SAMPLE ASSESSMENT TASKS

MATHEMATICS SPECIALIST

ATAR YEAR 11

Acknowledgement of Country

Kaya. The School Curriculum and Standards Authority (the Authority) acknowledges that our offices are on Whadjuk Noongar boodjar and that we deliver our services on the country of many traditional custodians and language groups throughout Western Australia. The Authority acknowledges the traditional custodians throughout Western Australia and their continuing connection to land, waters and community. We offer our respect to Elders past and present.

Disclaimer

Any resources such as texts, websites and so on that may be referred to in this document are provided as examples of resources that teachers can use to support their learning programs. Their inclusion does not imply that they are mandatory or that they are the only resources relevant to the course. Teachers must exercise their professional judgement as to the appropriateness of any they may wish to use.

Sample assessment task

Mathematics Specialist – ATAR Year 11

Task 2 – Unit 1

Assessment type:	Response
Conditions:	Time for the task: up to 55 minutes In class, under test conditions
Materials required:	Section one: calculator-free Standard writing equipment Section two: calculator assumed Calculator with CAS capability (to be provided by the student)
Other materials allowed:	Drawing templates, one A4 page of notes in Section two
Marks available:	**61 marks** Section one: calculator-free – 32 marks Section two: calculator assumed – 29 marks
Task weighting:	**7%**

Section one: calculator-free (32 marks)

Time allowed: 30 minutes

Question 1 **(8 marks)**

The position vectors of points A and B and C are $\mathbf{a} = \begin{pmatrix} 7 \\ -4 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ respectively.

(a) Determine:

(i) $\overrightarrow{AC}$

(ii) $\mathbf{a} - 3\mathbf{b}$

(iii) the value(s) of k so that $|\mathbf{a} + k\mathbf{c}| = 5$

(b) Show that $\mathbf{a} \bullet (\mathbf{b} + \mathbf{c}) = \mathbf{a} \bullet \mathbf{b} + \mathbf{a} \bullet \mathbf{c}$

Question 2 **(3 marks)**

Given the points F = (3, 7) and G = (9, 5), determine the exact value that the vector $\overrightarrow{FG}$ must be multiplied by so that the length of the vector is 20 units.

Question 3 **(6 marks)**

A cyclist has an initial position vector of $3\mathbf{i} - \mathbf{j}$ km with respect to Perth. She cycles towards Midland with a constant velocity of $8\mathbf{i} + 6\mathbf{j}$ km h^{-1}.

(a) State her speed.

(b) Determine her position with respect to Perth after 1½ hours.

(c) Determine her distance from Perth.

Question 4 **(7 marks)**

Given the position vectors $\mathbf{a} = 5\mathbf{i} + 2\mathbf{j}$ and $\mathbf{b} = 3\mathbf{i} + 4\mathbf{j}$ determine:

(a) a vector parallel to $\mathbf{a} + \mathbf{b}$ and 2.5 units in length

(b) a unit vector perpendicular to $\mathbf{b}$

Question 5 (4 marks)

The vectors **a, b, c** and **d** are shown on the grid below.

(a) Draw and label the vector

    (i) **a** + **c**

    (ii) $3\mathbf{d} - \mathbf{b}$

(b) Express the vector **b** in terms of **a, c** and **d.**
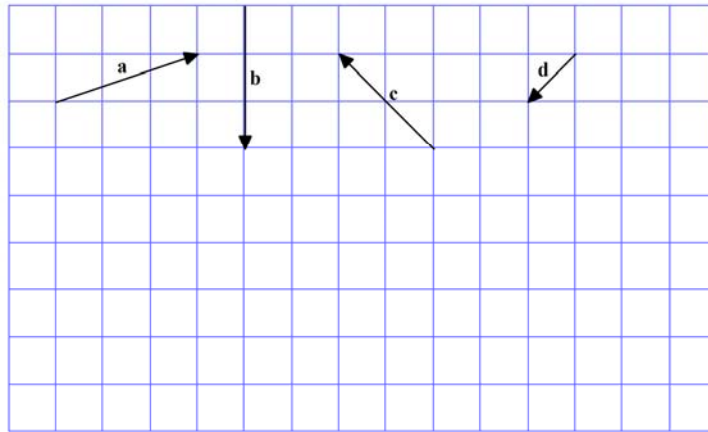
Question 6 (4 marks)

