Government of **Western Australia**
School Curriculum and Standards Authority

SAMPLE ASSESSMENT TASKS

MATHEMATICS SPECIALIST
ATAR YEAR 11

Sample assessment task

Mathematics Specialist – ATAR Year 11

Unit 1

School Name

Sample test 3

Student name: _____ **Teacher name:** _____

Class: _____

Task type:	**Response**	
Time allowed for this task:	55 minutes, in-class, under test conditions	
	Section One: Calculator-free 35 minutes	(28 marks)
	Section Two: Calculator-assumed 20 minutes	(21 marks)
Materials required:	Calculator with CAS capability (to be provided by the student)	
Standard items:	Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters	
Special items:	Drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations	
Marks available:	**49 marks**	
Task weighting:	**6%**	

Section One: Calculator-free (28 marks)

Time allowed: 35 minutes

Question 1 (Section 1) 1.1.5 **(7 marks)**

A group of 67 Year 9 students responded to a survey stating they owned one or more of the following items.

A: An internet connected mobile phone
B: A tablet or iPad
C: A laptop computer

The following information was recorded using set notation
n(A) = 37, n(B) = 28 and n(C) = 32 and the following notes were made.

There were as many students who owned all three as those who owned a phone and iPad but not a laptop.
There were as many students who owned all three as those who owned a laptop and iPad but not a phone.
There were twice as many students who owned a phone and laptop but not an iPad as those who owned all three.

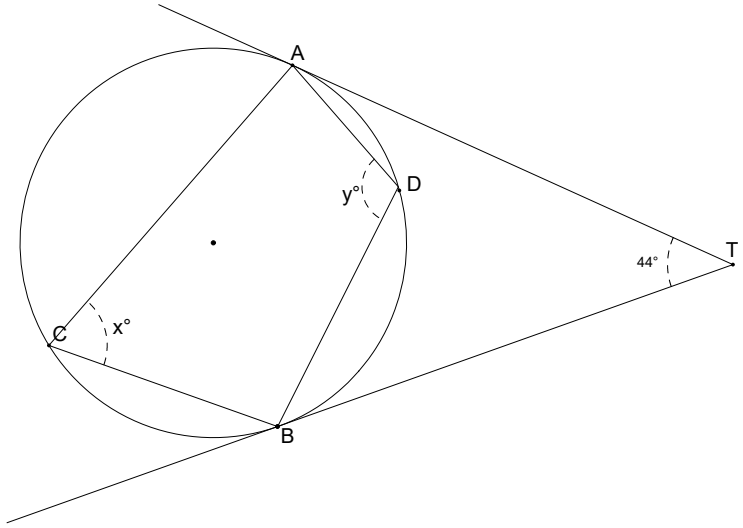
Calculate how many students owned all three devices.
Calculate how many students owned only one of these devices.

Question 2 (Section 1) 1.1.9 **(6 marks)**

Given $1.001=1+0.001$ calculate the expansion $(1.001)^4$ using the coefficients of Pascal's triangle.

Question 3 (Section 1) 1.3.7, 1.3.9 **(7 marks)**

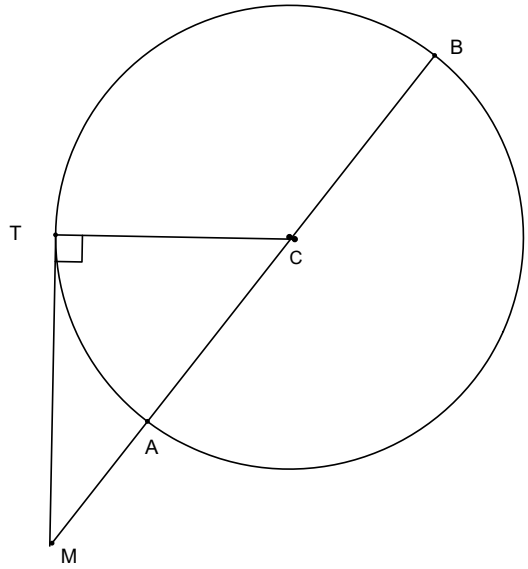
Two tangents are drawn from point T to points A and B on a circle as shown in the diagram. Given the angle between the tangents is 44^0 calculate the size of $\angle ADB$ and $\angle ACB$ showing proof of each result.



