Sample Assessment Tasks

Mathematics Specialist

ATAR Year 12

**Acknowledgement of Country**

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Sample assessment task

Mathematics Specialist – ATAR Year 12

Task 1 – Unit 3

**Assessment type:** Response

**Time allowed:** 55 minutes

Section one – calculator-free section – 35 minutes

Section two – calculator assumed section – 20 minutes

**Conditions:** In class, under supervised conditions

Section two – Drawing instruments, templates, notes on two unfolded sheets of A4 paper and up to three calculators suitable for use in ATAR course examinations

**Task weighting:** 9% of the school mark for this pair of units

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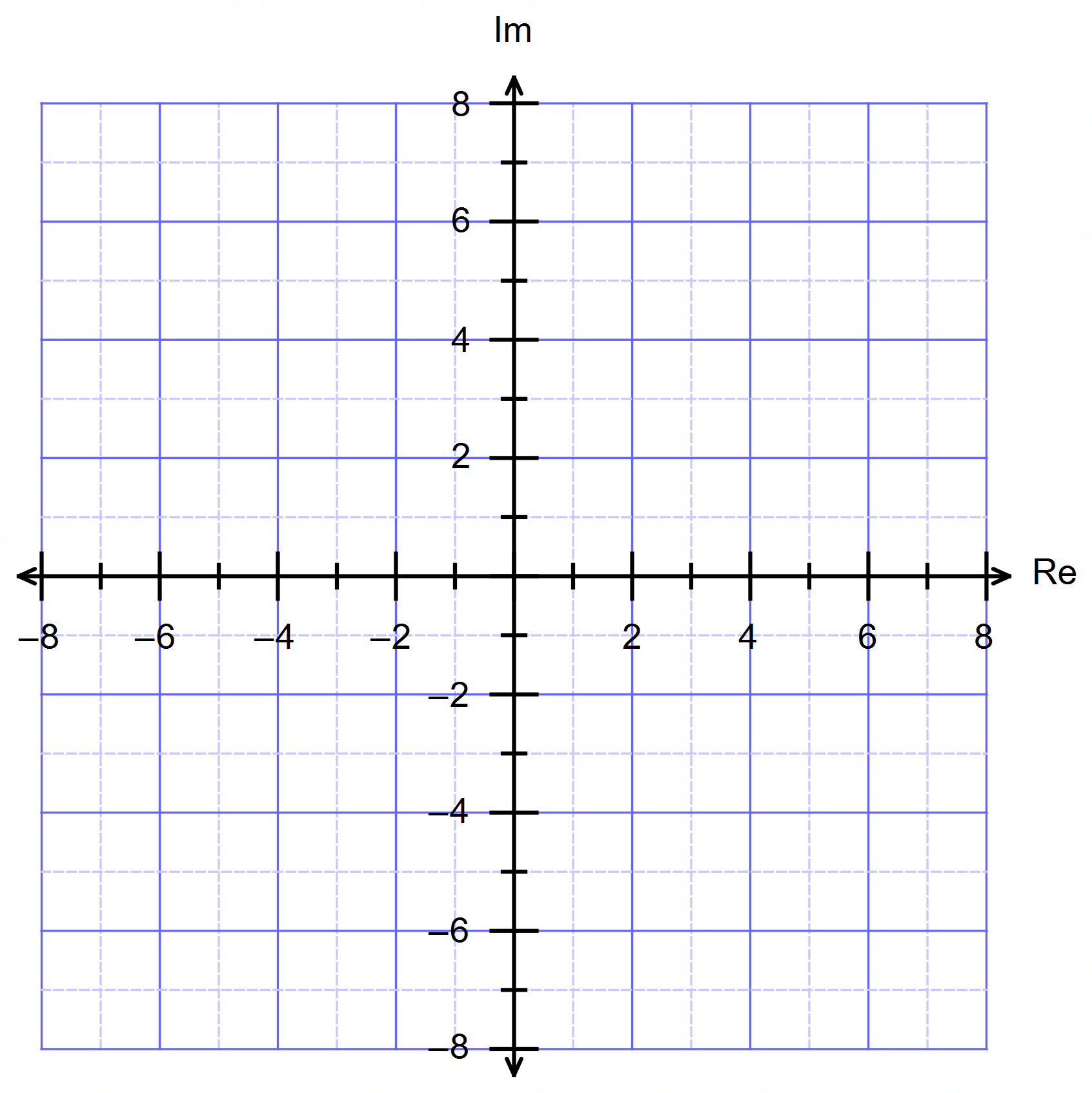
Section one – calculator-free section (26 marks)