KLM is a scalene triangle where $\overrightarrow{KL} = 2\boldsymbol{a}$, $\overrightarrow{KM} = 3\boldsymbol{b}$ and $\overrightarrow{LN} = \frac{1}{4}\overrightarrow{LM}$.
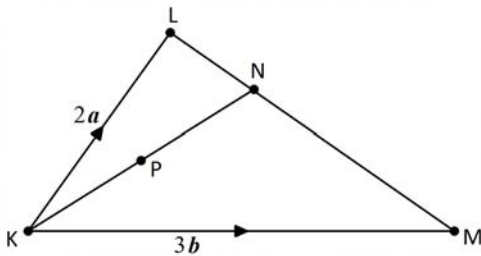
Point P is positioned such that $\overrightarrow{KP} = \overrightarrow{PN}$

Show that $\overrightarrow{KP} = \frac{3}{8}(2\boldsymbol{a} + \boldsymbol{b})$

Section two: calculator assumed (29 marks)

Time allowed: 25 minutes

Question 7 (6 marks)

Two sides of a parallelogram are defined by the vectors $\overrightarrow{AB}$ and $\overrightarrow{AD}$ where $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ and $\overrightarrow{OD} = \begin{pmatrix} 9 \\ 1 \end{pmatrix}$

(a) Determine the vectors $\overrightarrow{BC}$ and $\overrightarrow{CD}$ that define the other two sides.

(b) Determine the size of the internal obtuse angle of the parallelogram.

Question 8 (4 marks)

A surfer is paddling in the ocean at 3.1 km h⁻¹ parallel to the shore. The current is moving away from the shore at 2.6 km h⁻¹ on an angle of 39° to the shore. If the surfer's speed is decreased by the current, determine the true direction and speed of the surfer.

Question 9 **(10 marks)**

A hiker leaves camp and walks 4 kilometres on a bearing of $035°$. He then turns and walks on a bearing of $119°$ for 5 kilometres.

(a) Calculate the distance and direction he will need to travel to get back to camp. (4 marks)

(b) Calculate the **i** and **j** components for stage one and stage two of his walk. (2 marks)

(c) Show clearly the equivalence, in **i** and **j** component form, between the solution in (a) and your answers in (b). (4 marks)
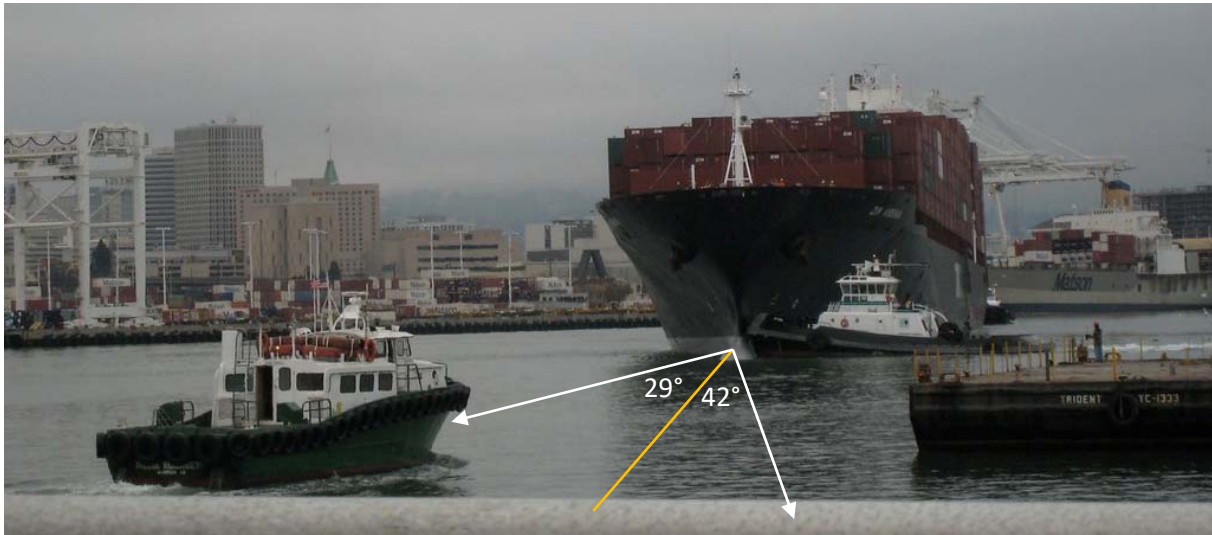
Question 10 **(5 marks)**

