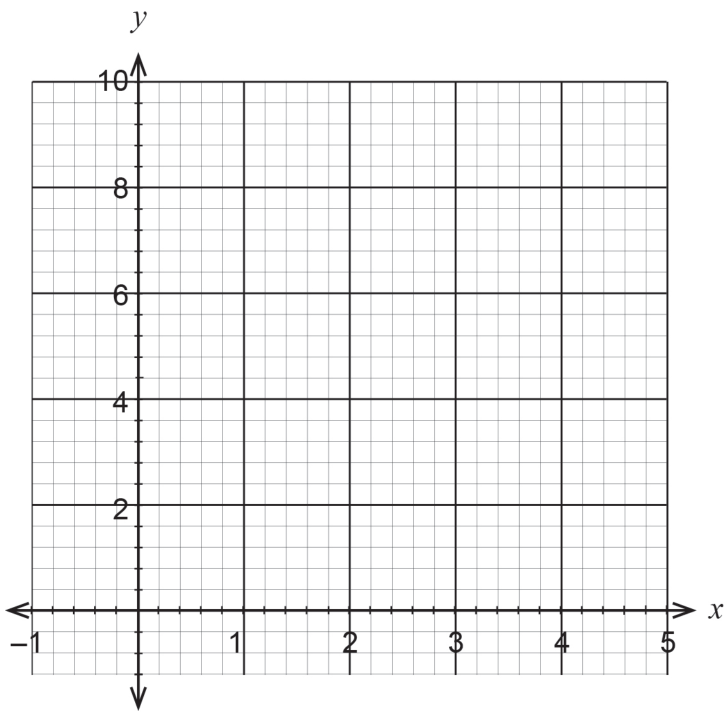
Question 13 **(10 marks)**

(a) Determine $\frac{d}{dx}(x^2 \ln x)$. (2 marks)

(b) Using your answer from part (a), show that the graph of $y = x^2 \ln x$ has only one stationary point. (3 marks)

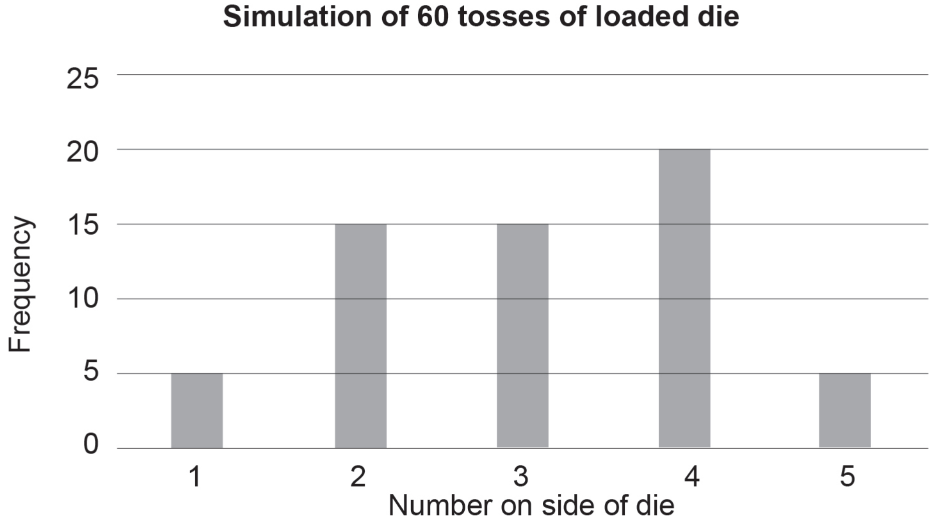
(c) Sketch the graph of $y = x^2 \ln x$, showing all features. (3 marks)



(d) Calculate the area bounded by the graph of $y = x^2 \ln x$, the x axis, $x = 1$ and $x = e$. (2 marks)

Question 14 **(9 marks)**

The simulation of a loaded (unfair) five-sided die rolled 60 times is recorded with the following results.



(a) Calculate the proportion of prime numbers recorded in this simulation. (2 marks)

(b) Determine the mean and standard deviation for the sample proportion of prime numbers in 60 tosses, using the results above. (2 marks)

(c) It has been decided to create a confidence interval for the proportion of prime numbers using the simulation results on page 8. The level of confidence will be chosen from 90% or 95%. Explain which level of confidence will give the smallest margin of error. State this margin of error. (3 marks)

This simulation of 60 rolls of the die is performed another 200 times, with the proportion of prime numbers recorded each time and graphed.

(d) Comment briefly on the key features of this graph. (2 marks)

Question 15 **(6 marks)**

A tetrahedral die has the numbers 1 to 4 on each face. When thrown, each side is equally likely to land facedown. Let X be defined as the sum of the numbers on the facedown side when the die is thrown twice.

(a) Complete the following table. (1 mark)

		Roll two			
Roll one	Sum of two rolls	1	2	3	4
	1	1 + 1 = 2	3		
	2	3			
	3		5		
	4				

(b) (i) Hence, or otherwise, complete the probability distribution of X, which is given by the following table. (1 mark)

x	**2**	**3**	**4**	**5**	**6**	**7**	**8**
$P(X = x)$	$\frac{1}{16}$						$\frac{1}{16}$

(ii) Calculate the probability of obtaining a sum of five or less. (2 marks)

(iii) Determine the mean and standard deviation for X. (2 marks)

Question 16 **(10 marks)**

An automated milk bottling machine fills bottles uniformly to between 247 ml and 255 ml. The label on the bottle states that it holds 250 ml.