Question 4 (Section 1) 1.3.13 **(8 marks)**

Consider point M external to the circumference of the circle centre C. A secant is drawn from M passing through the centre C also intersecting the circumference of the circle at 2 points A and B. A line is also drawn from M to the circle at a point of tangency T.

Prove that $\overline{TM}^2 = \overline{AM} \times \overline{MB}$.



Section Two: Calculator-assumed (21 marks)

Time allowed: 20 minutes

Question 5 (Section 2) 1.2.4, 1.2.7, 1.2.8 **(12 marks)**

Two forces act on an object in a flat plane. $\mathbf{F_1}$ has a magnitude of 4 newtons and acts on a bearing of 035^0 and $\mathbf{F_2}$ with a magnitude of 5 newtons acts on a bearing 113^0.

(a) Use the triangle rule to calculate the magnitude and direction of the resultant force to an accuracy of two (2) decimal places. (7 marks)

(b) Calculate the **i** and **j** components of $\mathbf{F_1}$ and $\mathbf{F_2}$ given the unit vector **i** is on a bearing 0^0 T and the unit vector **j** is on a bearing 90^0 T (3 marks)

(c) Evaluate $\mathbf{F_1} + \mathbf{F_2}$ in **i** and **j** component form. (2 marks)

Question 6 (Section 2) 1.2.13 **(9 marks)**

The vectors $\mathbf{a} = 3\mathbf{i} + 5\mathbf{j}$ and $\mathbf{b} = 5\mathbf{i} + \mathbf{j}$ calculate the projection of

(i) **a** onto the x-axis and

(ii) **b** onto the y-axis

(iii) **a** onto **b**.

Solutions for sample test 3

Question 1 (Section 1) 1.1.5 **(7 marks)**

A group of 67 Year 9 students responded to a survey stating they owned one or more of the following items.

A: An internet connected mobile phone
B: A tablet or iPad
C: A laptop computer

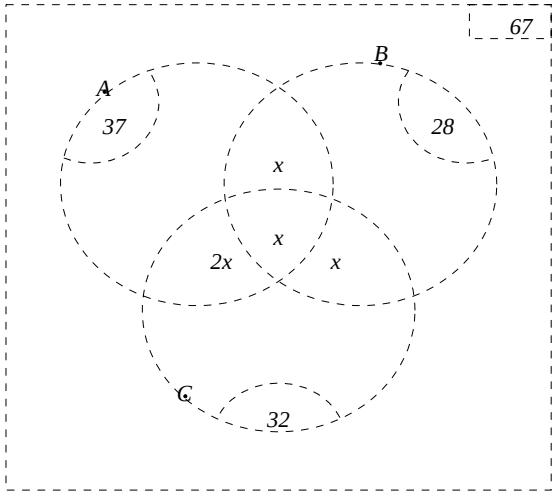
The following information was recorded using set notation
$n(A)$ = 37, $n(B)$ = 28 and $n(C)$ = 32 and the following notes were made.

There were as many students who owned all three as those who owned a phone and iPad but not a laptop.
There were as many students who owned all three as those who owned a laptop and iPad but not a phone.
There were twice as many students who owned a phone and laptop but not an iPad as those who owned all three.

(a) Calculate how many students owned all three devices.

(b) Calculate how many students owned only one of these devices.