Given the vectors **a** and **b** are such that $\mathbf{a} = 3\mathbf{i} + 5\mathbf{j}$ and $\mathbf{b} = 5\mathbf{i} - \mathbf{j}$

(a) determine the scalar projection of

    (i) **a** onto the x-axis

    (ii) **b** onto the y-axis

(b) determine the exact projection vector of **a** on **b**

Question 11 (4 marks)

Two tugboats are towing a barge into the harbour. The first tug is pulling with a force of 2825 N at an angle of 42° to the left of the direction that the barge is travelling in, while the second tug is pulling the barge at an angle of 29° to the right of the bow, as shown in the diagram below. Calculate, to the nearest 10 N, the magnitude of the force that needs to be applied by the second barge for the barge to continue in a straight line. Hence, determine the magnitude of the resulting force applied to the barge.

Hjem. (2007). Tugs [adapted image].

Retrieved from https://search.creativecommons.org/photos/683eb7e4-fe1c-423a-921d-77a33e4c10c6

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Solutions for sample response Unit 1

Section one: calculator-free (32 marks)

Question 1(a) (5 marks)

Solution
(i) $\begin{pmatrix} -8 \\ 6 \end{pmatrix}$
(ii) $\begin{pmatrix} 1 \\ -19 \end{pmatrix}$
(iii) $\mathbf{a} + k\mathbf{c} = \begin{pmatrix} 7 - k \\ -4 + 2k \end{pmatrix} \implies \sqrt{(7-k)^2 + (-4+2k)^2} = 5$
$65 - 30k + 5k^2 = 25$
$5(k-4)(k-2) = 0$
$\therefore \ k = 4 \ or \ k = 2$

Behaviours	Marks
(i) Determines the vector correctly	1
(ii) Determines the vector correctly	1
(iii) Determines the vector expression for **a** + *k***c** correctly	1
Writes an expression for the magnitude of **a** + *k***c** correctly	1
Correctly identifies the two solutions for *k*	1

Question 1(b) (3 marks)

Solution
$\mathbf{b} + \mathbf{c} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$ $\therefore$ $\begin{pmatrix} 7 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 7 \end{pmatrix} = \begin{pmatrix} 7 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} 7 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 2 \end{pmatrix}$
$7 + (-28) = \ 14 + (-20) + (-7) + (-8)$
$-21 \ = \ -21$

Behaviours	Marks
Determines the correct vector for **b** + **c**	1
Demonstrates the use of the dot product to simplify the left-hand side correctly	1
Demonstrates the use of the dot product to simplify the right-hand side correctly	1

Question 2 (3 marks)

Solution
$\overrightarrow{FG} = \begin{pmatrix} 9 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ 7 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$ $\qquad \therefore \quad \left\|\overrightarrow{FG}\right\| = \sqrt{6^2 + (-2)^2}$ $= \sqrt{40}$ $= 2\sqrt{10}$ $\therefore \; \overrightarrow{FG}$ must be multiplied by $\sqrt{10}$

Behaviours	Marks
Determines the vector $\overrightarrow{FG}$ correctly	1
Calculates the magnitude of $\overrightarrow{FG}$ correctly	1
States the correct scale factor	1

Question 3(a) (2 marks)

Solution
$\|8\mathbf{i} + 6\mathbf{j}\|$ $= \sqrt{8^2 + 6^2}$ $= \sqrt{100}$ $= 10 \text{ km/hr}$

Behaviours	Marks
Calculates the magnitude correctly	1
States the speed correctly, including units	1

Question 3(b) (2 marks)

