Government of **Western Australia**
**School Curriculum and Standards Authority**

## ATAR course examination, 2017

## Question/Answer booklet

# MATHEMATICS SPECIALIST

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

## Section One: Calculator-free

Student number: In figures

In words

### Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable):

### Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet
Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

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## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|---|---|---|---|---|---|
| Section One: Calculator-free | 8 | 8 | 50 | 53 | 35 |
| Section Two: Calculator-assumed | 11 | 11 | 100 | 97 | 65 |
| | | | | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2017*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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**See next page**

**Section One: Calculator-free** **35% (53 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

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**Question 1** **(4 marks)**

Let $z = a - bi$, where $a > 0, b > 0$. Consider $w = z + i\overline{z}$.

Determine the possible value(s) for $\arg(w)$.

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**Question 2** **(6 marks)**

Consider $f(z) = 2z^3 - 5z^2 + 4z - 10$ where $z$ is a complex number.

(a) Show that $(z - \sqrt{2}i)$ is a factor of $f(z)$. (2 marks)

(b) Given that $(z - \sqrt{2}i)$ is a factor of $f(z)$, state another factor of $f(z)$. (1 mark)

(c) Solve the equation $2z^3 - 5z^2 + 4z - 10 = 0$. (3 marks)

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**Question 3** **(7 marks)**

Consider the definite integral $\int_0^1 \frac{x^2}{(1+x^2)^2}\,dx$.

(a) By using the substitution $x = \tan u$, show that $\int_0^1 \frac{x^2}{(1+x^2)^2}\,dx = \int_a^b \sin^2 u\,du$ and state the values of $a$, $b$. (4 marks)

(b) Hence evaluate $\int_0^1 \frac{x^2}{(1+x^2)^2}\,dx$ exactly. (3 marks)

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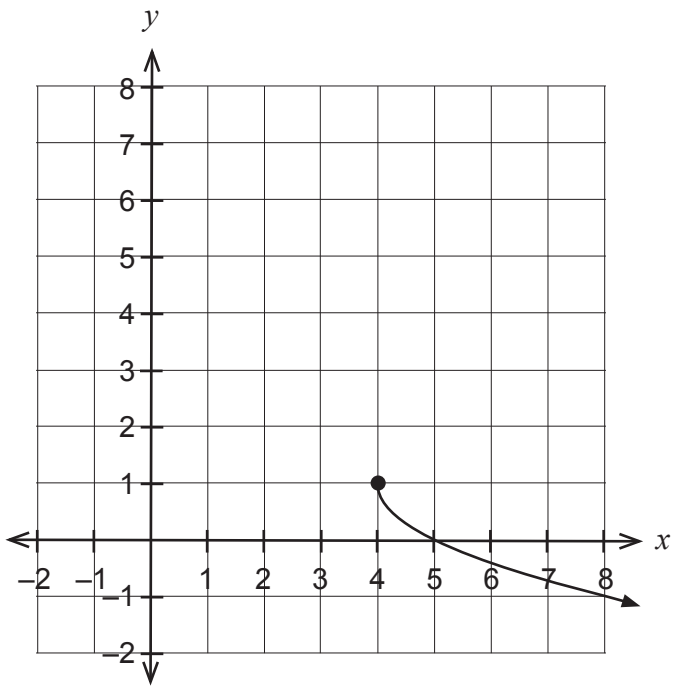
**Question 4** **(9 marks)**

Function $f$ is defined as $f(x) = 1 - \sqrt{x-4}$. The graph of $y = f(x)$ is shown below.




(a) Sketch the graph of $y = f^{-1}(x)$ on the axes above. (2 marks)

(b) Determine the defining rule for $y = f^{-1}(x)$ and state its domain. (3 marks)

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Function $g$ is defined as $g(x) = \frac{1}{x^2}$.