Solution



$n\left(\overline{A \cup B \cup C}\right) = 0$

Let $n(A \cap B \cap C) = x$

then $n\left(A \cap B \cap \overline{C}\right) = n\left(B \cap C \cap \overline{A}\right) = x$

and $n\left(A \cap C \cap \overline{B}\right) = 2x$

$n(A) + n(B) + n(c) - n(A \cup B \cup C) = 97 - 67 = 6x$

$x = 5$

$n(A \cap B \cap C) = 5$

$n(\text{One only}) = 42$

Specific behaviours	Mark allocation	Item classification
Defines $n(A \cap B \cap C) = x$	1	simple
Sets up an equation in x for the intersection areas	3	simple
Sets up an equation to solve for x	1	complex
Correctly evaluates the number of students owned all three devices	1	complex
Correctly evaluates the number of students owned one device	1	complex

Question 2 (Section 1) 1.1.9 **(6 marks)**

Given $1.001=1+0.001$ calculate the expansion $(1.001)^4$ using the coefficients of Pascal's triangle.

Solution

Given 1.001=1 + 0.001 calculate the expansion $(1.001)^4$ using the coefficients of Pascal's triangle.

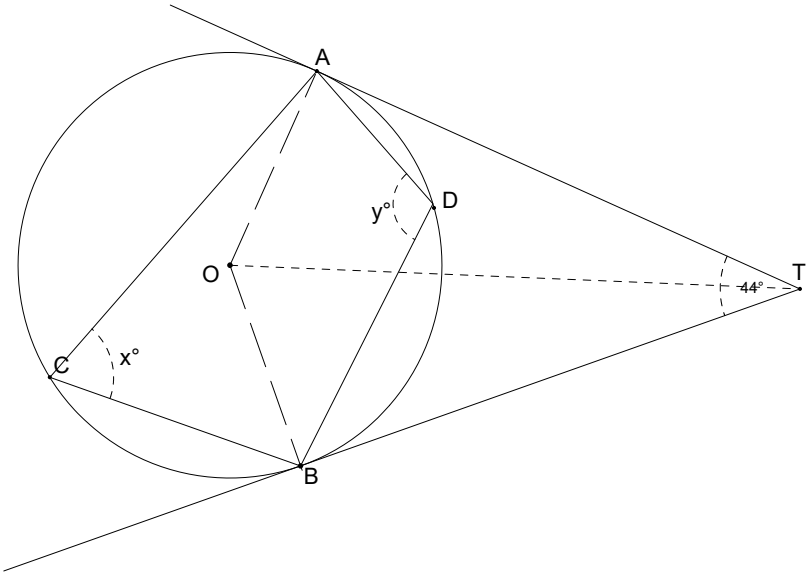
$$(1.001)^4 = (1+0.001)^4$$
$$= 1^4 + 4\times 1^3 \times 0.001^1 + 6\times 1^2 \times 0.001^2 + 4\times 1^1 \times 0.001^3 + 0.001^4$$
$$= 1 + 0.004 + 0.000006 + 0.000000004 + 0.000000000001$$
$$= 1.004006004001$$

Specific behaviours	Mark allocation	Item classification
Expands the term with the correct coefficients	1	simple
Calculates each decimal term correctly	4	simple
Adds the terms accurately	1	simple

Question 3 (Section 1) 1.2.4, 1.2.7, 1.2.8 **(7 marks)**

Two tangents are drawn from point T to points A and B on a circle as shown in the diagram. Given the angle between the tangents is 44^0 calculate the size of $\angle ADB$ and $\angle ACB$ showing proof of each result.

Solution



Construct a radius from the centre of the circle O to each of the points A and B

$\Delta ATO \equiv \Delta BTO$ $\{RHS\}$

$\angle ATO = \angle BTO = 22^o$ $\{\text{congruent } \Delta s\}$

$\angle AOT = \angle BOT = 68^o$ $\{\text{Right } \Delta s\}$

$\angle AOB = 68^o + 68^o = 136^o$ $\{\text{adjacent } \angle s\}$

$\angle ACB\left(x^o\right) = 68^o$ $\{\angle \text{ at circumference on arc ADB}\}$

$\angle ADB\left(y^o\right) = 112^o$ $\{\text{opposite } \angle s \text{ of cyclic quadrilateral}\}$