Solution
$\mathbf{r}_c = 1.5(8\mathbf{i} + 6\mathbf{j}) + (3\mathbf{i} + \mathbf{j})$ $= 12\mathbf{i} + 9\mathbf{j} + 3\mathbf{i} - \mathbf{j}$ $= 15\mathbf{i} + 8\mathbf{j} \text{ km from Perth}$

Behaviours	Marks
Writes a correct expression for the position of the cyclist after 1½ hours	1
States the position correctly, including units	1

Question 3(c) (2 marks)

Solution
$\|15\mathbf{i} + 8\mathbf{j}\|$
$= \sqrt{15^2 + 8^2}$
$= \sqrt{289}$
$= 17$ km from Perth

Behaviours	Marks
Calculates the magnitude of $\mathbf{r}_c$ correctly	1
States the distance correctly including units	1

Question 4(a) (3 marks)

Solution
$\mathbf{a} + \mathbf{b} = 5\mathbf{i} + 2\mathbf{j} + 3\mathbf{i} + 4\mathbf{j}$
$\qquad = 8\mathbf{i} + 6\mathbf{j}$
$\|\mathbf{a} + \mathbf{b}\| = \sqrt{8^2 + 6^2}$
$\qquad = \sqrt{100}$
$\qquad = 10 \qquad \qquad \therefore$ must be divided by 4
the vector is $\dfrac{1}{4}(8\mathbf{i} + 6\mathbf{j}) = 2\mathbf{i} + 1.5\mathbf{j}$

Behaviours	Marks
Correctly determines $\mathbf{a} + \mathbf{b}$	1
Correctly determines the magnitude of $\mathbf{a} + \mathbf{b}$	1
Correctly scales the vector by $\dfrac{1}{4}$	1

Question 4(b) (4 marks)

Solution

$$\binom{3}{4} \bullet \binom{x}{y} = 0 \qquad\qquad \text{also } x^2 + y^2 = 1$$

$$3x + 4y = 0 \qquad\qquad (\frac{-4y}{3})^2 + y^2 = 1$$

$$\therefore \ x = \frac{-4y}{3} \qquad\qquad y^2 = \frac{9}{25}$$

$$\therefore \ y = \frac{3}{5} \text{ or } y = \frac{-3}{5}$$

$$\therefore \text{ the vector is either } \frac{-4}{5}\mathbf{i} + \frac{3}{5}\mathbf{j} \ or \ \frac{4}{5}\mathbf{i} - \frac{3}{5}\mathbf{j}$$

Behaviours	Marks
Determines a correct equation for the dot product of the two perpendicular vectors	1
Determines a correct equation for the magnitude of the unit vector	1
Solves the simultaneous equations to correctly determine a possible value for y	1
Determines a vector that satisfies both conditions correctly	1

Question 5(a) (2 marks)

Solution
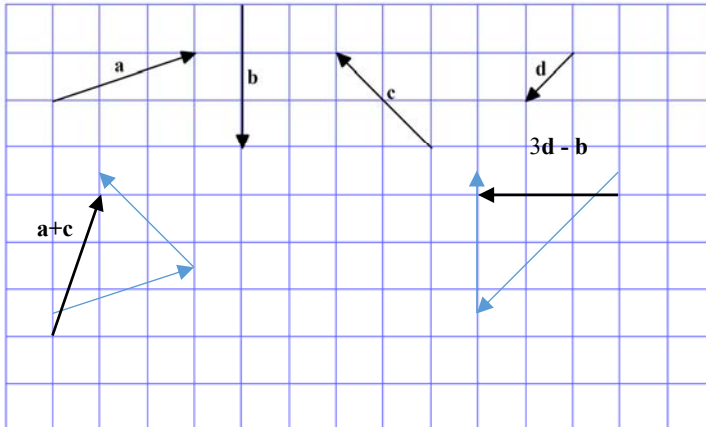
Behaviours	Marks
Draws and labels the vector to represent $\mathbf{a} + \mathbf{c}$ correctly	1
Draws and labels the vector to represent $3\mathbf{d} - \mathbf{b}$ correctly	1

Question 5(b) (2 marks)