(c) Determine an expression for $f \circ g(x)$. (1 mark)

(d) For $f \circ g(x)$, determine the domain. (3 marks)

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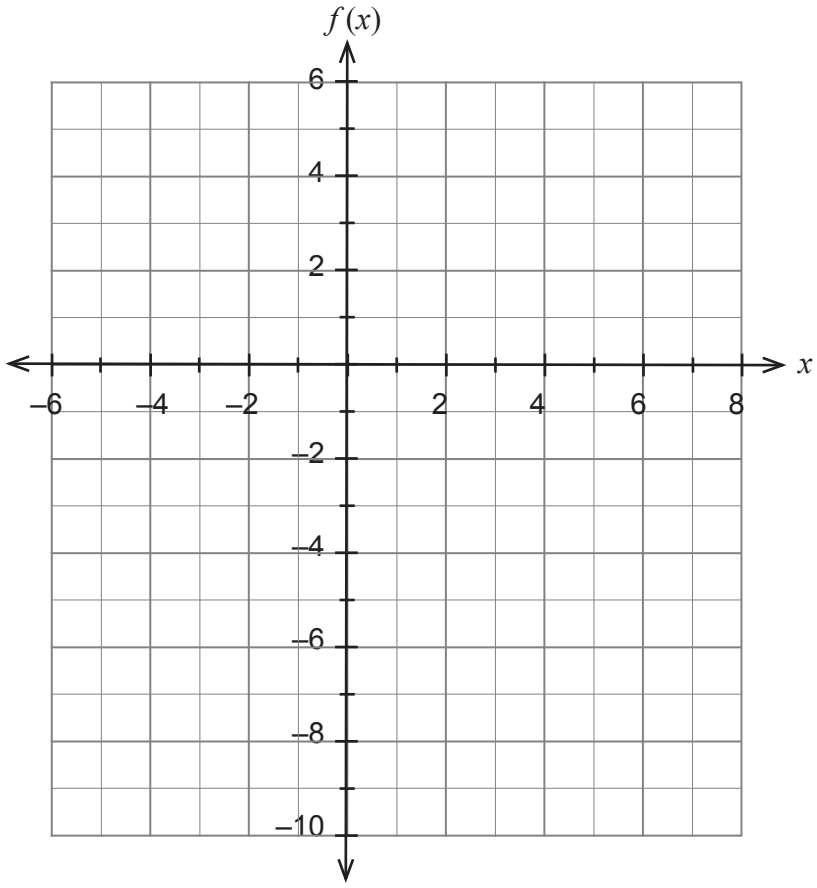
**Question 5** **(6 marks)**

Sketch the graph of $f(x) = -\dfrac{4(x-3)(x+1)}{x^2 - 2x - 8}$ on the axes below.

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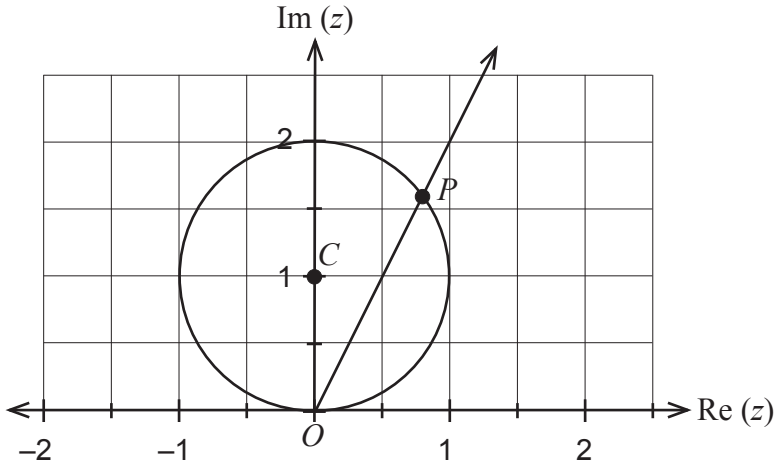
**Question 6** **(6 marks)**

A circle and a ray are indicated in the complex plane. The ray has equation $\arg(z) = \tan^{-1}(2)$. Point $C$ is the centre of the circle. Point $P$ is the intersection of the circle and the ray.




(a) Determine the equation for the circle. (2 marks)

Point $P$ determines a complex number $w = r\,cis\theta$.