Question 1 (8 marks)

1. Given evaluate giving the answer in Cartesian form. (2 marks)
2. Given and evaluate the following in exact Cartesian form  
    (4 marks)
3. (ii) (iii)
4. Solve for in exact form. (2 marks)

**Question 2**  **(6 marks)**

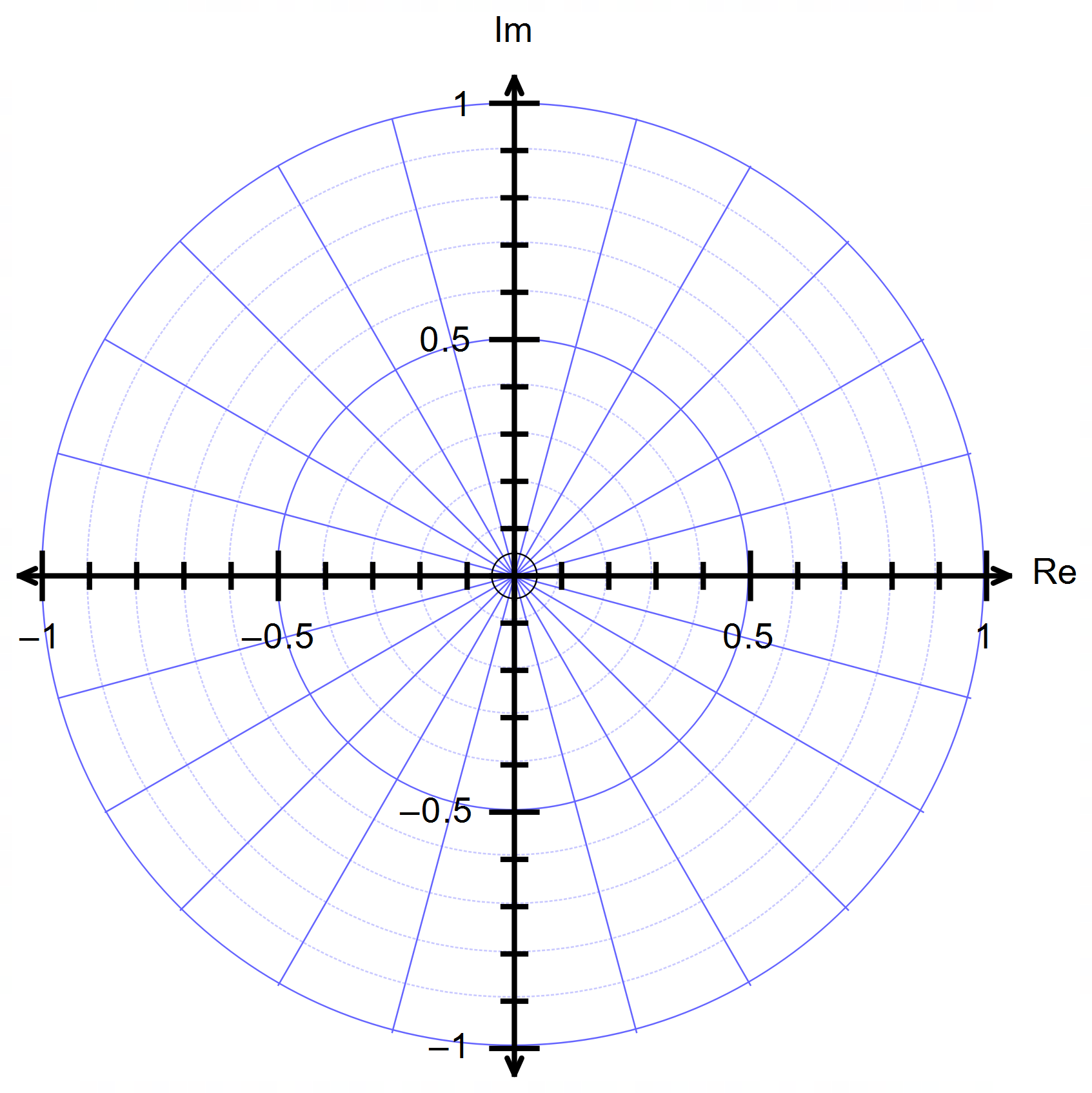
1. Sketch the set of points defined by . (4 marks)