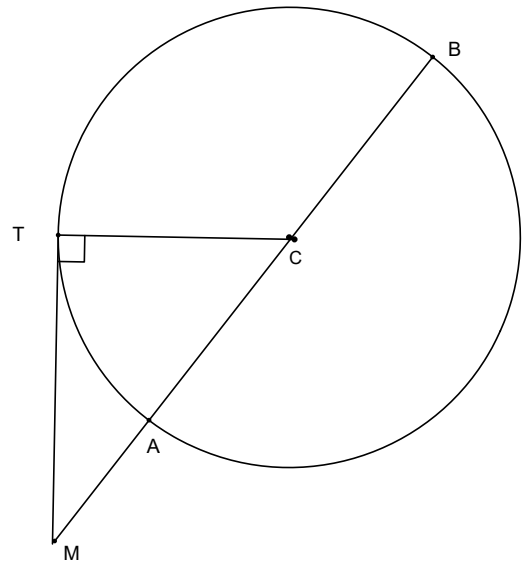
Specific behaviours	Mark allocation	Item classification
Draws the two radii OA OB and line segment OT	3	simple
Draws quadrilateral ADBC	1	simple
Determines $\Delta AOB = 136^o$	1	simple
Determines $\angle ACB = 68^o$	1	simple
Correctly determines the angles $\angle BDA = 112^o$	1	simple

Question 4 (Section 1) 1.3.13 **(8 marks)**

Consider the point M external to the circumference of the circle whose centre is at C. A secant is drawn from M passing through the centre C also intersecting the circumference of the circle at two points A and B.
A tangent is drawn from M to the circle at T.

$TM^2 = \overline{AM} \times \overline{MB}$

Solution



Prove that $\overline{TM}^2 = \overline{AM} \times \overline{MB}$

In $\triangle MTC$: $\overline{MC}^2 = \overline{TM}^2 + \overline{CT}^2$ $\{\text{Pythagoras}\}$ (1)

$\overline{MC}^2 = \left(\overline{MA} + \overline{AC}\right)^2$

$= \overline{AM}^2 + 2\overline{AC} \times \overline{AM} + \overline{AC}^2$

$= \overline{AM}^2 + \overline{AB} \times \overline{AM} + \overline{AC}^2$ $\left\{\overline{AB} = 2\overline{AC}\ (\text{radius})\right\}$

$= \overline{AM}\left(\overline{MA} + \overline{AB}\right) + \overline{AC}^2$

$= \overline{AM}\left(\overline{MB}\right) + \overline{AC}^2$ (2)

Equating RHS of equation (1) = RHS of equation (2)

$\overline{TM}^2 + \overline{CT}^2 = \overline{AM}\left(\overline{MB}\right) + \overline{AC}^2$ $\{\text{AC=CT=radius}\}$

$\overline{TM}^2 = \overline{AM}\left(\overline{MB}\right)$

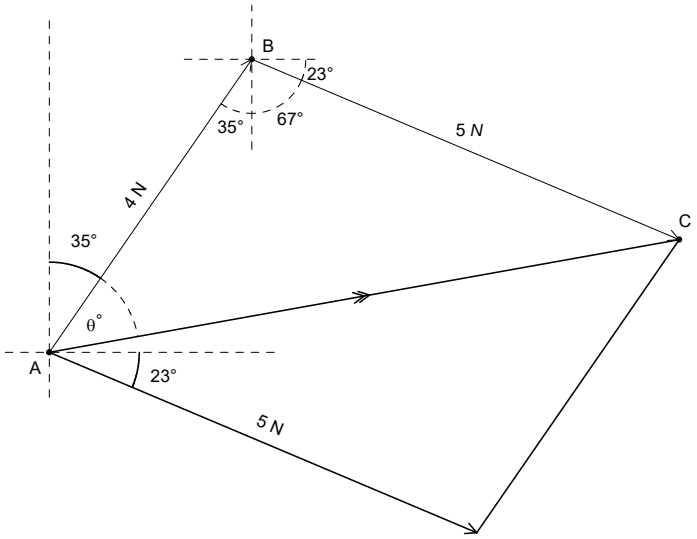
Specific behaviours	Mark allocation	Item classification
Uses Pythagoras in ΔMTC as in equation 1	1	complex
Expresses $\overline{MC} = \left(\overline{MA} + \overline{AC}\right)$	1	complex
Expands $\overline{MC}^2 = \left(\overline{MA} + \overline{AC}\right)^2$	1	complex
Determines equation 2 correctly	2	complex
Equates RHS of equation 1 to RHS of equation 2	1	complex
Simplifies to get the required expression $\overline{TM}^2 = \overline{AM}\left(\overline{MB}\right)$	2	complex