Solution
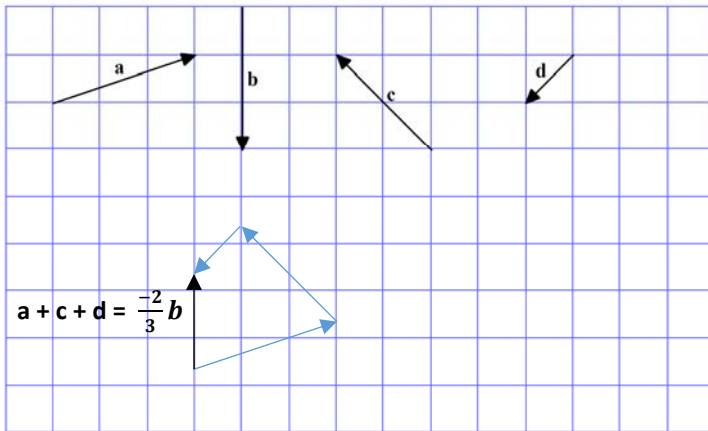
$$\mathbf{a} + \mathbf{c} + \mathbf{d} = \frac{-2}{3}\,\mathbf{b}$$

$$\mathbf{b} = \frac{-3(\mathbf{a} + \mathbf{c} + \mathbf{d})}{2}$$

Behaviours	Marks
Expresses vector **b** in terms of **a**, **c** and **d** correctly (multiple solutions possible)	1
Justifies the solution either algebraically or graphically	1

Question 6 (4 marks)

Solution

$\overrightarrow{KP} = \frac{1}{2}\overrightarrow{KN}$

$\overrightarrow{KN} = \overrightarrow{KL} + \frac{1}{4}\overrightarrow{LM}$

$\qquad = 2\mathbf{a} + \frac{1}{4}(-2\mathbf{a} + 3\mathbf{b})$

$\qquad = \frac{3}{2}\mathbf{a} + \frac{3}{4}\mathbf{b}$

$\therefore \overrightarrow{KP} = \frac{1}{2}\left(\frac{3}{2}\mathbf{a} + \frac{3}{4}\mathbf{b}\right)$

$\qquad = \frac{3}{4}\mathbf{a} + \frac{3}{8}\mathbf{b}$

$\qquad = \frac{3}{8}(2\mathbf{a} + \mathbf{b})$

Behaviours	Marks
Writes an expression for $\overrightarrow{KN}$ in terms of $\overrightarrow{LM}$, $\overrightarrow{KL}$ and $\overrightarrow{KM}$ correctly	1
Expresses $\overrightarrow{LM}$ in terms of **a** and **b** correctly	1
Expresses $\overrightarrow{KN}$ in terms of **a** and **b** correctly	1
Correctly manipulates the expression formed to show equivalence	1

Section two: calculator-assumed (29 marks)

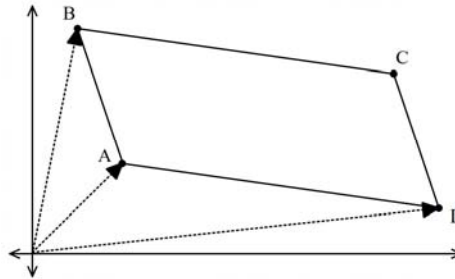**Question 7(a)** (3 marks)

Solution

$\overrightarrow{BC} = \overrightarrow{AD}$

$= \begin{pmatrix} 9 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$\overrightarrow{BC} = 7\mathbf{i} - \mathbf{j}$

$\overrightarrow{CD} = \overrightarrow{BA}$

$= \begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \end{pmatrix}$

$\overrightarrow{CD} = \mathbf{i} - 3\mathbf{j}$

Behaviours	Marks
Identifies two pairs of parallel vectors	1
Determines $\overrightarrow{BC}$ correctly	1
Determines $\overrightarrow{CD}$ correctly	1

Question 7(b) (3 marks)

Solution

$\overrightarrow{BA} \bullet \overrightarrow{AD}$

$\begin{pmatrix} 1 \\ -3 \end{pmatrix} \bullet \begin{pmatrix} 7 \\ -1 \end{pmatrix} = 10 \quad \Rightarrow \quad 10 = |\overrightarrow{BA}||\overrightarrow{BC}|\cos\theta$