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1. Determine the maximum value of . (2 marks)

Question 3 (6 marks)

1. Determine and locate all solutions in the Argand plane to the equation . (4 marks)

****

1. The solution located in the fourth quadrant above is also a solution to the equation . Determine the solution to the equation that lies in the first quadrant. (2 marks)

**Question 4 (6 marks)**

Given

1. Evaluate and (2 marks)
2. Hence, find all roots of the equation (4 marks)

Section two – calculator assumed section (27 marks)

Question 5 (10 marks)

1. Expand and simplify the expression . (2 marks)
2. Hence, express the in terms of . (3 marks)
3. Use to solve the equation and express the solutions in polar form.  
    (5 marks)

Question 6 (7 marks)

The complex numbers and are shown on the Argand diagram below.

****

1. On the Argand diagram, draw and clearly label the position of (4 marks)
2. (ii) (iii)
3. On the diagram below, sketch the set of complex numbers such that .   
    (3 marks)



Question 7 (10 marks)

Given

1. Express in simplified polar form. (4 marks)
2. Show and hence prove . (6 marks)

Marking key for sample assessment task 1

Section one – calculator-free section (26 marks)

Question 1 (8 marks)

1. Given evaluate giving the answer in Cartesian form. (2 marks)

|  |  |
| --- | --- |
| **Solution** | |
| Given evaluate  =  or | |
| **Behaviours** | **Marks** |
| Expands the Cartesian form of *z*6  Simplifies correctly | 1  1 |
| **or** | |
| Expresses *z*6 in polar form  Expresses the answer in Cartesian form | 1  1 |
| **Subtotal** | **/2** |

1. Given and evaluate the following in exact Cartesian form (4 marks)

(i)  (ii) (iii)

|  |  |
| --- | --- |
| **Solution** | |
| (i) (ii) (iii) | |
| **Behaviours** | **Marks** |
| 1. Writes the Cartesian form of correctly | 1 |
| 1. Writes the Cartesian form of correctly | 1 |
| 1. Expresses polar term for required term correctly | 1 |
| Writes the Cartesian form correctly | 1 |
| **Subtotal** | **/4** |

1. Solve for in exact form. (2 marks)

|  |  |  |
| --- | --- | --- |
| **Solution** | | |
|  | Or | |
| **Behaviours** | | **Marks** |
| Completes the square correctly  Solves the equation using the exact form | | 1  1 |
| **or** | | |
| Substitutes into the quadratic formula correctly  Simplifies the expressions to the correct exact form | | 1  1 |
| **Subtotal** | | **/2** |

Question 2 (6 marks)

1. Sketch the set of points defined by . (4 marks)

|  |  |
| --- | --- |
| **Solution** | |
|  | |
| **Behaviours** | **Marks** |
| Draws a circle with a solid line  Has the correct centre,  Has the correct radius  Shades the interior of the circle | 1  1  1  1 |
| **Subtotal** | **/4** |

1. Determine the maximum value of . (2 marks)

|  |  |  |
| --- | --- | --- |
| **Solution** | | |
|  | Maximum distance | |
| **Behaviours** | | **Marks** |
| Determines distance from centre of circle to (0, 0) correctly  Adds the length of the radius | | 1  1 |
| **Subtotal** | | **/2** |

Question 3 (6 marks)

1. Determine and locate all solutions in the Argand plane to the equation . (4 marks)

|  |  |  |
| --- | --- | --- |
| **Solution** | | |
|  |  | |
| **Behaviours** | | **Marks** |
| Expresses one solution correctly and locates it on the polar graph | | 1 |
| Identifies that there are five solutions, with angular separation of | | 1 |
| Expresses all five solutions correctly | | 1 |
| Locates the solutions on the polar graph | | 1 |
| **Subtotal** | | **/4** |