Question 5 (Section 2) 1.2.4, 1.2.7, 1.2.8 **(12 marks)**

Two forces act on an object in a flat plane. $\mathbf{F_1}$ has a magnitude of 4 newtons and acts on a bearing of 035^0 and $\mathbf{F_2}$ with a magnitude of 5 newtons acts on a bearing 113^0.

(a) Use the triangle rule to calculate the magnitude and direction of the resultant force to an accuracy of two (2) decimal places. (7 marks)

Solution



In ΔABC $\quad AC^2 = 4^2 + 5^2 - 2\times 4\times 5\times \cos 102^o$

$$AC \approx 7.02$$

$$\frac{7.02}{\sin 102} = \frac{5}{\sin \theta}$$

$$\theta \approx 44.14^o$$

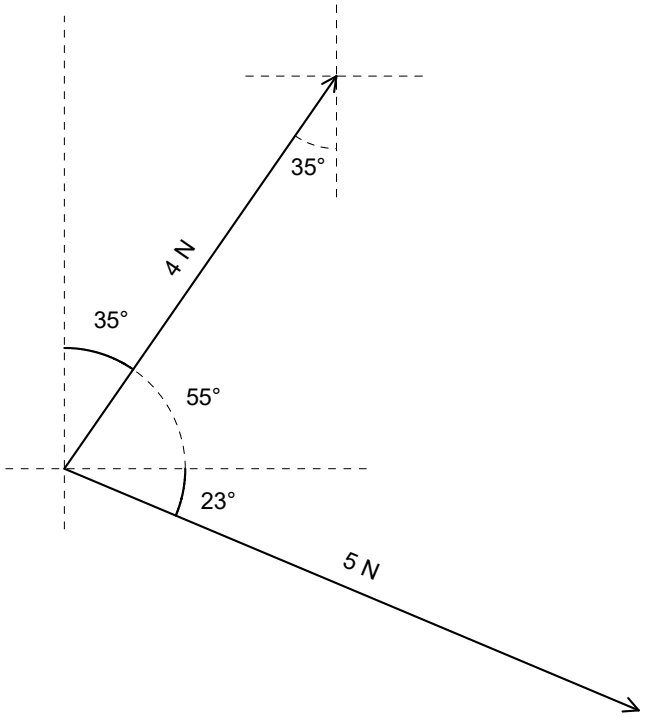
Bearing $= 35 + 44 = 79^0$

Specific behaviours	Mark allocation	Item classification
Constructs a triangle of forces	1	simple
Correctly evaluates the included angle = 102^0	1	simple
Uses the cosine rule to evaluate the magnitude	1	simple
States the correct magnitude in newtons	1	simple
Uses the sine rule to calculate angle θ	1	simple
Evaluates the angle between F_1 and the resultant	1	simple
Calculates the correct bearing of the resultant	1	simple

(b) Calculate the **i** and **j** components of $\mathbf{F_1}$ and $\mathbf{F_2}$ to 2 decimal places, given the unit vector **i** is on a bearing 0^0 T and the unit vector **j** is on a bearing 90^0 T (4 marks)

(c) Evaluate $\mathbf{F_1} + \mathbf{F_2}$ in **i** and **j** component form. (1 mark)

Solution



$\mathbf{F}_1 = 4\cos 55^\circ \mathbf{i} + 4\sin 55^\circ \mathbf{j}$
$= 2.29\mathbf{i} + 3.28\mathbf{j}$

$\mathbf{F}_2 = 5\cos(-23)^\circ \mathbf{i} + 5\sin(-23)^\circ \mathbf{j}$
$= 4.60\mathbf{i} + \text{-}1.95\mathbf{j}$

Evaluate $\mathbf{F_1} + \mathbf{F_2}$ in **i** and **j** component form.