$10 = \sqrt{10}\sqrt{50}\cos\theta$

$\cos\theta = \frac{10}{\sqrt{500}}$

$\therefore \quad \theta = 63.43°$

angle required is $180 - 63.43° = 116.57°$

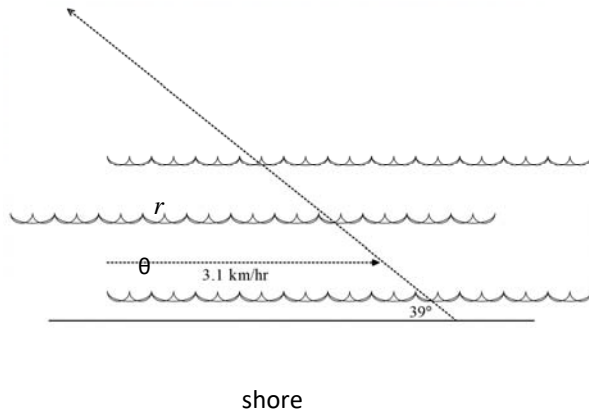| Behaviours	Marks
Calculates the dot product correctly	1
Calculates the acute angle correctly	1
Gives the obtuse angle between the vectors	1

Question 8 (4 marks)

Solution

$r^2 = (3.1)^2 + (2.6)^2 - 2(3.1)(2.6)\cos 39°$

$r = 1.96$

$\dfrac{\sin\theta}{2.6} = \dfrac{\sin 39}{1.96}$

$\theta = 56.6°$

Surfer is moving at 1.96 km h^{-1}

at an angle of 56.6° away from the shore

shore

Behaviours	Marks
Creates a diagram to correctly add the vectors	1
Determines the resulting speed of the surfer	1
Determines the resulting vector angle of the surfer	1
Expresses the solution in terms of a direction with respect to the shore	1

Question 9(a) (4 marks)

Solution

$d^2 = (4)^2 + (5)^2 - 2(4)(5)\cos 96°$

$d = 6.72$

$\dfrac{\sin\theta}{4} = \dfrac{\sin 96}{6.72}$

$\theta = 36.3°$
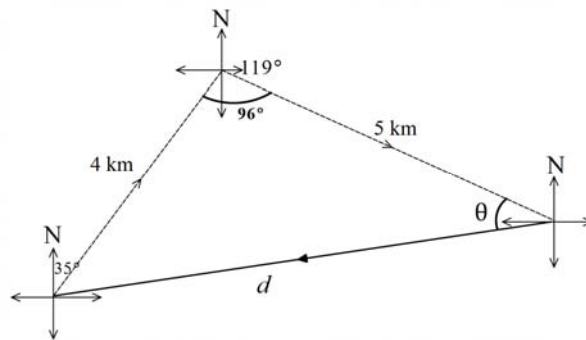
The hiker will need to travel 6.72 km on a bearing of 262.7°

Behaviours	Marks
Creates a diagram to add the vectors and determines the included angle correctly	1
Calculates the distance back to the camp	1
Calculates the angle between the last leg and return journey	1
Determines the direction back to camp as a correct bearing	1

Question 9(b)   (2 marks)

Solution	
Stage one: $4\cos 55° \mathbf{i} + 4\sin 55° \mathbf{j}$ $= 2.29\mathbf{i} + 3.28\mathbf{j}$	Stage two: $5\cos(-29°)\mathbf{i} + 5\sin(-29°)\mathbf{j}$ $= 4.37\mathbf{i} - 2.42\mathbf{j}$

Behaviours	Marks
States the stage one components correctly	1
States the stage two components correctly	1

Question 9(c)   (4 marks)

Solution
Return journey from (a): 6.72 km on a bearing of 262.7° $\qquad = 6.72\cos(-172.7°)\mathbf{i} + 6.72\sin(-172.7°)\mathbf{j}$ $\qquad = -6.66\mathbf{i} - 0.85\mathbf{j}$ Stage one + stage two (b): $(2.29\mathbf{i} + 3.28\mathbf{j}) + (4.37\mathbf{i} - 2.42\mathbf{j})$ $\qquad = 6.66\mathbf{i} + 0.85\mathbf{j}$ Back to camp will be reverse direction: $-(6.66\mathbf{i} + 0.85\mathbf{j})$, which is equivalent to the return journey of 6.72 km on a bearing of 262.7°, $(-6.66\mathbf{i} - 0.85\mathbf{j})$.