(b) Determine the exact values for $r$, $\theta$. (4 marks)

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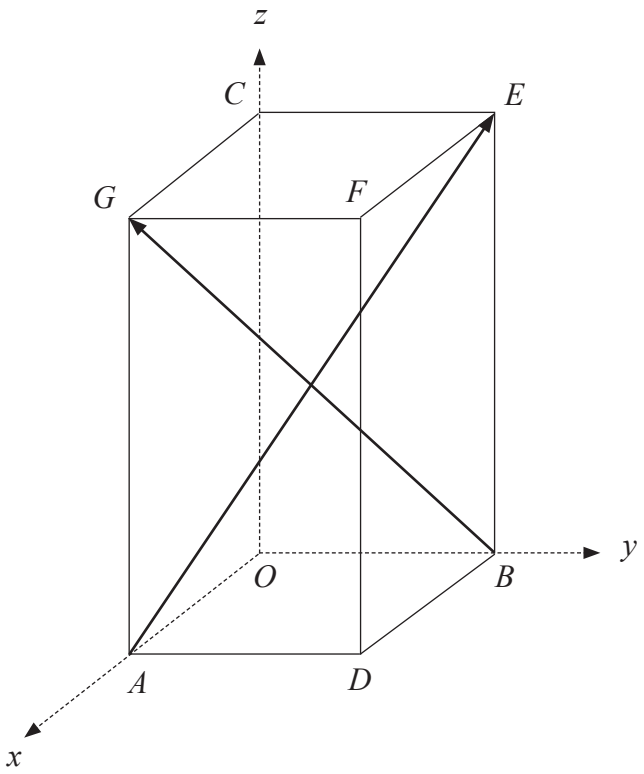
**Question 7** **(10 marks)**

A right rectangular prism, with square base $OADB$, is shown below. Point $O$ is the origin and points $A,B,C$ have respective position vectors $\begin{pmatrix}4\\0\\0\end{pmatrix}$, $\begin{pmatrix}0\\4\\0\end{pmatrix}$, $\begin{pmatrix}0\\0\\c\end{pmatrix}$ where $c > 0$.




(a) Determine, in terms of $c$, the:

(i) vector equation for the line containing points $A$ and $E$. (3 marks)

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(ii) Cartesian equation for the plane $ADEC$. (4 marks)

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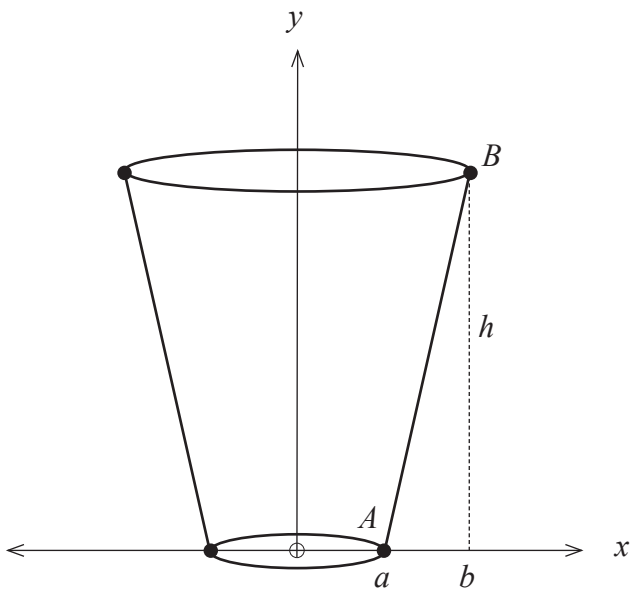
In general, the main diagonals $\overrightarrow{AE}$, $\overrightarrow{BG}$ are not perpendicular to each other.

(b) Determine the value of $c$ so that the main diagonals of the prism are perpendicular to each other. (3 marks)

**See next page**

**Question 8** **(5 marks)**

The inner surface of a drinking glass can be modelled by rotating the line segment $\overline{AB}$ about the $y$ axis, as shown in the diagram below. The radius of the glass at the bottom is $a$ cm and the radius at the top is $b$ cm. The height of the glass is $h$ cm.




The equation for $\overline{AB}$ is $y = \left(\frac{x - a}{b - a}\right)h$.

(a) Write an expression, in terms of a definite integral, for the volume of liquid contained by the glass when it is full. (2 marks)

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(b) By using an anti-derivative, obtain a simplified expression/formula (in terms of $a$, $b$ and $h$) for the volume of liquid contained by the glass when it is full. (3 marks)

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**End of section**

Supplementary page

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