Solution

$\mathbf{F}_1 + \mathbf{F}_2 = 6.90\mathbf{i} + 1.32\mathbf{j}$

Specific behaviours	Mark allocation	Item classification
Calculates the **i** and **j** components of $\mathbf{F}_1$	2	simple
Calculates the **i** and **j** components of $\mathbf{F}_2$	2	simple
Adds $\mathbf{F}_1$ and $\mathbf{F}_2$ accurately	1	simple

Question 6 (Section 2) 1.2.13 **(9 marks)**

The vectors **a** = 3**i** + 5**j** and **b** = 5**i** + **j** calculate the projection of

(i) **a** onto the x-axis and
(ii) **b** onto the y-axis
(iii) **a** onto **b**.

$\mathbf{a}=3\mathbf{i}+5\mathbf{j}\Leftrightarrow|\mathbf{a}|=\sqrt{3^2+5^2}=\sqrt{34}$ and $\cos\theta=\frac{3}{\sqrt{34}}$ where θ is the angle between vector **a** and the x-axis where $\overrightarrow{OX}=k\mathbf{i}$

$\mathbf{b}=5\mathbf{i}+\mathbf{j}\Leftrightarrow|\mathbf{a}|=\sqrt{5^2+1^2}=\sqrt{26}$ and $\cos\phi=\frac{1}{\sqrt{26}}$ where ϕ is the angle between vector **b** and the y-axis where $\overrightarrow{OY}=k\mathbf{j}$

(i) The projection of **a** onto the $x-$axis $=|\mathbf{a}|\cos\theta\mathbf{i}=\sqrt{34}\times\frac{3}{\sqrt{34}}\mathbf{i}=3\mathbf{i}$

(ii) The projection of **b** onto the $y-$axis $=|\mathbf{b}|\cos\phi\mathbf{i}=\sqrt{26}\times\frac{1}{\sqrt{26}}\mathbf{j}=\mathbf{j}$

(iii) The projection of **a** onto $\mathbf{b}=|\mathbf{a}|\cos\varphi\mathbf{b}=\sqrt{34}\times\frac{\begin{pmatrix}3\\5\end{pmatrix}\bullet\begin{pmatrix}5\\1\end{pmatrix}}{\sqrt{34}\times\sqrt{26}}\times\frac{1}{\sqrt{26}}(5\mathbf{i}+\mathbf{j})$

where φ is the angle between **a** and **b**

$=\frac{20}{26}(5\mathbf{i}+\mathbf{j})$

Specific behaviours	Mark allocation	Item classification
Correctly determines $\|\mathbf{a}\|=\sqrt{34}$	1	simple
Correctly determines $\cos\theta=\frac{3}{\sqrt{34}}$	1	simple
Correctly calculates the projection of **a** onto the x-axis	1	simple
Correctly determines $\|\mathbf{b}\|=\sqrt{26}$	1	simple
Correctly determines $\cos\phi=\frac{1}{\sqrt{26}}$	1	simple
Correctly calculates the projection of **b** onto the y-axis	1	simple
Correctly determines $\cos\varphi=\frac{20}{\sqrt{34}\times\sqrt{26}}$	1	complex
Correctly calculates $\mathbf{b}$	1	complex
Correctly simplifies the expression for projection of **a** onto **b**	1	simple

Question	1	2	3	4	5	6	Total
Simple	4	6	7	0	12	7	36
Complex	3	0	0	8	0	2	13
	Comb	Comb	Geom	Geom	Vector	Vector	