Behaviours	Marks
Expresses the return journey from (a) in component form correctly	1
Adds vectors for stage one and two in component form	1
States the reverse direction required for the sum of stage one and two	1
States equivalence, commenting on rounding error if required	1

Question 10(a)   (2 marks)

Solution
(i)   3   (ii) -1

Behaviours	Marks
States the projection on the axis as the correct scalar value	1
States the projection on the axis as the correct scalar value	1

Question 10(b) (3 marks)

Solution

Scalar projection $= \mathbf{a} \bullet \hat{\mathbf{b}}$

$\hat{\mathbf{b}} = \dfrac{5}{\sqrt{26}}\mathbf{i} - \dfrac{1}{\sqrt{26}}\mathbf{j}$

$\mathbf{a} \bullet \hat{\mathbf{b}} = (3\mathbf{i} + 5\mathbf{j}) \bullet \left(\dfrac{5}{\sqrt{26}}\mathbf{i} - \dfrac{1}{\sqrt{26}}\mathbf{j}\right)$

$= \dfrac{10}{\sqrt{26}}$

Projection vector is $\dfrac{10}{\sqrt{26}}\left(\dfrac{5}{\sqrt{26}}\mathbf{i} - \dfrac{1}{\sqrt{26}}\mathbf{j}\right) = \dfrac{25}{13}\mathbf{i} - \dfrac{5}{13}\mathbf{j}$

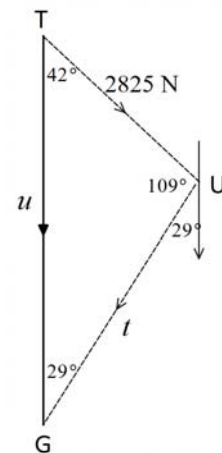| Behaviours	Marks
Determines the unit vector $\hat{\mathbf{b}}$ correctly	1
Calculates the scalar projection correctly	1
Determines the projection vector	1

Question 11 (4 marks)

Solution

$\dfrac{t}{\sin 42°} = \dfrac{2825}{\sin 29°}$

$t = 3900 \text{ N}$

$\dfrac{u}{\sin 109°} = \dfrac{2825}{\sin 29°}$

$u = 5510 \text{ N}$

The second tugboat must pull with a force of 3900 N

and the resulting force applied will have a magnitude of 5510 N

Behaviours	Marks
Creates a diagram to show the addition of the vectors and determines the included angle correctly	1
Calculates the second force, t, correctly	1
Calculates the resulting force being applied to the barge, u	1
Rounds all solutions to the nearest ten correctly	1

Sample assessment task
Mathematics Specialist – ATAR Year 11
Task 7 – Unit 2

Assessment type: Investigation

Conditions: The investigation will be completed over one week. Students will be encouraged to work independently to complete the task and may use any appropriate technology.

Note: while the Authority provides sample assessment tasks for guidance, it is the expectation of the Authority that teachers will develop tasks customised to reflect their school's context and the needs of the student cohort. This resource is available on a public website and use of the resource without modification may affect the integrity of the assessment.

Task weighting: 7% of the school mark for this pair of units

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Complex numbers and the complex plane   **(36 marks)**

Given $z = a + bi$, investigate the nature of the curve formed by joining together in succession on the Complex Plane, the points formed by z^n, for different values of a and b .