1. The solution located in the fourth quadrant above is also a solution to the equation . Determine the solution to the equation that lies in the first quadrant. (2 marks)

|  |  |
| --- | --- |
| **Solution** | |
| Using E:  Solution in first quadrant is | |
| **Behaviours** | **Marks** |
| Identifies the solution in the fourth quadrant from (a)  Determines the first quadrant solution correctly | 1  1 |
| **Subtotal** | **/2** |

Question 4 (6 marks)

1. Evaluate and . (2 marks)

|  |  |
| --- | --- |
| **Solution** | |
|  | |
| **Behaviours** | **Marks** |
| Evaluates correctly  Evaluates correctly | 1  1 |
| **Subtotal** | **/2** |

1. Hence, find all roots of the equation (4 marks)

|  |  |
| --- | --- |
| **Solution** | |
| Given from part is a factor of and is a factor of is a factor of  Then equating the constant:  And equating coefficients of :  Hence, the roots to are | |
| **Behaviours** | **Marks** |
| Uses the factor theorem to give factors | 1 |
| Identifies that the conjugate is a factor | 1 |
| Determines the remaining factors | 1 |
| Correctly writes all the roots | 1 |
| **Subtotal** | **/4** |

Section two – calculator assumed section (27 marks)

Question 5 (10 marks)

1. Expand and simplify the expression (2 marks)

|  |  |
| --- | --- |
| **Solution** | |
|  | |
| **Behaviours** | **Marks** |
| Shows the real and imaginary terms correctly | 2 |

1. Hence, express the in terms of . (3 marks)

|  |  |
| --- | --- |
| **Solution** | |
|  | |
| **Behaviours** | **Marks** |
| Writes the real part of | 1 |
| Substitutes for | 1 |
| Simplifies to give the correct expression for | 1 |
| **Subtotal** | **/3** |

1. Use to solve the equation and express the solutions in polar form. (5 marks)

|  |  |
| --- | --- |
| **Solution** | |
| from part (b) | |
| **Behaviours** | **Marks** |
| Uses de Moivre’s theorem to state | 1 |
| Makes the substitution in polynomial | 1 |
| Replaces the polynomial with | 1 |
| Solves =1 in terms of | 1 |
| Gives all five solutions in terms of *x* | 1 |
| **Subtotal** | **/5** |

Question 6 (7 marks)

The complex numbers and are shown on the Argand diagram below.

1. On the Argand diagram, draw and clearly label the position of (4 marks)
2. (ii) (iii)

|  |  |
| --- | --- |
| **Solution** | |
|  | |
| **Behaviours** | **Marks** |
| Shows the location of on the Argand diagram | 1 |
| Shows the location of on the Imaginary axis | 1 |
| Locates with correct modulus | 1 |
| Shows the location of on the Argand diagram | 1 |
| **Subtotal** | **/4** |

1. On the diagram below, sketch the set of complex numbers such that .  
    (3 marks)

|  |  |
| --- | --- |
| **Solution** | |
|  | |
| **Behaviours** | **Marks** |
| Draws the perpendicular bisector of the line joining the and  Draws a dotted line  Shades the diagram correctly | 1  1  1 |
| **Subtotal** | **/3** |

**Question 7 (10 marks)**

1. Express in simplified polar form. (4 marks)

|  |  |
| --- | --- |
| **Solution** | |
|  | |
| **Behaviours** | **Marks** |
| Rewrites the expression with and in polar form  Simplifies both numerator and denominator correctly  Writes the correct final term | 1  2  1 |
| **Subtotal** | **/4** |

1. Show and hence prove . (6 marks)

|  |  |
| --- | --- |
| **Solution** | |
| (1)  (2)  (1) (2) | |
| **Behaviours** | **Marks** |
| Rewrites and using double angle form | 1 |
| Gathers terms and simplifies | 1 |
| Rewrites and using double angle form | 1 |
| Gathers terms and simplifies | 1 |
| Equates both equations | 1 |
| Writes correct final expression | 1 |
| **Subtotal** | **/6** |

Sample assessment task

Mathematics Specialist – ATAR Year 12

Task 6 – Unit 4

**Assessment type:** Investigation

**Conditions:** The investigation will be completed over one week. Students will be encouraged to work independently to complete the task and may use any appropriate technology.