(a) Determine the probability that a bottle selected randomly from the conveyor belt of this machine contains less than the labelled amount. (3 marks)

(b) Calculate the mean and standard deviation of the amount of milk in the bottles. (4 marks)

A worker selects bottles from the conveyor belt, one at a time.

(c) Determine the probability that it takes the selection of 15 bottles before five bottles containing less than the labelled amount have been selected. (3 marks)

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Question 17 **(7 marks)**

A school has analysed the examination scores for all its Year 12 students taking Methods as a subject. Let X = the examination percentage scores of all the Methods Year 12 students at the school. The school found that the mean was 75 with a standard deviation of 22.

Determine the following.

(a) $E(X+5)$ (1 mark)

(b) $Var(25-2X)$ (2 marks)

The school has decided to scale the results using the transformation $Y = aX + b$ where a and b are constants and Y = the scaled percentage scores. The aim is to change the mean to 60 and the standard deviation to 15.

(c) Determine the values of a and b. (4 marks)

Question 18 **(6 marks)**

The waiting times at a Perth Airport departure lounge have been found to be normally distributed. It is observed that passengers wait for less than 55 minutes, 5% of the time, while there is a 13% chance that the waiting times will be greater than 100 minutes.

(a) Determine the mean and standard deviation for the waiting times at Perth Airport departure lounge. (5 marks)

(b) Determine the probability that the waiting time will be between 75 and 90 minutes. (1 mark)

Question 19 **(8 marks)**

The displacement in centimetres of a particle from the point O in a straight line is given by $x(t) = \frac{1}{3}\left(\frac{t}{2} - 4\right)^2 - 2$ for $0 \le t \le 10$, where t is measured in seconds.

Calculate the:

(a) time(s) that the particle is at rest. (2 marks)

(b) displacement of the particle during the fifth second. (2 marks)

(c) maximum speed of the particle and the time when this occurs. (2 marks)

(d) total distance travelled in the first 10 seconds. (2 marks)

Question 20 **(14 marks)**

A chocolate factory produces chocolates of which 80% are pink. Each box of chocolates contains exactly 30 pieces.

(a) Identify the probability distribution of $X =$ the number of pink chocolates in a single box and also give the mean and standard deviation. (3 marks)

(b) Determine the probability, to three decimal places, that there are at least 27 pink chocolates in a randomly selected box. (3 marks)

Quality Control collects samples sizes of 20 boxes and counts the number of pink chocolates in total.

(c) Determine a 95% confidence interval for the proportion of pink chocolates in a sample of 20 boxes, using the assumption that 80% of chocolates in the sample are pink. (2 marks)

(d) Quality Control collects three samples and determines a 95% confidence interval each time. Determine the probability that only one of these intervals will **not** contain the true value 0.8 of the proportion of pink chocolates. (2 marks)

(e) Using your 95% confidence interval in part (c), determine the range in which the expected number of pink chocolates in a sample of 20 boxes would lie. (2 marks)

Quality Control counted the number of pink chocolates in five samples as shown below.

Sample	**1**	**2**	**3**	**4**	**5**
Number of pink chocolates	433	463	482	473	566

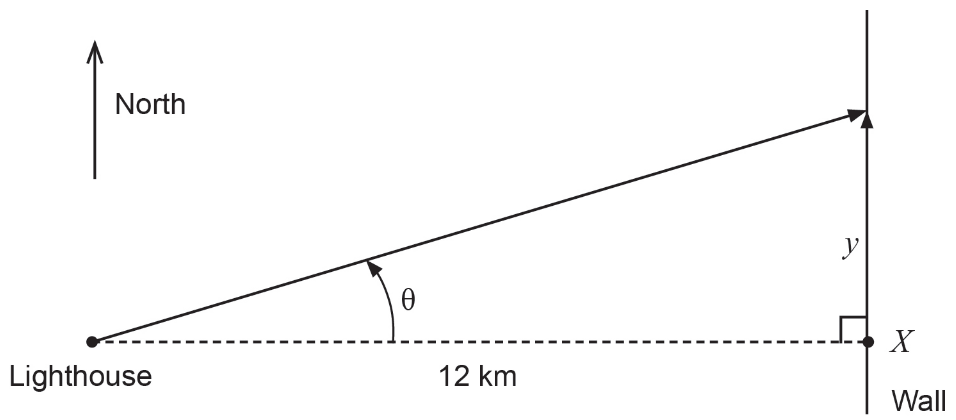
(f) Decide which samples lie outside the 95% confidence interval, if any. Justify. (2 marks)

Question 21 **(6 marks)**

A lighthouse is situated 12 km away from the shoreline, opposite point X as seen in the diagram below. A long brick wall is placed along the shoreline and at night the light from the lighthouse can be seen moving along this wall.

Let y = displacement of light on the wall from point X and θ = angle of the rotating light from the lighthouse.

The light is revolving anticlockwise at a uniform rate of three revolutions per minute ($\frac{d\theta}{dt} = 6\pi$ radians/minute).



(a) Show that $\dfrac{dy}{d\theta} = \dfrac{12}{\cos^2\theta}$. (3 marks)

(b) Determine the velocity, in kilometres per minute, of the light on the wall when the light is 5 km north of point X. (3 marks)

(Hint: $\frac{dy}{dt} = \frac{dy}{d\theta} \times \frac{d\theta}{dt}$)

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End of questions

Additional working space

Question number: ____________________

Additional working space

Question number: ________________

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