As you complete this task, take care to clearly

- identify and organise relevant information   **(7 marks)**

- choose effective models and methods and carry through correctly   **(7 marks)**

- follow mathematical convention and attend to accuracy   **(9 marks)**

- link mathematical results to data and context to reach conclusions   **(7 marks)**

- communicate mathematical reasoning, results and conclusions   **(6 marks)**
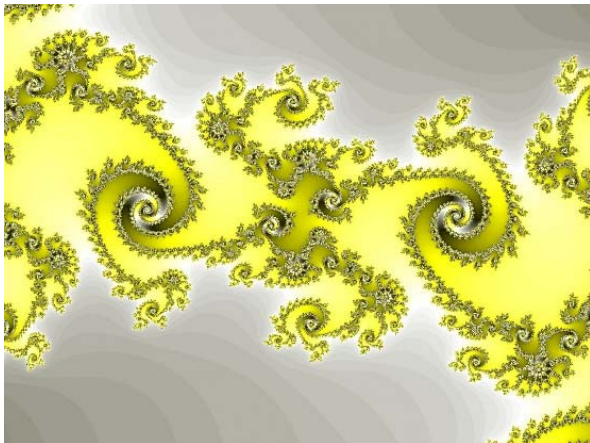
Fractal, rendering, spiral, abstract, yellow [Image]. Public domain.

Marking key for sample assessment task 7 – Unit 2

Identifies and organises relevant information

Behaviours	Marks
Writes a detailed outline of the investigation, identifying all the important elements of the problem and showing an understanding of the relationship between them; defines relevant parameters for the variables, explains an appropriate and systematic strategy to be applied and identifies appropriate assumptions	6–7
Writes a detailed outline of the investigation, accurately documenting mathematical content related to the task, defines relevant parameters for the variables and explains the process to be applied	5
Writes a detailed outline of the investigation, accurately documenting mathematical content related to the task and defines relevant parameters for the variables	3–4
Writes an outline of the investigation, identifying mathematical content supplied with the task	2
Writes a simple restatement of the task	1
Subtotal	**/7**

Chooses effective models and methods and carries through correctly (including appropriate use of technology)

Behaviours	Marks
Chooses an efficient and systematic method to investigate the problem, including appropriate and effective use of technology for the mathematical context being considered; and applies the method correctly and consistently throughout the task	5–7
Chooses an efficient method to investigate the problem, including some use of technology, and applies the method consistently throughout the task	3–4
Chooses a simple method and applies the method consistently throughout the task	2
Chooses a simple method to make some progress towards a solution	1
Subtotal	**/7**

Follows mathematical conventions and attends to accuracy

Behaviours	Marks
Applies mathematical procedures with accuracy, demonstrates complete understanding of the mathematical concepts and principles and the relationships between the important elements of the problem Provides appropriate interpretation and consistent use of mathematical terminology, symbols and conventions in graphing and calculations	8–9
Applies mathematical procedures with accuracy, demonstrates understanding of the mathematical concepts and principles and the relationships between the important elements of the problem Provides appropriate interpretation and use of mathematical terminology, symbols and conventions in graphing and calculations	6–7
Applies mathematical procedures with some accuracy, demonstrates understanding of some mathematical concepts and principles in attempts to identify relationships Provides appropriate interpretation and makes some use of mathematical terminology, symbols and conventions in graphing and calculations	4–5
Applies mathematical procedures with some accuracy, and attempts to identify relationships between elements of the problem	2–3
Attempts to apply mathematical procedures to investigate the problem	1
Subtotal	**/9**

Links mathematical results to data and contexts to reach reasonable conclusions

Behaviours	Marks
Organises and links findings to make clear connections between all algebraic and graphical representations and makes inferences from analysis to draw conclusions across all domains	6–7
Organises and links findings to make clear connections between all algebraic and graphical representations and draws conclusions for the domain of the task	5
Organises and links findings to make clear connections between algebraic and graphical representations and draws some valid conclusions	3–4
Links findings to make connections between some algebraic and graphical representations	1–2
Subtotal	**/7**

Communicates mathematical reasoning, results and conclusions

Behaviours	Marks
Provides a complete response that summarises the findings with clear, unambiguous explanations and/or descriptions using mathematical language and including supporting diagrams linked back to the context of the original problem Includes logical justification of the possible limitation of the results and recognition of assumptions made Provides a coherent and organised presentation of the investigation report, including acknowledgement of resources where appropriate	5–6
Communicates findings in a systematic and concise way using mathematical language, linking the solution to the original problem and acknowledging resources where appropriate	3–4
Communicates findings in a systematic way but with poor use of mathematical language	2
Presents simple conclusions that are not organised or supported by data or calculations	1
Subtotal	**/6**
Total	**/36**