Note: while the Authority provides sample assessment tasks for guidance, it is the expectation of the Authority that teachers will develop tasks customised to reflect their school’s context and the needs of the student cohort. This resource is available on a public website and use of the resource without modification may affect the integrity of the assessment.

**Task weighting:** 12% of the school mark for this pair of units

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Centrepiece (40 marks)

Your task is to design a mould for the production of a unique shaped candle that will be used as the centre piece on each table at a graduation dinner.

The curve(s) defining the shape for the candle must pass through the points (2, 2) and (3, 5) and the mould must hold 1L of molten wax.

Write a report to present your design to the graduation committee. As you write your report, take care to clearly identify the underlying mathematics used throughout the process.

Your report should include the following:

* An **introduction**, that clearly defines the purpose of the task, identifies key information, any assumptions made and an outline of your strategy (10 marks)
* **Evidence of the application of mathematical model and strategies**, including calculations and results using appropriate representations (graphs, tables, formulae etc.) (10 marks)
* Your final design communicated in a systematic and concise manner, including **analysis and interpretation** in the context of the problem and consideration of the reasonableness and limitations of the results. (15 marks)
* Use of correct mathematical conventions, symbols and terminology. (5 marks)

The format of the report may be written or digital.

Note: the creativity, complexity and quality of your solution, along with attention to mathematical detail will be assessed in this task.

Marking key for sample assessment task 6 – Unit 4

This marking key may be adjusted based on the conditions of the task

**Introduction**

|  |  |
| --- | --- |
| **Behaviours** | **Marks** |
| Succinctly writes a general introduction that accurately summarises all aspects of the investigation | 1–2 |
| Identifies and documents the need to determine volumes of revolution | 1 |
| States a suitable scale to represent the parameters provided | 1 |
| Describes or sketches a proposed design | 1 |
| Produces a proposed design that:   * uses curves * includes more than one function * is rotationally symmetrical. | 3 |
| Identifies assumptions made, e.g. rotation can be about either axis **or** mould will be filled to be level with the top of the container **or** any function or relation can be used to model the container **or** the mould design needs to be rotationally symmetrical | 2 |
| **Subtotal** | **/10** |

**Application of the mathematical model and strategy**

|  |  |
| --- | --- |
| **Behaviours** | **Marks** |
| Clearly identifies the axis of rotation used | 1 |
| Selects more than one type of function to model the proposed curve | 1 |
| Includes a function other than a polynomial | 1 |
| Accurately determines points of intersection as boundaries of each function | 2 |
| Demonstrates accurate use of integration to determine the volume of revolution for each curved surface | 2 |
| Determines total volume of proposed shape | 1 |
| Shows refinement of solution to meet volume requirements and justifies any rounding of the solution | 2 |
| **Subtotal** | **/10** |

**Analysis and interpretation**

|  |  |
| --- | --- |
| **Behaviours** | **Marks** |
| Presents an accurate graph of the final function/s | 1–3 |
| Clearly labels all key points on the graph (intersections, end points) | 1–3 |
| Presents an accurate sketch of the final product formed by the rotation about the axis | 1–2 |
| Clearly demonstrates that the proposed model meets the requirements for volume | 1–2 |
| Clearly demonstrates that the proposed model passes through the prescribed points | 1–2 |
| Discusses the reasonableness of the model within the given context and with reference to assumptions made | 1–3 |
| **Subtotal** | **/15** |

**Use of mathematical conventions, symbols and terminology**

|  |  |
| --- | --- |
| **Behaviours** | **Marks** |
| Graphs are correctly labelled and displayed appropriately (sometimes = 1 mark, consistently = 2 marks) | 1–2 |
| Uses mathematical language throughout the investigation (sometimes = 1 mark, consistently = 2 marks) | 1–2 |
| Presents investigation in a systematic and concise way | 1 |
| **Subtotal** | **/5** |
| **Total** | **/40** |